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ABOUT THE DESIGN OF PRESTRESSED FRAMES WITH THE INFINITELY RIGID BEAM, SUBJECTED TO HORIZONTAL FORCES

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Abstract: A design method of frames with the infinitely rigid beam and prestressed columns by high strength tie bars, is presented. After a brief reminder of the equivalent section method the calculation formulas of displacement stiffness for the basic systems are established. The conclusion is reached that the displacement stiffness of these bars can be determined by the formula $\overline{k_p} = \overline{k} \cdot \overline{\rho}$ where $\overline{k}$ is the stiffness known for unprestressed bars (without tie bars) and $\overline{\rho}$ a correction factor, which depends on the prestressing degree and type of basic system. The other parameters which appear in the calculation relationship of the factor $\overline{\rho}$, have a very small importance. The relationships corresponding to these correction factors of the displacement stiffness of the prestressed columns are given.

Key words: building-in moment, equivalence coefficient, prestressing, tie-bar, stiffness, stress.

1,GENERALITIES. CALCULATION HYPOTHESES

The columns of the frames with an infinite bending stiffness of beams, subjected to important horizontal forces, can be conceived as prestressed elements by the high-strength tie-bars (Fig. 1).

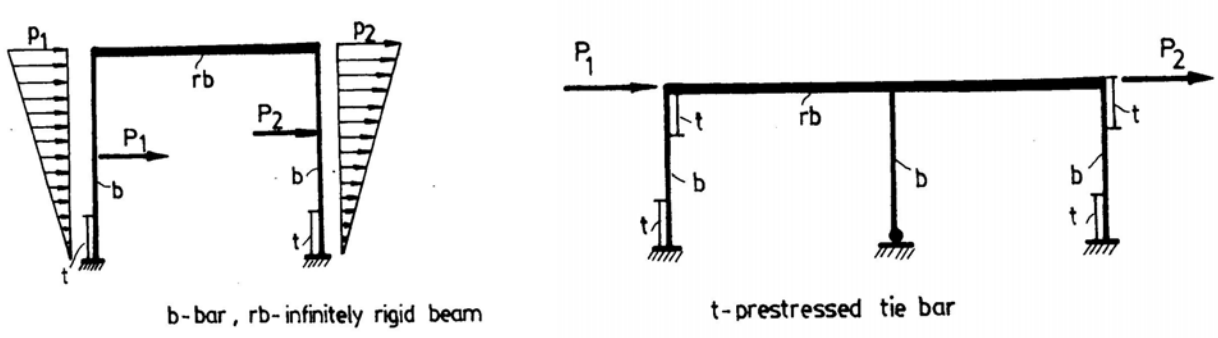


Fig. 1 – Examples of Prestressed Frames with the Infinitely Rigid Beam

It is convenient to make the design of these structures by taking advantage of the equivalent section method. As shown in 2, this design method is based on the following principle: a prestressed section (Fig. 2a.) can be replaced in the calculation,– from the point of view of strength condition – by unprestressed section capable of the same bending moment (Fig. 2b). Between these two sections there are the following dependence relationships

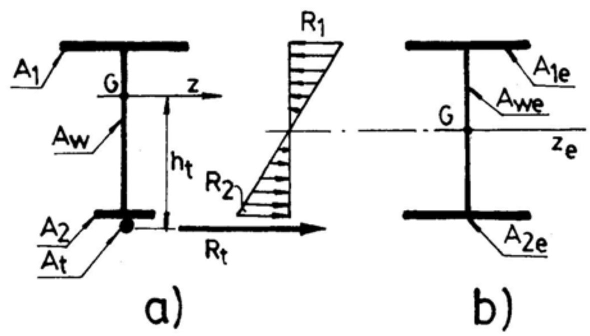


Fig. 2 a). Prestressed Section; b). Equivalent Section

$$A_{1e} = A_1 \qquad A_{we} = A_w \qquad A_{2e} = A_2 + A_{te} \quad \text{where} \qquad \text{(a)}$$

$$A_{te} = \alpha A_t \qquad \alpha = \frac{R_t}{A_2} \qquad \text{(b), (c)}$$

So, the unprestressed equivalent section must be dimensioned at the total bending moment from the calculation section (overtaken by the bar+ tie bar assembly):

$$M = M_b + x \cdot h_t \qquad \text{(d)}$$

According to this method, the design of the section of the prestressed structures is made with the following calculation stages:

1.The total bending moment M is determined.

2.The necessary equivalent section is dimensioned using the calculation know to unprestressed structures (using the relationships of optimal dimensioning of unprestressed section).

3.According to the quality of the material in the bar and tie-bar, the equivalence coefficient is calculated (rel. (c)). By choosing the ratio A_{te}/A_{2e} (usually $0.8 \div 0.9$) the equivalent area of the tie-bar, A_{te}, and then its area A_t (rel. (b)) are determined.

4.The tensioned area of the equivalent section is reduced with A_{te} and there results the prestressed section which has:

$$A_1 = A_{1e}\ , \qquad A_w = A_{we}\ , \qquad A_2 = A_{2e} - A_{te} \qquad \text{(a')}$$

The section and tie bar thus obtained exactly satisfy the strength condition under the service forces. As it follows, only the checking of the structure at the prestressing of tie-bar is necessary.

5.The total effort in the tie-bar, X, results from:

- the given effort at its prestressing, X_0, and -the effort produced by the service forces, X_1

$$X = X_0 + X_1 \qquad \text{(e)}$$

X_1 is the effort of the structure subjected to the service forces, having the tie-bars assembled but unprestressed.

6.Out of the relationships (e) we get

$$X_0 = X - X_1 \qquad \text{(f)}$$

which must check the structure.

For determining the total bending moment of the structure, necessary in the first stage of calculation:

- the moment of perfect building-in, and - the displacement stiffness of the basic systems must be known.

The first problem is solved in **1**, **3** where it is shown that the moment of perfect building-in, in a such system is given by the relation:

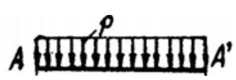
$$\overline{M_A} = c_{MA} \cdot M_A \qquad \overline{M_{A'}} = C_{MA'} \cdot M_{A'}\ , \qquad \text{where (g), (g')}$$

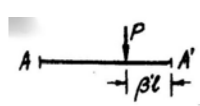
M_A, $M_{A'}$ are the moment of perfect building-in of the and A, respectively A' of the bar without the tie bar

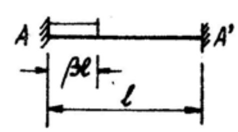
c_{MA}, $c_{MA'}$ correction coefficients – which can be tabulated according to the prestressing degree of the section and the type of basic system-given by the relationships:

$$c_{MA} = \frac{1}{1-\beta(4-3\beta)K_M} \qquad \text{(h)}$$

$$c_{MA'} = \eta - c_{MA} \cdot \beta(2-3\beta)K_M \qquad \text{(i)}$$

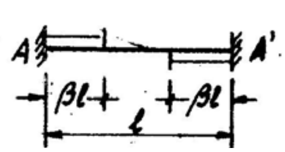
$\eta = 1$ for the loading case 

$\eta = \dfrac{1-\beta'}{\beta'}$ for the loading case 

for basic system 

$$c_{MA} = \frac{1}{1-2\beta K_M} \qquad \text{(h')}$$

for both previous loading cases (β'=0.5)

for basic system 

As it follows, the second problem is analyzed for the basic system of these structures:

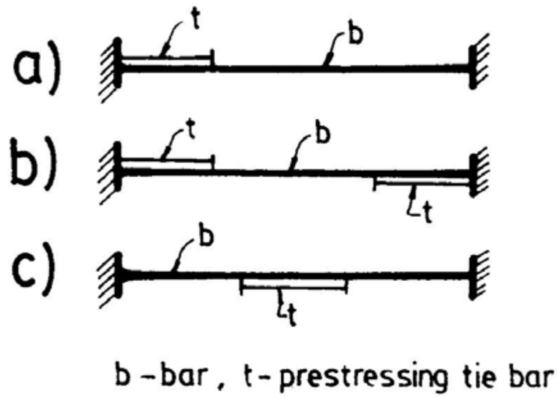


Fig. 3 – Basic System;

a). Prestressed Bar at One of Ends b). Prestressed Bar at Both Ends c). Prestressed Bar in the Span

The following calculation hypotheses are admitted:

a). for the structure

- the beam of the frame has an infinite stiffness;
- the columns are built-in at its both ends;
- the service forces are horizontal and may be distributed by any law;

b). for the basic system:

- the bar is straight and has a constant section along;
- the prestressing tie-bar is rectilinear and parallel to the longitudinal axis;
- the material of the bar and tie bar obeys the hypotheses of linear – elastic calculation.

2. THE ESTABLISHING OF CALCULATION RELATIONSHIPS

Bar with tie bar placed at one of its ends

Considering the bar with assembled, but unprestressed tie bar, we apply a unitary displacement between its both ends (Fig. 4a).From the compatibility condition of displacements, expressed on the system in fig. 4b.

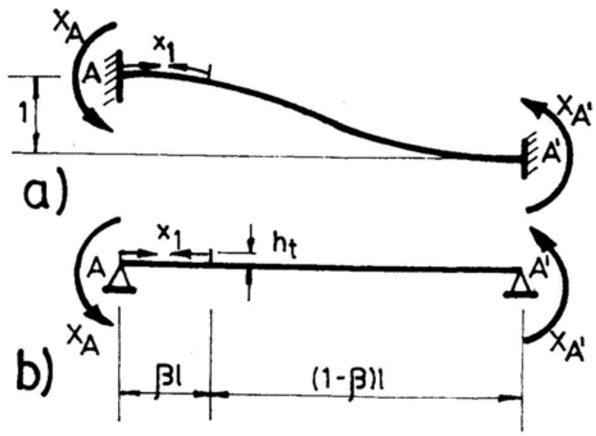


Fig. 4 – Prestressed Bar at One of Its Ends

a). Deformed Axis b). Calculation System

$$[\delta]\{X\}=\{\Delta_0\}$$ where (1)

$$[\delta]=\frac{l}{EI}\begin{bmatrix} \frac{1}{3} & -\frac{1}{6} & -\frac{2-\beta}{2}\beta h_t \\ -\frac{1}{6} & \frac{1}{3} & \frac{\beta^2}{2}h_t \\ -\frac{2-\beta}{2}\beta h_t & \frac{\beta^2}{2}h_t & \beta(h_t^2+\frac{I}{A}) \end{bmatrix},\ \{X\}=\begin{Bmatrix} X_A \\ X_{A'} \\ X_1 \end{Bmatrix},\ \{\Delta_0\}=\begin{Bmatrix} \frac{1}{l} \\ \frac{1}{l} \\ \overline{\Delta_t} \end{Bmatrix} \quad (2)$$

$$\overline{\Delta_t}=-\frac{X_1\beta l}{E_tA_t}=-\gamma\frac{\beta l X_1}{EI} \qquad \gamma=\frac{EI}{E_tA_t}\frac{\alpha EI}{E_tA_{te}}$$ we get: (3)

$$X_A=\frac{6EI}{l^2}\frac{h_t^2(1-\beta^2)+\frac{I}{A}+\gamma}{h_t^2\left[1-\beta\left(4-6\beta+3\beta^2\right)\right]+\frac{I}{A}+\gamma} \qquad X_{A'}=\frac{6EI}{l^2}\frac{h_t^2\left[1-\beta(2-\beta)\right]+\frac{I}{A}+\gamma}{h_t^2\left[1-\beta\left(4-6\beta+3\beta^2\right)\right]+\frac{I}{A}+\gamma} \quad (4), (5)$$

$$X_1=\frac{6EI}{l^2}\frac{h_t(1-\beta)}{h_t^2\left[1-\beta\left(4-6\beta+3\beta^2\right)\right]+\frac{I}{A}+\gamma} \quad (6)$$

Noting:

$\overline{k_{\rho A1}}$ - the displacement stiffness of the extremity A (with tie bar)

$\overline{k_{\rho A'1}}$ - the displacement stiffness of the extremity A' (without tie bar) and having in view that $6EI/l^2$ presents the displacement stiffness of the unprestressed bar (without tie bar), $\bar{k}$, from (4) (5) relationships, it results:

$$\overline{k_{PA1}} = \bar{k} \cdot \overline{\rho_{A1}} \qquad \overline{k_{PA'1}} = \bar{k} \cdot \overline{\rho_{A'1}} \tag{7),(8}$$

$\overline{\rho_{A1}}$ and $\overline{\rho_{A'1}}$ being the correction factors, whose expressions are:

$$\overline{\rho_{A1}} = \frac{h_t^2\left(1-\beta^2\right)+\frac{I}{A}+\gamma}{h_t^2\left[1-\beta\left(4-6\beta+3\beta^2\right)\right]+\frac{I}{A}+\gamma} \qquad \overline{\rho_{A'1}} = \frac{h_t^2\left[1-\beta(2-\beta)\right]+\frac{I}{A}+\gamma}{h_t^2\left[1-\beta\left(4-6\beta+3\beta^2\right)\right]+\frac{I}{A}+\gamma} \tag{9),(10}$$

<u>Bar with the tie bars at both ends</u> (Fig. 5)

In the same way to the precedent case, we get:

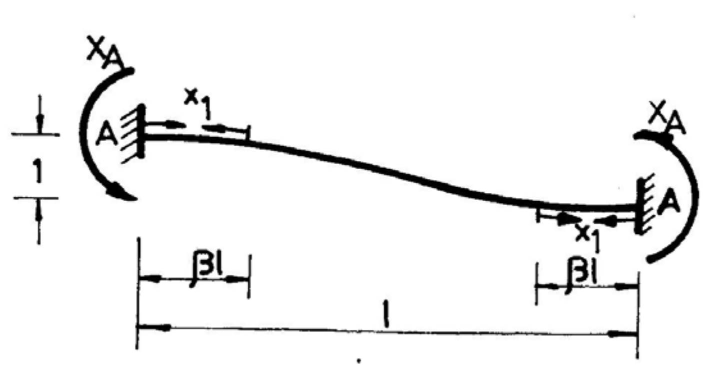


Fig. 5 – Deformed Axis of Prestressed Bar at Both Ends

$$\overline{k_{PA2}} = \overline{k_{PA'2}} = \bar{k}\overline{\rho_{A2}} \tag{7'}$$

the correction factor $\overline{\rho_{A2}}$ having the expressions:

$$\overline{\rho_{A2}} = \frac{h_t^2+\frac{I}{A}+\gamma}{h_t^2\left[1-6\beta(1-\beta)^2\right]+\frac{I}{A}+\gamma} \tag{9'}$$

3. DISCUSSION OF THE CORRECTION FACTORS OF STIFFNESS

Out of the definition relationships (9), (10), (9'), it follows that the correction factors $\overline{\rho_{ij}}$, i=A,A', j=1,2, depend on the degree of prestressing of the section and on the quality of the material in the bar and tie bar (I/A, γ) on the length of the bar (β) as well as the distance between the tie bar and the axis of the bar (h_t).

4. CONCLUSSIONS

Out of the results obtained the following may be concluded:

1.The displacement stiffness of the built-in prestressed bar is bigger then that of the unprestressed bar with the same section. The growth of stiffness increases with the prestressed degree, reaching up to 14% for the prestressed bar at both ends. For the bars prestressed in the span, the stiffness change is smaller, maximum 1%.

2.The lower the resistance of the prestressing tie-bar, the higher is the stiffness of the prestressed bar. If the resistance of the tie bar is between 1000-1500 MPa the modification of the correction factors is under 1 % .

3.The modification of factor β (and $\overline{\beta}$) produced by the change of the distribution law of cross loads has also a small influence on the displacement stiffness (max 4.5 %).

As a conclusion, if the prestressing is achieved by the high-strength tie-bars, placed on the face of the bar, the correction factors of stiffness practically depend only on the prestressing degree and on the type of the basic system. If the prestressing degree is below 0.5, the stiffness of the prestressed bar differs insignificantly from that of the unprestressed bar.

Notations: A – area of the bar section; A_t – area of the tie-bar section; A_{te} – equivalent area of the tie-bar section; EI – bending stiffness of the bar section; E_tA_t – stiffness of the tie bar; h_t - tie-bar eccentricity; $\overline{k}$ - displacement stiffness; M - total bending moment produced by the service forces on the structure; M_b - bending moment in the tie bar section; X - total effort in the tie-bar; X_A - building-in moment at the end A; $X_{A'}$ - building-in moment at the end A'; α - equivalence coefficient regarding the stresses; $\overline{\rho}$ - correction factors of unprestressed bar stiffness.

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