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The analysis of frame structures with non-prismatic beams

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Summary

This paper studies the static analysis of frame structures with non-prismatic beam elements. The flexibility coefficients of the non-prismatic elements are obtained by numerical integration of the equation of deflection of beams in bending The paper studies the possibility of approximation of the behavior of the non-prismatic beams by subdividing them in a minimal number of prismatic elements.

The results of this study are very useful in the practice, when special analysis software for non-prismatic beams is not available.

KEYWORDS: non-prismatic beam elements, frame structures.

1. INTRODUCTION

For a beam loaded only at the two ends, the end rotations can be computed with the help of the flexibility coefficients [1], [2]:

$$\varphi_1 = \frac{l}{EI_0}(M_1 c_1 + M_2 c_3),$$
$$\varphi_2 = \frac{l}{EI_0}(M_1 c_3 + M_2 c_2). \qquad (1)$$

The flexibility coefficients computed for unit end moments (figure 1.) are:

$$c_1 = \frac{I_0}{l}\int_0^l \frac{m_1^2}{I}dx; \quad c_2 = \frac{I_0}{l}\int_0^l \frac{m_2^2}{I}dx; \quad c_3 = \frac{I_0}{l}\int_0^l \frac{m_1 m_2}{I}dx; \qquad (2)$$

2. THE EQUIVALENT BEAM

The idea is to replace the given variation *I(x)* of the moment of inertia with two constant values I_1 and I_2 as it is shown in figure 2. In order to find these unknown values together with the non-dimensional length λ, the following notations are used:

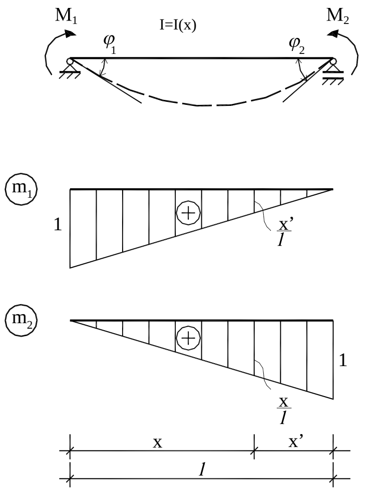

Figure 1. The deflection of the beam

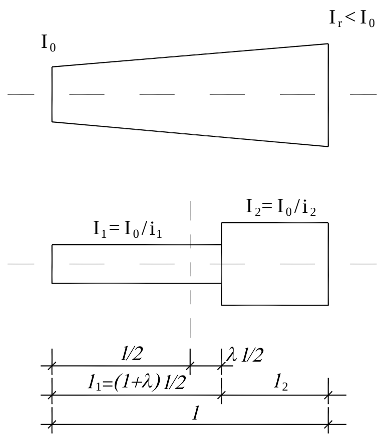

Figure 2. The equivalent beam

$$r_1 = \frac{I_0}{2}\int_{-1}^{1}\frac{\overline{m}_1^2}{I}d\xi = c_1 + c_2 + 2c_3;$$

$$r_2 = \frac{3I_0}{2}\int_{-1}^{1}\frac{\overline{m}_2^2}{I}d\xi = 3(c_1 + c_2 - 2c_3); \tag{3}$$

$$r_3 = I_0\int_{-1}^{1}\frac{\overline{m}_1\overline{m}_2}{I}d\xi = 2(c_2 - c_1).$$

The bending moment diagrams $\overline{m}_1$ and $\overline{m}_2$ are given in figure 3.

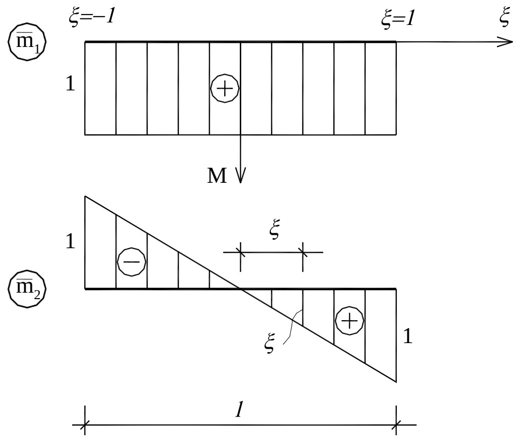

Figure 3. Bending moment diagrams $\overline{m}_1$ and $\overline{m}_2$

Introducing the step variation of the moment of inertia in relation (3) results a system of three equations:

$$\begin{cases}(1+\lambda)i_1+(1-\lambda)i_2=2r_1\\(1+\lambda^3)i_1+(1-\lambda^3)i_2=2r_2\\(-1+\lambda^2)i_1+(1-\lambda^2)i_2=2r_3\end{cases} \tag{4}$$

Here $i_1=I_0/I_1$ and $i_2=I_0/I_2$.
The solution of the system (4) is:

$$\lambda=\frac{r_2-r_1}{r_3};\qquad i_1=r_1-\frac{r_3}{\lambda+1};\qquad i_2=r_1-\frac{r_3}{\lambda-1}. \tag{5}$$

3. NUMERICAL EXAMPLE

In the figure 4 is presented a beam with I cross section. The height of the beam has a linear variation. The equivalent beam gives exact results if the beam is loaded only at its ends.

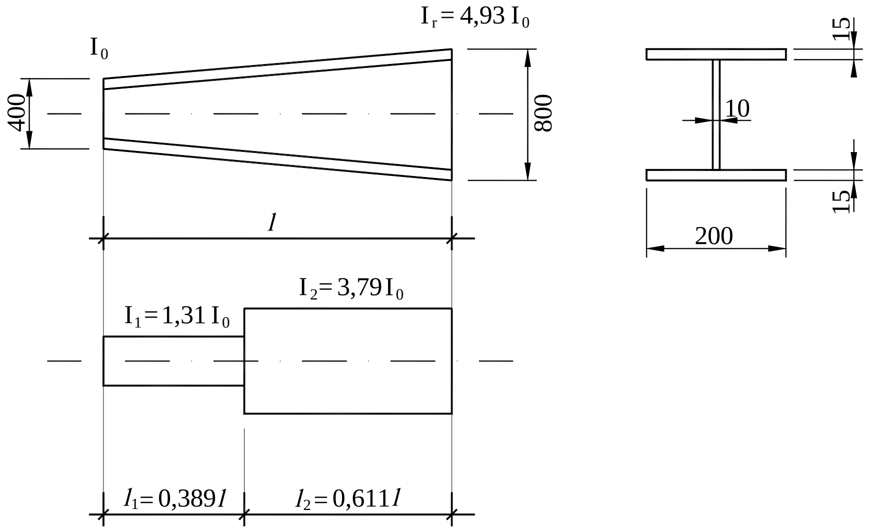

Figure 4. Numerical example

4. CONCLUSIONS

This paper introduces a very efficient way to treat the structures with non-prismatic beams. The given relations are based on the usual flexibility coefficients available in the literature or easy computable by numerical integration of the equation of deflection of the beams and permit to create simple structural models even when special software for non-prismatic beams is not available.

References

1. Avram, C.N. *Grinzi continue*, Editura tehnică, Bucureşti, 1981.
2. Caracostea, A.D., ş.a. *Manual pentru calculul construcţiilor*, Editura tehnică, Bucureşti, 1977, pag. 646-681.
3. Dósa, A. *Proiectare asistată de calculator*, îndrumător de lucrări, Reprografia Universităţii Transilvania din Braşov, 1999.