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RANDOM OSCILLATIONS OF LIQUID IN THE U-SHAPED PIPE WITH PRONOUNCED RUGOSITY AND THE NON-LINEAR DAMPING FORCE

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Abstract: *The present paper consists of discussion on dynamic response of oscillators under random load. This paper is concerned with forced oscillations of fluid. A major problem of fluid dynamics is that the equations of motion are non-linear. Therefore, it is necessary to replace the nonlinear system with an equivalent linear system. Method of equivalent linearization has been extensively used in these engineering applications. A substantial reference list of workin this area can be found in Roberts and Spanos. In general, and especially in random vibration analysis, it is difficult to obtain a closed form solution for dynamic response of a nonlinear system.*
Keyword**:** *Random oscillations, nonlinear damping, natural frequency, the power spectral density.*

1. INTRODUCTION

Nonlinear dynamic systems subject to random excitations are frequently met in engineering practice. The basic idea of the statistical linearization approach is to replace the original nonlinear system by a linear one. They are random processes and commonly described by spectral density functions. A major problem of fluid dynamics is that the equations of motion are non-linear. This implies that an exact general solution of these equations is not available.

2. SYSTEM MODEL

We consider a U-shaped pipe with rough inner surface with constant section diameter d, in which there is a liquid with known density ρ_0 . If there is a random disturbance occurs at a certain time t elevation h (t). Decrease in the first branch [4,5] and the second liquid fluid up. We believe that rubbing the tube is nonlinear.
The ordinary differential [1,2] equation of the motion can be written as:

$$m\ddot{h}(t)+c\dot{h}^{3}(t)+k\,h(t)=W(t) \tag{1}$$

where m is the mass, c is the viscous damping coefficient, $W(t)$ is the external excitation signal with zero mean and $h(t)$ is the displacement response of the system. $S_h(\omega)$ and $S_W(\omega)$ are the power spectral density for $h(t)$ and the external excitation $W(t)$ respectively.
Dividing the equation by m , the equation of motion can be rewritten as:

$$\ddot{h}(t)+2\xi p\dot{h}^{3}(t)+\frac{2g}{l}h(t)=w(t) \tag{2}$$

where ξ is the critical damping factor and p is the undamped natural frequency, for the linear system.
By linearization [2,3] of the equations of motion we find the following linear equation:

$$\ddot{h}(t)+\phi_{ech}\dot{h}(t)+\frac{2g}{l}h(t)=w(t)\,. \tag{3}$$

This linearization system introduces an error that has to be as low as possible so minimal. The difference is the difference between the nonlinear stiffness and linear stiffness terms [2,3,6], which is

$$\varepsilon = 2\xi p \dot{h}^{3}(t) - \phi_{ech}\,\dot{\eta}(t) \tag{4}$$

The value of ϕ_{ech} can be obtained by minimizing [2,6,7] the expectation of the square error

$$\frac{\partial}{\partial\ \phi_{ech}} E[\varepsilon^2]=0 \tag{5}$$

Because

$$E\{\varepsilon^2\}=4\xi^2 p^2 E\{\dot{h}^{6}\}+\phi_{ech}{}^{2}E\{\dot{h}^{2}\}-4\xi p\phi_{ech}E\{\dot{h}^{4}\}, \tag{6}$$

we obtain next equation

$$\phi_{ech}E\{\dot{h}^{2}\}-2\xi pE\{\dot{h}^{4}\}=0\,. \tag{7}$$

Because

$$\phi_{ech}=2\xi p\frac{E\{\dot{h}^{4}\}}{E\{\dot{h}^{2}\}}\,, \tag{8}$$

obtain

$$\ddot{h}(t)+c\frac{E\{\dot{h}^{4}\}}{E\{\dot{h}^{2}\}}\dot{h}(t)+\frac{2g}{l}h(t)=w(t)\,. \tag{9}$$

The displacement variance [1,2,7] of the system under Gaussian white noise excitation can be expressed as

$$\sigma^2_{\ h}=R_h(0)=\int_{-\infty}^{\infty}\left|H(\omega)\right| mS_0 d\omega \tag{10}$$

Because we have the following equation for the transfer function [1,7,8]

$$H(\omega)=\frac{1/m}{p^2-\omega^2+2i\omega\xi p\dfrac{E\{\dot{h}^{4}\}}{E\{\dot{h}^{2}\}}}\,, \tag{11}$$

the displacement variance can be expressed [1,8,9]

$$\sigma^2_{\ h}=\int_{-\infty}^{\infty}\frac{4S_0}{\rho l\pi d^2}\frac{1}{\left(p^2-\omega^2\right)^2+4\omega^2\xi^2p^2\left[1+\left(\dfrac{E\{\dot{h}^{4}\}}{E\{\dot{m}^{2}\}}\right)\right]^2}d\omega=\frac{\pi S_0}{\rho l\xi p^3\dfrac{\pi d^2}{2}\left(1+\dfrac{E\{\dot{h}^{4}\}}{E\{\dot{h}^{2}\}}\right)}, \tag{12}$$

where

$$c_e=2\xi pm\left(1+\frac{E\{\dot{h}^{4}\}}{E\{\dot{h}^{2}\}}\right) \tag{13}$$

The set of conditions that guarantee the existence of the Fourier transform is the Dirichlet conditions, which may be expressed as: the signal $h(t)$ has a finite number of finite discontinuities and the signal $h(t)$ contains a finite number of maxima and minima.

Using the transfer function [1,8,9] we obtain the answer as a function of frequency

$$\bar{h}(\omega)=H(\omega)\bar{W}(\omega)\,, \tag{14}$$

where

$$\bar{h}(\omega)=\mathrm{F}\,(h(t))\,, \tag{15}$$

$$\bar{W}(\omega)=\mathrm{F}\ \,(W(t))\,. \tag{16}$$

In this way, the power spectral density of the response for the system is

$$S_h(\omega)=\frac{S_W(\omega)}{m^2\left[p^2-\omega^2+2i\omega\xi p\frac{E\{\dot{h}^4\}}{E\{\dot{h}^2\}}\right]}. \tag{17}$$

3. THE NUMERICAL RESULTS

The Duffing oscillator has been used to illustrate this procedure here. Figure 1 was obtained for spectral density of excitation $S_F=0,5N^2\cdot s$, with parameters $l=23cm,\ \rho=910\frac{kg}{m^3}, d=1,5cm$.

The broadening of the first resonant peak is described very satisfactorily by the approximate solution.
Obtain in this case for the displacement variance

$$\sigma_h^2=\frac{\pi S_0}{2\xi pk\left[1+\frac{E\{\dot{h}^4\}}{E\{\dot{h}^2\}}\right]} \tag{18}$$

The power spectral density of the response $S_h(\omega)\,[m^2\cdot s]$, for Gaussian white noise with spectral density function $S_0=0,5\frac{N\cdot m}{s}$, is plotted for the differents parameters in fig 1.

By adding the formula

$$E\{\dot{h}^4\}=\frac{45}{4}\sigma_h^2 E\{\dot{h}^2\}, \tag{19}$$

we obtain the equation of the system with solution $\sigma_h^2=0,006827m^2$.

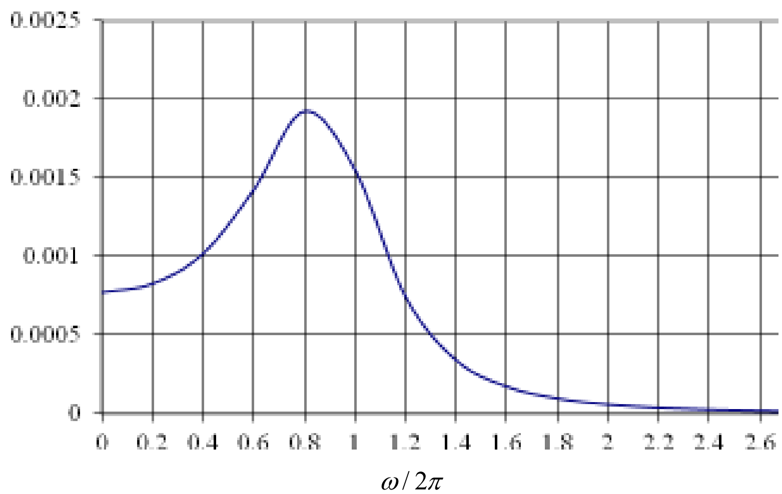

Figure 1. The power spectral density of the response $S_h(\omega)\,[m^2\cdot s]$ for $l=23cm,\ \rho=910\frac{kg}{m^3}, d=1,5cm.$

The power spectral density of the response $S_h(\omega)\,[m^2\cdot s]$, for Gaussian white noise with spectral density function $S_0=0,5\frac{N\cdot m}{s}$, is plotted for $l=23cm,\ \rho=850\frac{kg}{m^3}, d=1,5cm$ in fig 2. we obtain the equation of the system with solution $\sigma_h^2=0,009827m^2$.

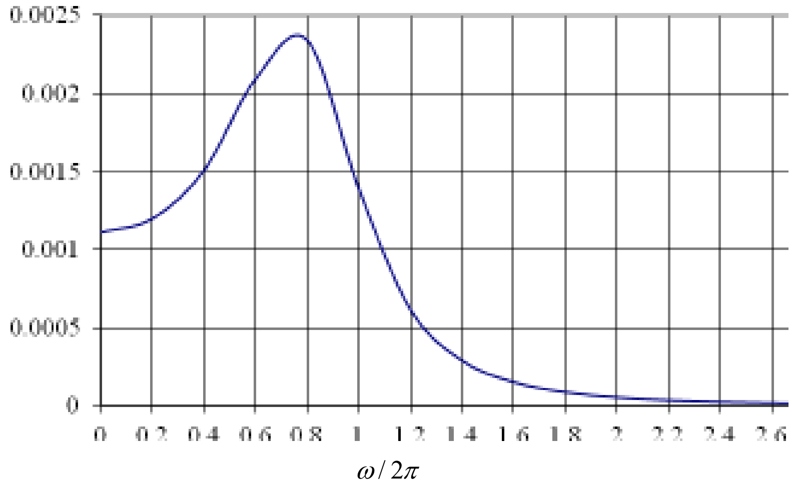

Figure 2. The power spectral density of the response $S_h(\omega)\,[m^2 \cdot s]$ for $l=23cm,\ \rho=850\dfrac{kg}{m^3}, d=1,5cm$.

4. CONCLUSIONS

Figure 1-2 show a good agreement between theory and experiment. Detailed numerical results are presented for of nonlinear oscillators under white noise excitation. The power spectral density of the response will not have a large spectral content at low frequencies and the skewness will be zero.

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