



MATHEMATICAL MODELS FOR THE VIRTUAL ANALYSIS OF THE MECHANICAL SYSTEMS – Part II

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Abstract: The paper presents a method for the calculus of the motion of an mechanical system. For a system with five elements are obtained the relations for the calculus of the velocities and accelerations. The body free diagram is used to obtain the mbasket-ball player.

Keywords: mechanical system, mathematical model, virtual analysis

1. INTRODUCTION

The paper presents a mechanical model for the study of the motion of a basket-ball player [1],[2],[3]. The model use the free body diagram of the system in order to obtain the motion equations for the player [4],[5],[6].

2. KINEMATICS AND CONSTRAINS

The positions of the mass center of the elements are:

$$x_{C1} = \alpha_1 l_1 c_1 ;$$

$$y_{C1} = \alpha_1 l_1 s_1 ;$$

$$x_{C2} = l_1 c_1 + \alpha_2 l_2 c_2 ;$$

$$y_{C2} = l_1 s_1 + \alpha_2 l_2 s_2 ;$$

$$x_{C3} = l_1 c_1 + l_2 c_2 + \alpha_3 l_3 c_3 ;$$

$$y_{C3} = l_1 s_1 + l_2 s_2 + \alpha_3 l_3 s_3 ;$$

$$x_{C4} = l_1 c_1 + l_2 c_2 + l_3 c_3 + \alpha_4 l_4 c_4 ;$$

$$y_{C4} = l_1 s_1 + l_2 s_2 + l_3 s_3 + \alpha_4 l_4 s_4 ;$$

$$x_{C5} = l_1 c_1 + l_2 c_2 + l_3 c_3 + l_4 c_4 + \alpha_5 l_5 c_5 ;$$

$$y_{C5} = l_1 s_1 + l_2 s_2 + l_3 s_3 + l_4 s_4 + \alpha_5 l_5 s_5 ;$$

$$x_6 = l_1 c_1 + l_2 c_2 + l_3 c_3 + l_4 c_4 + l_5 c_5 ;$$

$$y_6 = l_1 s_1 + l_2 s_2 + l_3 s_3 + l_4 s_4 + l_5 s_5 ;$$



Figure 1. Mechanical model

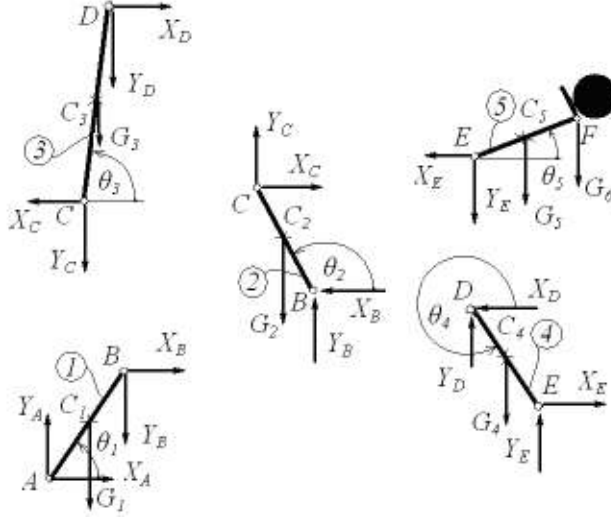


Figure 2. Free body diagram

$$\begin{aligned}
 \dot{x}_{C1} &= -\alpha_1 l_1 s_1 \omega_1 ; \\
 \dot{y}_{C1} &= \alpha_1 l_1 c_1 \omega_1 ; \\
 \dot{x}_{C2} &= -l_1 s_1 \omega_1 - \alpha_2 l_2 s_2 \omega_2 ; \\
 \dot{y}_{C2} &= l_1 c_1 \omega_1 + \alpha_2 l_2 c_2 \omega_2 ; \\
 \dot{x}_{C3} &= -l_1 s_1 \omega_1 - l_2 s_2 \omega_2 - \alpha_3 l_3 s_3 \omega_3 ; \\
 \dot{y}_{C3} &= l_1 c_1 \omega_1 + l_2 c_2 \omega_2 + \alpha_3 l_3 c_3 \omega_3 ; \\
 \dot{x}_{C4} &= -l_1 s_1 \omega_1 - l_2 s_2 \omega_2 - l_3 s_3 \omega_3 - \alpha_4 l_4 s_4 \omega_4 ; \\
 \dot{y}_{C4} &= l_1 c_1 \omega_1 + l_2 c_2 \omega_2 + l_3 c_3 \omega_3 + \alpha_4 l_4 c_4 \omega_4 ; \\
 \dot{x}_{C5} &= -l_1 s_1 \omega_1 - l_2 s_2 \omega_2 - l_3 s_3 \omega_3 - l_4 s_4 \omega_4 - \alpha_5 l_5 s_5 \omega_5 ; \\
 \dot{y}_{C5} &= l_1 c_1 \omega_1 + l_2 c_2 \omega_2 + l_3 c_3 \omega_3 + l_4 c_4 \omega_4 + \alpha_5 l_5 c_5 \omega_5 ; \\
 \dot{x}_6 &= -l_1 s_1 \omega_1 - l_2 s_2 \omega_2 - l_3 s_3 \omega_3 - l_4 s_4 \omega_4 - l_5 s_5 \omega_5 ; \\
 \dot{y}_6 &= l_1 c_1 \omega_1 + l_2 c_2 \omega_2 + l_3 c_3 \omega_3 + l_4 c_4 \omega_4 + l_5 c_5 \omega_5 ; \\
 \ddot{x}_{C1} &= -\alpha_1 l_1 s_1 \varepsilon_1 - \alpha_1 l_1 c_1 \omega_1^2 ; \\
 \ddot{y}_{C1} &= \alpha_1 l_1 c_1 \varepsilon_1 - \alpha_1 l_1 s_1 \omega_1^2 ; \\
 \ddot{x}_{C2} &= -l_1 s_1 \varepsilon_1 - l_1 c_1 \omega_1^2 - \alpha_2 l_2 s_2 \varepsilon_2 - \alpha_2 l_2 c_2 \omega_2^2 ; \\
 \ddot{y}_{C2} &= l_1 c_1 \varepsilon_1 - l_1 s_1 \omega_1^2 + \alpha_2 l_2 c_2 \varepsilon_2 - \alpha_2 l_2 s_2 \omega_2^2 ; \\
 \ddot{x}_{C3} &= -l_1 s_1 \varepsilon_1 - l_1 c_1 \omega_1^2 - l_2 s_2 \varepsilon_2 - l_2 c_2 \omega_2^2 - \alpha_3 l_3 s_3 \varepsilon_3 - \alpha_3 l_3 c_3 \omega_3^2 ; \\
 \ddot{y}_{C3} &= l_1 c_1 \varepsilon_1 - l_1 s_1 \omega_1^2 + l_2 c_2 \varepsilon_2 - l_2 s_2 \omega_2^2 + \alpha_3 l_3 c_3 \varepsilon_3 - \alpha_3 l_3 s_3 \omega_3^2 ; \\
 \ddot{x}_{C4} &= -l_1 s_1 \varepsilon_1 - l_1 c_1 \omega_1^2 - l_2 s_2 \varepsilon_2 - l_2 c_2 \omega_2^2 - l_3 s_3 \varepsilon_3 - l_3 s_3 \omega_3^2 - \alpha_4 l_4 s_4 \varepsilon_4 - \alpha_4 l_4 c_4 \omega_4^2 ; \\
 \ddot{y}_{C4} &= l_1 c_1 \varepsilon_1 - l_1 s_1 \omega_1^2 + l_2 c_2 \varepsilon_2 - l_2 s_2 \omega_2^2 + l_3 c_3 \varepsilon_3 - l_3 s_3 \omega_3^2 + \alpha_4 l_4 c_4 \varepsilon_4 - \alpha_4 l_4 s_4 \omega_4^2 ; \\
 \ddot{x}_{C5} &= -l_1 s_1 \varepsilon_1 - l_1 c_1 \omega_1^2 - l_2 s_2 \varepsilon_2 - l_2 c_2 \omega_2^2 - l_3 s_3 \varepsilon_3 - l_3 c_3 \omega_3^2 - l_4 s_4 \varepsilon_4 - l_4 c_4 \omega_4^2 - \alpha_5 l_5 s_5 \varepsilon_5 - \alpha_5 l_5 c_5 \omega_5^2 ; \\
 \ddot{y}_{C5} &= l_1 c_1 \varepsilon_1 - l_1 s_1 \omega_1^2 + l_2 c_2 \varepsilon_2 - l_2 s_2 \omega_2^2 + l_3 c_3 \varepsilon_3 - l_3 s_3 \omega_3^2 + l_4 c_4 \varepsilon_4 - l_4 s_4 \omega_4^2 + \alpha_5 l_5 c_5 \varepsilon_5 - \alpha_5 l_5 s_5 \omega_5^2 ;
 \end{aligned}$$

The previously obtained relation can be written as:

$$\begin{Bmatrix} \dot{x}_{c1} \\ \dot{y}_{c1} \\ \omega_1 \\ \dot{x}_{c2} \\ \dot{y}_{c2} \\ \omega_2 \\ \dot{x}_{c3} \\ \dot{y}_{c3} \\ \omega_3 \\ \dot{x}_{c4} \\ \dot{y}_{c4} \\ \omega_4 \\ \dot{x}_{c5} \\ \dot{y}_{c5} \\ \omega_5 \\ \dot{x}_6 \\ \dot{y}_6 \end{Bmatrix} = \begin{bmatrix} -\alpha_1 l_1 s_1 & 0 & 0 & 0 & 0 \\ \alpha_1 l_1 c_1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ -l_1 s_1 & -\alpha_2 l_2 s_2 & 0 & 0 & 0 \\ l_1 c_1 & \alpha_2 l_2 c_2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ -l_1 s_1 & -l_2 s_2 & -\alpha_3 l_3 s_3 & 0 & 0 \\ l_1 c_1 & l_2 c_2 & \alpha_3 l_3 c_3 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ -l_1 s_1 & -l_2 s_2 & -l_3 s_3 & -\alpha_4 l_4 s_4 & 0 \\ l_1 c_1 & l_2 c_2 & l_3 c_3 & \alpha_4 l_4 c_4 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ -l_1 s_1 & -l_2 s_2 & -l_3 s_3 & -l_4 s_4 & -\alpha_5 l_5 s_5 \\ l_1 c_1 & l_2 c_2 & l_3 c_3 & l_4 c_4 & \alpha_5 l_5 c_5 \\ 0 & 0 & 0 & 0 & 1 \\ -l_1 s_1 & -l_2 s_2 & -l_3 s_3 & -l_4 s_4 & -l_5 s_5 \\ l_1 c_1 & l_2 c_2 & l_3 c_3 & l_4 c_4 & l_5 c_5 \end{bmatrix} \begin{Bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \\ \omega_4 \\ \omega_5 \end{Bmatrix}$$

and:

$$\begin{Bmatrix} \ddot{x}_{c1} \\ \ddot{y}_{c1} \\ \varepsilon_1 \\ \ddot{x}_{c2} \\ \ddot{y}_{c2} \\ \varepsilon_2 \\ \ddot{x}_{c3} \\ \ddot{y}_{c3} \\ \varepsilon_3 \\ \ddot{x}_{c4} \\ \ddot{y}_{c4} \\ \varepsilon_4 \\ \ddot{x}_{c5} \\ \ddot{y}_{c5} \\ \varepsilon_5 \end{Bmatrix} = \begin{bmatrix} -\alpha_1 l_1 s_1 & 0 & 0 & 0 & 0 \\ \alpha_1 l_1 c_1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ -l_1 s_1 & -\alpha_2 l_2 s_2 & 0 & 0 & 0 \\ l_1 c_1 & \alpha_2 l_2 c_2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ -l_1 s_1 & -l_2 s_2 & -\alpha_3 l_3 s_3 & 0 & 0 \\ l_1 c_1 & l_2 c_2 & \alpha_3 l_3 c_3 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ -l_1 s_1 & -l_2 s_2 & -l_3 s_3 & -\alpha_4 l_4 s_4 & 0 \\ l_1 c_1 & l_2 c_2 & l_3 c_3 & \alpha_4 l_4 c_4 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ -l_1 s_1 & -l_2 s_2 & -l_3 s_3 & -l_4 s_4 & -\alpha_5 l_5 s_5 \\ l_1 c_1 & l_2 c_2 & l_3 c_3 & l_4 c_4 & \alpha_5 l_5 c_5 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \varepsilon_4 \\ \varepsilon_5 \end{Bmatrix} +$$

$$+ \begin{bmatrix} -\alpha_1 l_1 c_1 & 0 & 0 & 0 & 0 \\ -\alpha_1 l_1 s_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ -l_1 c_1 & -\alpha_2 l_2 c_2 & 0 & 0 & 0 \\ -l_1 s_1 & -\alpha_2 l_2 s_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ -l_1 c_1 & -l_2 c_2 & -\alpha_3 l_3 c_3 & 0 & 0 \\ -l_1 s_1 & -l_2 s_2 & -\alpha_3 l_3 s_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ -l_1 c_1 & -l_2 c_2 & -l_3 c_3 & -\alpha_4 l_4 c_4 & 0 \\ -l_1 s_1 & -l_2 s_2 & -l_3 s_3 & -\alpha_4 l_4 s_4 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ -l_1 c_1 & -l_2 c_2 & -l_3 c_3 & -l_4 c_4 & -\alpha_5 l_5 c_5 \\ -l_1 s_1 & -l_2 s_2 & -l_3 s_3 & -l_4 s_4 & -\alpha_5 l_5 s_5 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} \omega_1^2 \\ \omega_2^2 \\ \omega_3^2 \\ \omega_4^2 \\ \omega_5^2 \\ \omega_6^2 \end{Bmatrix}$$

$$\{a\} = [A_1]\{\varepsilon\} + [A_2]\{\omega^2\}.$$

$$\begin{aligned}
M'_{21} &= (\alpha_2 m_1 + 3)m_1 l_1 l_2 s_{2-1} \quad ; \quad M'_{22} = 0 \quad ; \quad M'_{23} = (\alpha_3 + 2)m_3 l_2 l_3 s_{2-3} \quad ; \\
M'_{24} &= (\alpha_4 + 1)m_4 l_2 l_4 s_{2-4} \quad ; \quad M'_{25} = \alpha_5 m_5 l_2 l_5 s_{2-5} \quad ; \\
M'_{31} &= l_1 l_3 (\alpha_3 m_3 + m_4 + m_5) s_{3-1} \quad ; \\
M'_{32} &= l_2 l_3 (\alpha_3 m_3 + m_4 + m_5) s_{3-2} \quad ; \quad M'_{33} = 0 \\
M'_{34} &= l_3 l_4 (\alpha_4 m_4 + m_5) s_{3-4} \quad ; \quad M'_{35} = \alpha_5 m_5 l_3 l_5 s_{3-5} \\
M'_{41} &= l_1 l_4 (\alpha_4 m_4 + m_5) s_{4-1} \quad ; \quad M'_{42} = l_2 l_4 (\alpha_4 m_4 + m_5) s_{4-2} \quad ; \\
M'_{43} &= l_3 l_4 (\alpha_4 m_4 + m_5) s_{4-3} \quad ; \quad M'_{44} = 0 \quad ; \quad M'_{45} = \alpha_5 m_5 l_4 l_5 s_{4-5} \\
M'_{51} &= l_1 l_5 (\alpha_5 m_5) s_{5-1} \quad ; \quad M'_{52} = l_2 l_5 (\alpha_5 m_5) s_{5-2} \quad ; \\
M'_{53} &= l_3 l_5 (\alpha_5 m_5) s_{5-3} \quad ; \quad M'_{55} = 0 \quad ; \quad M'_{54} = \alpha_5 m_5 l_4 l_5 s_{5-4}
\end{aligned}$$

$$[M'] = \begin{bmatrix} 0 & (\alpha_2 m_2 + m_3 + m_4 + m_5) l_1 l_2 s_{1-2} & (\alpha_3 m_3 + m_4 + m_5) l_1 l_3 s_{1-2} & (\alpha_4 m_4 + m_5) l_1 l_4 s_{1-4} & \alpha_5 m_5 l_1 l_5 s_{1-5} \\ (\alpha_2 m_2 + m_3 + m_4 + m_5) l_1 l_2 s_{2-1} & 0 & (\alpha_3 m_3 + m_4 + m_5) l_2 l_3 s_{2-3} & l_2 l_4 (\alpha_4 m_4 + m_5) s_{2-4} & \alpha_5 m_5 l_2 l_5 s_{2-5} \\ l_1 l_3 (\alpha_3 m_3 + m_4 + m_5) s_{3-1} & l_2 l_3 (\alpha_3 m_3 + m_4 + m_5) s_{3-2} & 0 & (\alpha_4 m_4 + m_5) l_3 l_4 s_{3-4} & \alpha_5 m_5 l_3 l_5 s_{3-5} \\ l_1 l_4 (\alpha_4 m_4 + m_5) s_{4-1} & l_2 l_4 (\alpha_4 m_4 + m_5) s_{4-2} & l_3 l_4 (\alpha_4 m_4 + m_5) s_{4-3} & 0 & \alpha_5 m_5 l_4 l_5 s_{4-5} \\ l_1 l_5 (\alpha_5 m_5) s_{5-1} & l_2 l_5 (\alpha_5 m_5) s_{5-2} & l_3 l_5 (\alpha_5 m_5) s_{5-3} & \alpha_5 m_5 l_4 l_5 s_{5-4} & 0 \end{bmatrix}$$

Matrix $[M']$ is skew-symmetric.

The loads vector is:

$$\{Q\} = \begin{Bmatrix} X_A + X_B \\ Y_A - Y_B - G_1 \\ M_1 + X_A \alpha_1 l_1 s_1 - X_B (1 - \alpha_1) l_1 s_1 - Y_A \alpha_1 l_1 c_1 - Y_B (1 - \alpha_1) l_1 c_1 \\ - X_B + X_C \\ Y_B + Y_C - G_2 \\ M_2 - X_B \alpha_2 l_2 s_2 - X_C (1 - \alpha_2) l_2 s_2 + Y_B \alpha_2 l_2 c_2 - Y_C (1 - \alpha_2) l_2 c_2 \\ - X_C + X_D \\ - Y_C - Y_D - G_3 \\ M_3 - X_C \alpha_3 l_3 s_3 - X_D (1 - \alpha_3) l_3 s_3 + Y_C \alpha_3 l_3 c_3 - Y_D (1 - \alpha_3) l_3 c_3 \\ X_E - X_D \\ Y_E + Y_D - G_4 \\ M_4 + X_D \alpha_4 l_4 s_4 + X_E (1 - \alpha_4) l_4 s_4 - Y_D \alpha_4 l_4 c_4 - Y_E (1 - \alpha_4) l_4 c_4 \\ - X_E \\ - Y_E - G_5 - G_6 \\ M_5 - X_E \alpha_5 l_5 s_5 + Y_E \alpha_5 l_5 c_5 \end{Bmatrix}$$

If we premultiply this vector with $[A_1]^T$ we obtain:

$$\{Q'\} = [A_1]^T \{Q\} = \begin{Bmatrix} M_1 \\ M_2 \\ M_3 \\ M_4 \\ M_5 \end{Bmatrix} - \begin{Bmatrix} l_1 c_1 (\alpha_1 G_1 + G_2 + G_3 + G_4 + G_5 + G_6) \\ l_2 c_2 (\alpha_2 G_2 + G_3 + G_4 + G_5 + G_6) \\ l_3 c_3 (\alpha_3 G_3 + G_4 + G_5 + G_6) \\ l_4 c_4 (\alpha_4 G_4 + G_5 + G_6) \\ l_5 c_5 \alpha_5 (G_5 + G_6) \end{Bmatrix} = \{M^{ext}\} - \{M_G\}$$

It results the motion equations:

$$[M] \{\varepsilon\} + [M'] \{\omega^2\} = [A_1]^T \{Q\}$$

or:

$$[M] \{\varepsilon\} + [M'] \{\omega^2\} = \{M^{ext}\} - \{M_G\}$$

The vector of the external moments become:

$$\{M^{ext}\} = [M]\{\varepsilon\} + [M']\{\omega^2\} + \{M_G\} .$$

To compute the normal force acting on the horizontal plane it is necessary to sum the equations 2, 5, 8, 11, 14. To do this we premultiply the system with:

$$\{YA\} = [0 \ 1 \ 0 \ 0 \ 1 \ 0 \ 0 \ 1 \ 0 \ 0 \ 1 \ 0 \ 0 \ 1 \ 0]$$

$$\{YA\}[m][A_1]\{\varepsilon\} + \{YA\}[m][A_2]\{\omega^2\} = \{YA\}\{Q\}$$

from where:

$$Y_A = G_1 + G_2 + G_3 + G_4 + G_5 + \{YA\}[m][A_1]\{\varepsilon\} + \{YA\}[m][A_2]\{\omega^2\} .$$

4. CONCLUSIONS

The measurement made with the Kistler table show a good agreement between the computed results and the experimental results.

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