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MECHANICAL MODELS FOR THE VIRTUAL ANALYSIS OF THE MECHANICAL SYSTEMS – Part I

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Abstract: *This paper shows mechanic models for the virtual analysis of the mechanical systems but also computed the mechanical characteristic values of an elastic composite material using the concentration of component phases of the material. This kind of calculus is no so accurate but for a large class of problems can be enough*
Keywords*: Mechanical models, Virtual analysis, Composite material*

1. INTRODUCTION

A composite material made by two phases will be considered: one of phases with great elasticity call matrix and another phase, with a great strength, with the role to reinforce the material, call fiber. The fibers are considered cylindrical and parallel oriented along an axis denoted Ox_1. The properties of such material will be computed considering only the properties of the two phases and the ratios of the two components. The hypothesis of linear elasticity is used. From this material we will consider a small element, enough large to be considered homogeneous. [1]-[3]. We suppose that the horizontal plane surface of the element remain plane and parallel using liaison friction free. The resulting composite material will be transversally isotropic

2. METHOD

Consider now a specimen that contains so many fibers that can be considered homogeneous with an acceptable tolerance. If we consider the faces of the element plane and parallel, every section remain plane and the axial deformation is constant ε, the shear deformation $\bar{\gamma}_{12}$, $\bar{\gamma}_{31}$ are null and the other component of the deformation depend only on y and z.

$$\hat{v}_f(k_f e_f + l_f \varepsilon) + \hat{v}_m(k_m e_m + l_m \varepsilon) = k(\hat{v}_f e_f + \hat{v}_m e_m) + l\varepsilon$$
$$\hat{v}_f(l_f e_f + n_f \varepsilon) + \hat{v}_m(l_m e_m + n_m \varepsilon) = l(\hat{v}_f e_f + \hat{v}_m e_m) + n\varepsilon \tag{1}$$

where $e = \bar{\varepsilon}_y + \bar{\varepsilon}_z$ represents the transversal deformation. Moving all the term in the left hand Eqs. (1) can be written:

$$\hat{v}_f(k_f - k)e_f + \hat{v}_m(k_m - k)e_m + (\hat{v}_f l_f + \hat{v}_m l_m - l)\varepsilon = 0$$
$$\hat{v}_f(l_f - l)e_f + \hat{v}_m(l_m - l)e_m + (\hat{v}_f n_f + \hat{v}_m n_m - n)\varepsilon = 0 \tag{2}$$

The deformation e_f, e_m and ε are independents and it results the proportionality of the equation's coefficients:

$$\frac{k - k_f}{l - l_f} = \frac{k - k_m}{l - l_m} = \frac{l - \hat{v}_f l_f - \hat{v}_m l_m}{n - \hat{v}_f n_f - \hat{v}_m n_m} = \frac{k_f - k_m}{l_f - l_m} \tag{3}$$

Using the same method it can be written:

$$\frac{\frac{1}{k}-\frac{1}{k_f}}{\nu-\nu_f}=\frac{\frac{1}{k}-\frac{1}{k_m}}{\nu-\nu_m}=\frac{4(\nu-\hat{v}_f\nu_f-\hat{v}_m\nu_m)}{-(E-\hat{v}_fE_f-\hat{v}_mE_m)}=\frac{\frac{1}{k_f}-\frac{1}{k_m}}{\nu_f-\nu_m} \qquad (4)$$

or, in an equivalent form (the law of mixture in the harmonic average form):

$$(\nu-\hat{v}_f\nu_f-\hat{v}_m\nu_m)=\left(\frac{\nu_f-\nu_m}{\frac{1}{k_f}-\frac{1}{k_m}}\right)\left(\frac{1}{k}-\frac{\hat{v}_f}{k_f}-\frac{\hat{v}_m}{k_m}\right)=-\frac{1}{4}\left(\frac{\frac{1}{k_f}-\frac{1}{k_m}}{\nu_f-\nu_m}\right)\left(E-\hat{v}_fE_f-\hat{v}_mE_m\right) \qquad (5)$$

If we apply the principle of minimum potential energy it is easy to obtain the bounds obtained first by Voigt and Reuss:

$$k\le\hat{v}_fk_f+\hat{v}_mk_m=k_V \quad ; \quad \frac{1}{k}\le\frac{\hat{v}_f}{k_f}+\frac{\hat{v}_m}{k_m}=\frac{1}{k_R} \qquad (6)$$

The differences:

$$k_V-k_R=\frac{\hat{v}_f\hat{v}_m(k_f-k_m)^2}{\hat{v}_fk_m+\hat{v}_mk_f} \quad ; \quad \frac{1}{k_R}-\frac{1}{k_V}=\frac{\hat{v}_f\hat{v}_m\left(\frac{1}{k_f}-\frac{1}{k_m}\right)^2}{\frac{\hat{v}_f}{k_m}+\frac{\hat{v}_m}{k_f}} \qquad (7)$$

become small quantities of second order when the two phases have mechanical properties with appropriate values. It results good results when the two constituents of the composite have appropriate values and bad results when the mechanical properties are different.

Another way to demonstrate that the Young's modulus is greater that the modulus calculated with the mixture law will use the potential energy. For an axial traction after the Ox1 axis, if ε is the strain, the energy of a representative unit of volume (RVE) is:

$$W^*=\frac{1}{2}E\varepsilon^2 \qquad (8)$$

and is greater than the energy of the fiber and matrix having the same strain but without any liaison between them. This is:

$$W^t=\frac{1}{2}\left(\hat{v}_fE_f+\hat{v}_mE_m\right)\varepsilon^2 \qquad (9)$$

It results:

$$E-\hat{v}_fE_f-\hat{v}_mE_m=2(\nu_f-\nu_m)^2U\ge0 \qquad (10)$$

where the signification of the dislocation energy U is presented by Hill [2],[3],[4]. In the same way can be demonstrated the relations:

$$\frac{1}{k}-\frac{\hat{v}_f}{k_f}-\frac{\hat{v}_m}{k_m}=-\frac{1}{2}\left(\frac{1}{k_f}-\frac{1}{k_m}\right)^2U\le0 \qquad (11)$$

$$n-\hat{v}_fn_f-\hat{v}_mn_m=-2(l_f-l_m)^2V\le0 \qquad (12)$$

$$k-\hat{v}_fk_f-\hat{v}_mk_m=-2(k_f-k_m)^2V\le0 \qquad (13)$$

In the following the presented results will be used to analyze a composite material reinforced with cylindrical fibers. We consider a cylindrical fiber in the center of a cylindrical matrix loaded with a uniform lateral load P and with an axial stress T introduce by rigid constrains that impose to the basis plane of the cylinder to remain parallel. In this case from (4) we can obtain:

$$-P=ke+l\varepsilon \quad ; \quad T=le+n\varepsilon \qquad (14)$$

For such composite cylinder, with a great computation effort (Hill), it is possible to obtain the values of modulus k, l, n, E and the Poisson's ratio:

$$k = \frac{\hat{v}_f k_f (k_m + m_m) + \hat{v}_m k_m (k_f + m_m)}{\hat{v}_f (k_m + m_m) + \hat{v}_m (k_f + m_m)} \tag{15}$$

$$l = \frac{\hat{v}_f l_f (k_m + m_m) + \hat{v}_m l_m (k_f + m_m)}{\hat{v}_f (k_m + m_m) + \hat{v}_m (k_f + m_m)} \tag{16}$$

or in a form to express the fiber and matrix ratio:

$$\frac{1}{k + m_m} = \frac{\hat{v}_f}{k_f + m_m} + \frac{\hat{v}_m}{k_m + m_m} \tag{17}$$

$$\frac{1}{l + m_m} = \frac{\hat{v}_f}{l_f + m_m} + \frac{\hat{v}_m}{l_m + m_m} \tag{18}$$

and:

$$\nu = \nu_m + \frac{(\nu_f - \nu_m)\hat{v}_f \left(\frac{1}{k_m} + \frac{1}{m_m} \right)}{\left(\frac{\hat{v}_f}{k_m} + \frac{\hat{v}_m}{k_f} + \frac{1}{m_m} \right)} \tag{19}$$

$$n = \hat{v}_f n_f + \hat{v}_m n_m - \frac{\hat{v}_f \hat{v}_m (l_f - l_m)^2}{(\hat{v}_f k_m + \hat{v}_m k_f + m_m)} \tag{20}$$

$$E = \hat{v}_f E_f + \hat{v}_m E_m + \frac{4\hat{v}_f \hat{v}_m (\nu_f - \nu_m)^2}{(\frac{\hat{v}_f}{k_m} + \frac{\hat{v}_m}{k_f} + \frac{1}{m_m})} \tag{21}$$

The presented relations are exacts and present the values of the mechanical constants for a composite cylinder made by a cylindrical matrix around a cylindrical fiber.

In the following will be calculated the bulk modulus, the Young's modulus and the Poisson's ratio for three representative cases.

In the first case (study case 1) we have considered the composite made by an epoxy resin with the Young's modulus 2,7 MPa and the Poisson's ratio 0,35 and the fiber with the Young's modulus 72,4 and the Poisson's ratio equal with 0,22.

In the second case (study case 2) the matrix is considered with the Young's modulus 0.27 MPa and the Poisson's ratio 0,35 and the fiber made by the same material as in the study case 1.

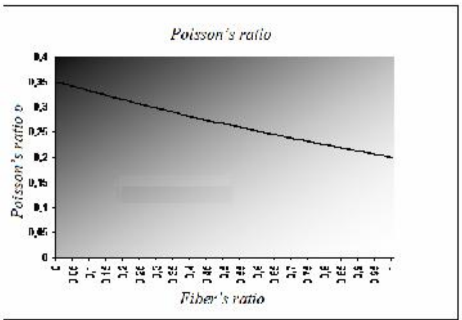

Figure 1. Poisson's ratio ν

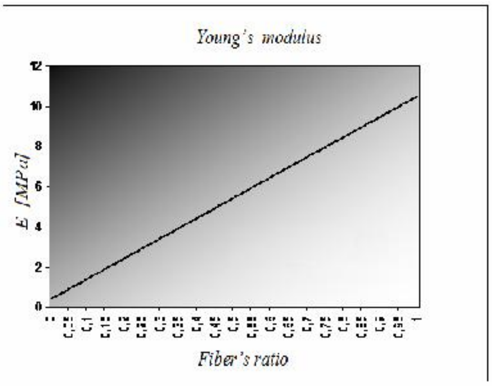

Figure 2. Young's modulus E

3. DISCUSSION

The theory conduces finally to a numerical approach and, in every case must be appreciate the best way to calculate the mechanical constants for a composite. The above example demonstrates that the method just presented can be applied analytically in a rather simple way, to a one-dimensional case. For bi-dimensional problems however, or for more complicated cases the analytical approach becomes a difficult task and therefore, the problem has to be analyzed numerically.

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