STRESSES IN VARIOUS COMPOSITE LAMINATES FOR GENERAL SET OF APPLIED IN-PLANE LOADS

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Abstract: *This paper presents numerical simulations regarding the determination of stresses in all plies of various composite laminates such as anti-symmetric, symmetric cross-ply, symmetric angle-ply and balanced angle-ply laminates for general set of in-plane loads. The basic laminate theory has been used considering that in all plies, the fibers and matrix properties, fibers content and ply thickness are same. Following input data have been used: stack of plies, fibers axis of each ply at specified angle to reference direction, axial and transverse Young's moduli and shear modulus both for fibers and matrix, axial-transverse Poisson ratio for fibers and matrix. Output data include longitudinal, transverse and shear stresses in all plies at angles varying between 0° and 90° to reference direction.*
Keywords: *composite laminates, anti-symmetric, symmetric cross-ply, symmetric angle-ply, balanced angle-ply*

1. INTRODUCTION

Fibers-reinforced polymer matrix composite materials are heterogeneous and anisotropic materials so that their mechanics is more complex than that of conventional materials. The basic element of a composite laminate structure is the individual layer (called lamina) unidirectional reinforced with fibers inserted into a resin system (called matrix). Basic assumptions in the description of the interaction between fibers and matrix in a unidirectional reinforced lamina subject to tensile loads are [1]:

- Both fibers and matrix acts as a linear elastic material;
- Initially, the lamina presents no residual stresses;
- Loads are applied parallel or perpendicular to the fibers direction;
- The matrix presents no voids and failures;
- The bond between fibers and matrix is perfect;
- The fibers are uniformly distributed in the matrix.

For in-plane stress condition by superimposing three loadings $\sigma_{\parallel}$, $\sigma_{\perp}$ and $\tau_{\#}$, following law of elasticity can be given:

$$\begin{bmatrix} \varepsilon_{II} \\ \varepsilon_{\perp} \\ \gamma_{\#} \end{bmatrix} = \begin{bmatrix} \frac{1}{E_{II}} & -\frac{\upsilon_{II\perp}}{E_{\perp}} & 0 \\ -\frac{\upsilon_{\perp II}}{E_{II}} & \frac{1}{E_{\perp}} & 0 \\ 0 & 0 & \frac{1}{G_{\#}} \end{bmatrix} \cdot \begin{bmatrix} \sigma_{II} \\ \sigma_{\perp} \\ \tau_{\#} \end{bmatrix}, \tag{1}$$

where the second matrix is called the compliances matrix.

Transverse contraction coefficients $\upsilon_{\parallel\perp}$ and $\upsilon_{\perp\parallel}$ are not independent of each other. If we assume the existence of small deformations and a linear elastic behavior of the composite material than, between the coefficients of transverse contraction υ and the Young's moduli E, there is following relationship:

$$\frac{\upsilon_{\perp II}}{E_{II}} = \frac{\upsilon_{II\perp}}{E_{\perp}}. \tag{2}$$

Relation (2) is called the Maxwell-Betti law. Therefore, the unidirectional reinforced lamina can be described by four basic elasticity terms: $E_{\|}$, $E_{\perp}$, $\upsilon_{\perp\|}$ and $G_{\#}$. If desired, the expression (1) in terms of stresses versus strains can be written as following:

$$\begin{bmatrix} \sigma_{II} \\ \sigma_{\perp} \\ \tau_{\#} \end{bmatrix} = \begin{bmatrix} \frac{E_{II}}{1-\upsilon_{\perp II}\cdot\upsilon_{II\perp}} & \frac{\upsilon_{II\perp}\cdot E_{\perp}}{1-\upsilon_{\perp II}\cdot\upsilon_{II\perp}} & 0 \\ \frac{\upsilon_{\perp II}\cdot E_{II}}{1-\upsilon_{\perp II}\cdot\upsilon_{II\perp}} & \frac{E_{\perp}}{1-\upsilon_{\perp II}\cdot\upsilon_{II\perp}} & 0 \\ 0 & 0 & G_{\#} \end{bmatrix} \cdot \begin{bmatrix} \varepsilon_{II} \\ \varepsilon_{\perp} \\ \gamma_{\#} \end{bmatrix}, \tag{3}$$

where the second matrix is called the stiffness matrix.

Expressing the strains versus stresses lead to the advantage to compute the compliances as a function of lamina's basic elastic properties. These basic elastic properties are also called technical constants. These constants can be determined from the lamina's micromechanics using the fibers and matrix elastic properties [1]:

$$E_{II} = E_F \cdot \varphi + E_M \cdot (1-\varphi), \tag{4}$$

$$\upsilon_{\perp II} = \varphi \cdot \upsilon_F + (1-\varphi) \cdot \upsilon_M , \tag{5}$$

$$E_{\perp} = \frac{E_M}{1-\upsilon_M^2} \cdot \frac{1+0{,}85\cdot\varphi^2}{(1-\varphi)^{1,25} + \frac{\varphi \cdot E_M}{\left(1-\upsilon_M^2\right)\cdot E_F}}, \tag{6}$$

$$\upsilon_{II\perp} = \upsilon_{\perp II} \cdot \frac{E_{\perp}}{E_{II}}, \tag{7}$$

$$G_{\#} = G_M \cdot \frac{1+0{,}6\cdot\varphi^{0,5}}{(1-\varphi)^{1,25} + \varphi \cdot \frac{G_M}{G_F}}. \tag{8}$$

Regarding the lamina being in the stress plane state, its strains can be expressed versus stresses using the transformed components of the compliances matrix:

$$\begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{12} & c_{22} & c_{23} \\ c_{13} & c_{23} & c_{33} \end{bmatrix} \cdot \begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \end{bmatrix}. \tag{9}$$

These transformed compliances can be computed as following [1]:

$$c_{11} = \frac{\cos^4\alpha}{E_{II}} + \frac{\sin^4\alpha}{E_{\perp}} + \frac{1}{4}\cdot\left(\frac{1}{G_{\#}} - \frac{2\cdot\upsilon_{\perp II}}{E_{II}}\right)\cdot\sin^2 2\alpha, \tag{10}$$

$$c_{22} = \frac{\sin^4\alpha}{E_{II}} + \frac{\cos^4\alpha}{E_{\perp}} + \frac{1}{4}\cdot\left(\frac{1}{G_{\#}} - \frac{2\cdot\upsilon_{\perp II}}{E_{II}}\right)\cdot\sin^2 2\alpha, \tag{11}$$

$$c_{33} = \frac{\cos^2 2\alpha}{G_{\#}} + \left(\frac{1}{E_{II}} + \frac{1}{E_{\perp}} + \frac{2\cdot\upsilon_{\perp II}}{E_{II}}\right)\cdot\sin^2 2\alpha, \tag{12}$$

$$c_{12} = \frac{1}{4}\cdot\left(\frac{1}{E_{II}} + \frac{1}{E_{\perp}} - \frac{1}{G_{\#}}\right)\cdot\sin^2 2\alpha - \frac{\upsilon_{\perp II}}{E_{II}}\cdot\left(\sin^4\alpha + \cos^4\alpha\right), \tag{13}$$

$$c_{13} = \left(\frac{2}{E_{\perp}} + \frac{2\cdot\upsilon_{\perp II}}{E_{II}} - \frac{1}{G_{\#}}\right)\cdot\sin^3\alpha\cdot\cos\alpha - \left(\frac{2}{E_{II}} + \frac{2\cdot\upsilon_{\perp II}}{E_{II}} - \frac{1}{G_{\#}}\right)\cdot\cos^3\alpha\cdot\sin\alpha, \tag{14}$$

$$c_{23} = \left(\frac{2}{E_{\perp}} + \frac{2\cdot\upsilon_{\perp II}}{E_{II}} - \frac{1}{G_{\#}}\right)\cdot\cos^3\alpha\cdot\sin\alpha - \left(\frac{2}{E_{II}} + \frac{2\cdot\upsilon_{\perp II}}{E_{II}} - \frac{1}{G_{\#}}\right)\cdot\sin^3\alpha\cdot\cos\alpha. \tag{15}$$

In case in which the stresses are expressed versus strains, the relations are:

$$\begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{12} & r_{22} & r_{23} \\ r_{13} & r_{23} & r_{33} \end{bmatrix} \cdot \begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{bmatrix}, \tag{16}$$

where r_{ij} represent the transformed components of the stiffness matrix.

A laminate composite structure is considered formed of N unidirectional reinforced laminae subjected to a general set of in-plane loads. The elasticity law of a unidirectional reinforced K lamina is:

$$\begin{bmatrix}\sigma_{xx\,K}\\ \sigma_{yy\,K}\\ \tau_{xy\,K}\end{bmatrix}=\begin{bmatrix}r_{11\,K} & r_{12\,K} & r_{13\,K}\\ r_{12\,K} & r_{22\,K} & r_{23\,K}\\ r_{13\,K} & r_{23\,K} & r_{33\,K}\end{bmatrix}\cdot\begin{bmatrix}\varepsilon_{xx\,K}\\ \varepsilon_{yy\,K}\\ \gamma_{xy\,K}\end{bmatrix},\tag{17}$$

where r_{ijK} represents the transformed stiffness, σ_{xxK} and σ_{yyK} are medium stresses of a K lamina on x and y axes, τ_{xyK} represent the medium shear stress according to x-y coordinate system. The laminate balance equations are:

$$n_{xx}=\underline{\sigma}_{xx}\cdot t=\sum_{K=1}^{N}\left(\sigma_{xxK}\cdot t_K\right)=\sum_{K=1}^{N}n_{xxK},\tag{18}$$

$$n_{yy}=\underline{\sigma}_{yy}\cdot t=\sum_{K=1}^{N}\left(\sigma_{yyK}\cdot t_K\right)=\sum_{K=1}^{N}n_{yyK},\tag{19}$$

$$n_{xy}=\underline{\tau}_{xy}\cdot t=\sum_{K=1}^{N}\left(\tau_{xyK}\cdot t_K\right)=\sum_{K=1}^{N}n_{xyK},\tag{20}$$

where n_{xx} and n_{yy} are the normal forces, n_{xy} is the shear force, $\underline{\sigma}_{xx}$ and $\underline{\sigma}_{yy}$ represent the normal stresses, $\underline{\tau}_{xy}$ is the shear stress of the composite laminate, t_K and t are the K lamina thickness respective the laminate thickness, n_{xxK} and n_{yyK} are normal forces on the unit length of the K lamina and n_{xyK} is the in-plane shear force on the unit length of the K lamina. With relations (17)-(20) the composite laminate elasticity law can be obtained:

$$\begin{bmatrix}\underline{\sigma}_{xx}\\ \underline{\sigma}_{yy}\\ \underline{\tau}_{xy}\end{bmatrix}=\begin{bmatrix}\sum\limits_{K=1}^{N}\left(r_{11\,K}\cdot\dfrac{t_K}{t}\right) & \sum\limits_{K=1}^{N}\left(r_{12\,K}\cdot\dfrac{t_K}{t}\right) & \sum\limits_{K=1}^{N}\left(r_{13\,K}\cdot\dfrac{t_K}{t}\right)\\ \sum\limits_{K=1}^{N}\left(r_{12\,K}\cdot\dfrac{t_K}{t}\right) & \sum\limits_{K=1}^{N}\left(r_{22\,K}\cdot\dfrac{t_K}{t}\right) & \sum\limits_{K=1}^{N}\left(r_{23\,K}\cdot\dfrac{t_K}{t}\right)\\ \sum\limits_{K=1}^{N}\left(r_{13\,K}\cdot\dfrac{t_K}{t}\right) & \sum\limits_{K=1}^{N}\left(r_{23\,K}\cdot\dfrac{t_K}{t}\right) & \sum\limits_{K=1}^{N}\left(r_{33\,K}\cdot\dfrac{t_K}{t}\right)\end{bmatrix}\cdot\begin{bmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \gamma_{xy}\end{bmatrix},\tag{21}$$

and from these relations, the composite laminate stiffness can be determined:

$$\underline{r}_{ij}=\sum_{K=1}^{N}\left(r_{ijK}\cdot\frac{t_K}{t}\right).\tag{22}$$

In these conditions, the composite laminate elasticity law becomes:

$$\begin{bmatrix}\underline{\sigma}_{xx}\\ \underline{\sigma}_{yy}\\ \underline{\tau}_{xy}\end{bmatrix}=\begin{bmatrix}\underline{r}_{11} & \underline{r}_{12} & \underline{r}_{13}\\ \underline{r}_{12} & \underline{r}_{22} & \underline{r}_{23}\\ \underline{r}_{13} & \underline{r}_{23} & \underline{r}_{33}\end{bmatrix}\cdot\begin{bmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \gamma_{xy}\end{bmatrix},\tag{23}$$

where $\underline{r}_{ij}$ are functions of the basic elastic properties of each lamina $E_{\parallel K}$, $E_{\perp K}$, $\upsilon_{\perp\parallel K}$, $G_{\#K}$ and of the fibers disposal angle. Analogue to stresses, a strains analysis can be carried out. From relation (23) the strains ε_{xx}, ε_{yy} and γ_{xy} can be computed. The individual strains of each lamina can be determined through transformation as following:

$$\begin{bmatrix}\varepsilon_{II\,K}\\ \varepsilon_{\perp K}\\ \gamma_{\#K}\end{bmatrix}=\begin{bmatrix}\cos^2\alpha_K & \sin^2\alpha_K & \sin\alpha_K\cos\alpha_K\\ \sin^2\alpha_K & \cos^2\alpha_K & -\sin\alpha_K\cos\alpha_K\\ -2\sin\alpha_K\cos\alpha_K & 2\sin\alpha_K\cos\alpha_K & \left(\cos^2\alpha_K-\sin^2\alpha_K\right)\end{bmatrix}\cdot\begin{bmatrix}\varepsilon_{xx\,K}\\ \varepsilon_{yy\,K}\\ \gamma_{xy}\end{bmatrix}.\tag{24}$$

Finally, from the strains presented in relation (24), the stresses in each individual lamina can be computed:

$$\sigma_{II\,K}=\frac{E_{II\,K}}{1-\upsilon_{\perp II\,K}\cdot\upsilon_{II\perp K}}\cdot\varepsilon_{II\,K}+\frac{\upsilon_{\perp II\,K}\cdot E_{\perp K}}{1-\upsilon_{\perp II\,K}\cdot\upsilon_{II\perp K}}\cdot\varepsilon_{\perp K},\tag{25}$$

$$\sigma_{\perp K}=\frac{\upsilon_{\perp II\,K}\cdot E_{\perp K}}{1-\upsilon_{\perp II\,K}\cdot\upsilon_{II\perp K}}\cdot\varepsilon_{II\,K}+\frac{E_{\perp K}}{1-\upsilon_{\perp II\,K}\cdot\upsilon_{II\perp K}}\cdot\varepsilon_{\perp K},\tag{26}$$

$$\tau_{\#K}=G_{\#}\cdot\gamma_{\#}.\tag{27}$$

To carry out a prediction regarding the failure of the individual laminae, a break criterion is usually used. The system of coordinates $\parallel$-$\perp$-z represents the local system of coordinates and is applied to each individual lamina. The x-y-z system of coordinates represents the global system of coordinates and is usually applied to the entire laminate. Some experimental results obtained on various composite laminates subjected to a wide range of loadings are presented in references [2], [3], [7-11] as well as numerical simulations to predict the elastic properties of some fibers-reinforced composite laminates are given in papers [4-6].

2. COMPUTATIONAL MICROMECHANICS OF FOUR LAMINATES

In order to compute stresses in each individual lamina of a composite laminate, four examples of laminates have been chosen. These laminates are: anti-symmetric laminate with following plies sequence: [30/0/0/-30]; symmetric cross-ply laminate with plies distribution: [90/0/0/90]; symmetric angle-ply laminate with plies sequence: [30/-30/-30/30]; balanced angle-ply laminate with following plies distribution: [30/30/-30/-30].

The computational method is based on the approach presented in reference [12]. For all types of laminates, the Tenax IMS65 carbon fibers have been taken into account as well as Huntsman XB3585 epoxy resin with following input data:

- Matrix axial and transverse Young's modulus: 3.2 GPa;
- Fibers axial Young's modulus: 290 GPa;
- Fibers transverse Young's modulus: 4.8 GPa;
- Matrix axial-transverse Poisson ratio: 0.3;
- Fibers axial-transverse Poisson ratio: 0.05;
- Matrix axial-transverse shear modulus: 1.15 GPa;
- Fibers axial-transverse shear modulus: 4.2 GPa;
- Fibers volume fraction: 0.51;
- Applied normal stress in x-direction: 2000 MPa;
- Applied normal stress in y-direction: 200 MPa;
- Applied shear stress in x-y plane: 100 MPa;
- Off-axis loading system: between 0° and 90°.

For these types of laminates, stresses in each lamina have been computed. Example of stresses in some plies in case of some considered laminates subjected to a general set of in-plane loads are presented in tables 1-2 and stresses distributions according to different off-axis loading angles are visualized in figures 1-8.

Table 1: Example of computational stresses in first ply in case of [30/0/0/-30] anti-symmetric laminate

Off-axis loading system	Normal stress $\sigma_{\parallel}$ [GPa]	Normal stress $\sigma_{\perp}$ [GPa]	Shear stress $\tau_{\#}$ [GPa]
0°	2179.08820396	-0.71697426	-35.75065488
10°	939.39407259	36.90536763	-49.96934150
20°	122.51085318	121.05810845	-26.13182244
30°	-173.03328210	241.59118570	32.88674569
40°	88.40865122	383.96653138	119.96785257
50°	875.30289753	531.01157756	224.60823152
60°	2092.73839739	664.99052153	334.18670852
70°	3593.87446208	769.74352531	435.48650202
80°	5197.65192780	832.63583067	516.28936191
90°	6710.63156295	846.08169739	566.84927077

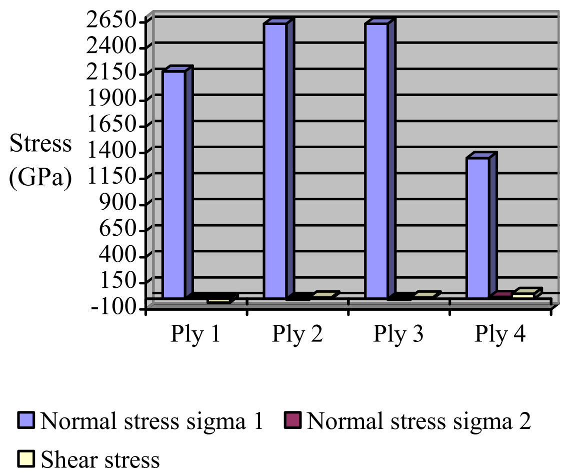

Figure 1: Stresses in plies' laminate [30/0/0/-30]

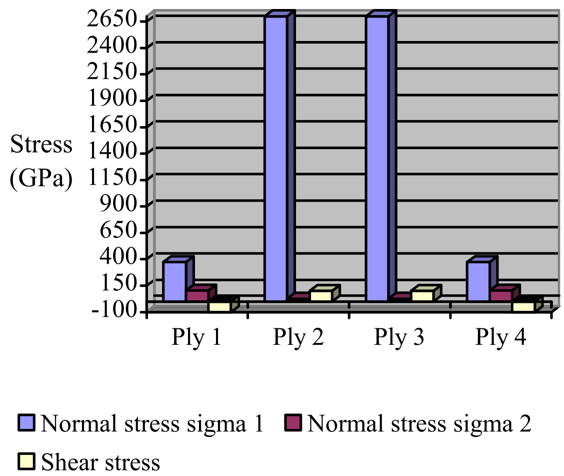

Figure 2: Stresses in plies' laminate [90/0/0/90]

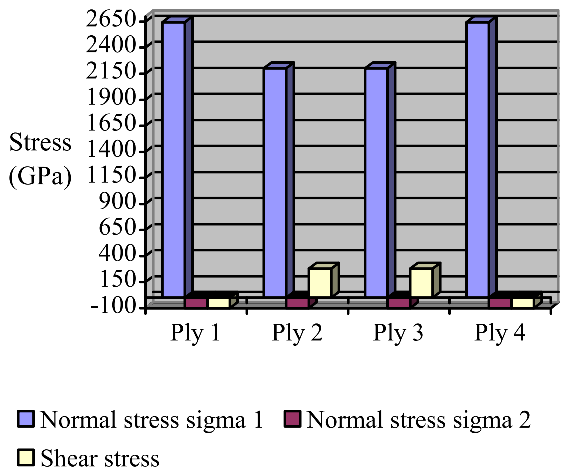

Figure 3: Stresses in plies' laminate [30/-30/-30/30]

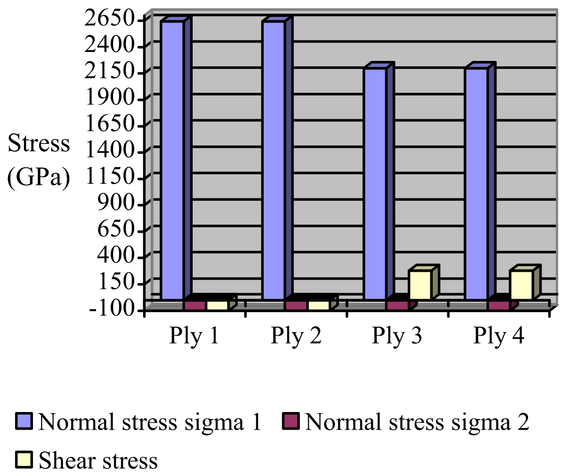

Figure 4: Stresses in plies' laminate [30/30/-30/-30]

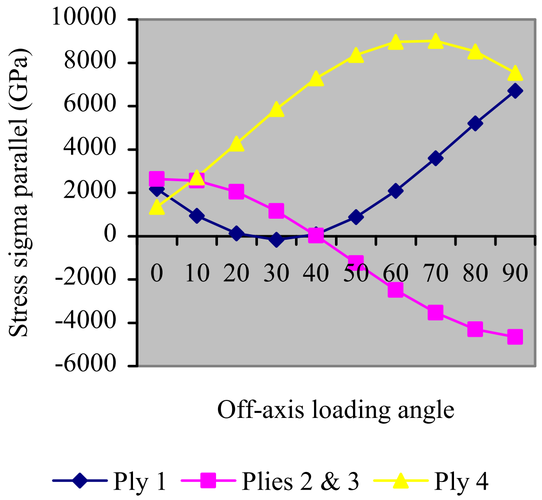

Figure 5: Stress $\sigma_{\parallel}$ in plies' laminate [30/0/0/-30]

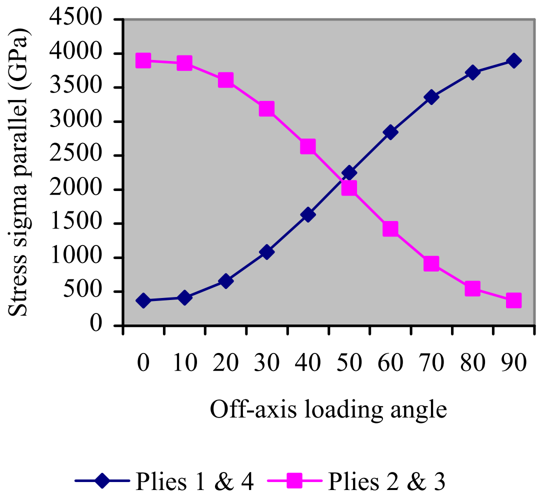

Figure 6: Stress $\sigma_{\parallel}$ in plies' laminate [90/0/0/90]

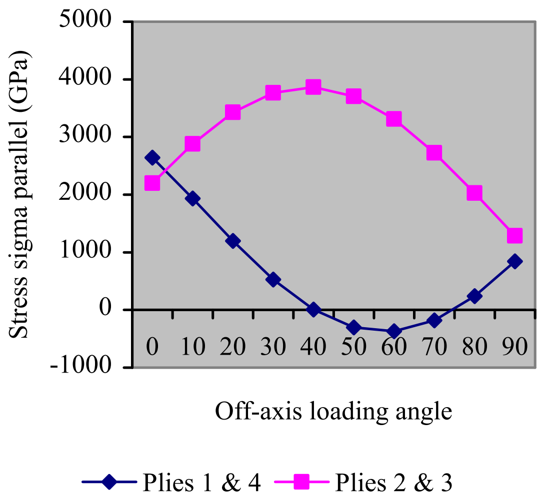

Figure 7: Stress $\sigma_{\parallel}$ in laminate [30/-30/-30/30]

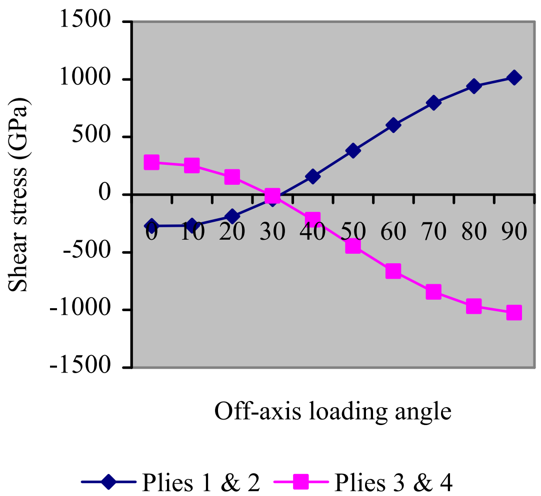

Figure 8: Shear stress in laminate [30/30/-30/-30]

Table 2: Example of computational stresses in first ply in case of [90/0/0/90] symmetric cross-ply laminate

Off-axis loading system	Normal stress $\sigma_{\parallel}$ [GPa]	Normal stress $\sigma_{\perp}$ [GPa]	Shear stress $\tau_{\#}$ [GPa]
0°	373.01363977	103.92906445	-100.00000000
10°	412.30478474	103.07095551	213.84886691
20°	659.32319009	97.67613434	501.90440441
30°	1084.27479052	88.39529598	729.42286341
40°	1635.90415140	76.34784651	868.96215995
50°	2247.67663063	62.98688614	903.69179548
60°	2845.80343841	49.92394388	829.42286342
70°	3358.14165431	38.73460334	655.11329304
80°	3722.89572819	30.76846414	401.78739109
90°	3896.07093555	26.98636024	100.00000002

3. CONCLUSION

A strong anisotropy at all considered laminates can be noticed. Stresses along fibers direction are up to ten times greater than those transverse to fibers direction. The theoretical approach can be used to determine computational stresses in various composite laminates, stresses that can be compared with experimental results obtained by different methods.

REFERENCES

[1] Schürmann H., Konstruiren mit Faser-Kunststoff-Verbunden, Springer, 2005.

[2] Teodorescu-Draghicescu H., Vlase S., Scutaru M.L., Serbina L., Calin M.R., Hysteresis Effect in a Three-Phase Polymer Matrix Composite Subjected to Static Cyclic Loadings, Optoelectron. Adv. Mater. – Rapid Comm. **5**(3), 273, 2011.

[3] Vlase S., Teodorescu-Draghicescu H., Motoc D.L., Scutaru M.L., Serbina L., Calin M.R., Behavior of Multiphase Fiber-Reinforced Polymers Under Short Time Cyclic Loading, Optoelectron. Adv. Mater. – Rapid Comm. **5**(4), 419, 2011.

[4] Vlase S., Teodorescu-Draghicescu H., Calin M.R., Serbina L., Simulation of the Elastic Properties of Some Fibre-Reinforced Composite Laminates Under Off-Axis Loading System, Optoelectron. Adv. Mater. – Rapid Comm. **5**(4), 424, 2011.

[5] Teodorescu-Draghicescu H., Stanciu A., Vlase S., Scutaru M.L., Calin M.R., Serbina L., Finite Element Method Analysis Of Some Fibre-Reinforced Composite Laminates, Optoelectron. Adv. Mater. – Rapid Comm. **5**(7), 782, 2011.

[6] Teodorescu-Draghicescu H., Vlase S., Homogenization and Averaging Methods to Predict Elastic Properties of Pre-Impregnated Composite Materials, Computational Materials Science, **50**(4), 1310, Elsevier, Feb. 2011.

[7] Stanciu A., Teodorescu-Draghicescu H., Vlase S., Scutaru M.L., Calin M.R., Mechanical Behavior of CSM450 and RT800 Laminates Subjected to Four-Point Bend Tests, Optoelectron. Adv. Mater. – Rapid Comm. **6**(3-4), 495, 2012.

[8] Vlase S., Teodorescu-Draghicescu H., Calin M.R., Scutaru M.L., Advanced Polylite composite laminate material behavior to tensile stress on weft direction, J. Optoelectron. Adv. Mater., **14**(7-8), 658, 2012.

[9] Teodorescu-Draghicescu H., Scutaru M.L., Rosu D., Calin M.R., Grigore P., New Advanced Sandwich Composite with twill weave carbon and EPS, J. Optoelectron. Adv. Mater., **15**(3-4), 199, 2013.

[10] Modrea A., Vlase S., Teodorescu-Draghicescu H., Mihalcica M., Calin M.R., Astalos C., Properties of Advanced New Materials Used in Automotive Engineering, Optoelectron. Adv. Mater. – Rapid Comm. **7**(5-6), 452, 2013.

[11] Vlase S., Purcarea R., Teodorescu-Draghicescu H., Calin M.R., Szava I., Mihalcica M., Behavior of a new Heliopol/Stratimat300 composite laminate, Optoelectron. Adv. Mater. – Rapid Comm. **7**(7-8), 569, 2013.

[12] Hull D., Clyne T.W., An Introduction to Composite Materials, CUP, 1996.