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ON NONLINEAR EFFECTS DUE TO THERMO-MECHANICAL BEHAVIOUR IN COMPUTATIONAL DYNAMICS OF ELASTOMERIC ISOLATORS

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Abstract: *This paper deals with heating regime analysis during ordinary and intensive behaviour for a set of anti-vibratory elements based on composite elastomeric materials. The research started from observations and measurements in practice, and contains simulation scenario for some critical cases. Basic aim of this study frames the necessity of dignify the main parameters and their evolution with strength influences on vibration isolation characteristic. It was used classical rheological models with non-linear influence terms. Computational dynamics has performed using finite element method implementation onto the Matlab© FEA toolbox. The results have shown good performances of numerical models according to the experimental tests.*
Keywords: *Vibration isolation, Thermo-dynamics, Dissipative characteristics, Elastomeric composites*

1. INTRODUCTION

Traceable and measurable reality of dynamic behaviour of the vibration isolators during their regular working cycle based on the experimental remarks reveals continuous changing of dynamic characteristics. Many measurements developed on various structural configuration composites-based elements shown two main directions in influences factors as follows: thermal phenomenon inside the basic material during the exploitation, and nonlinear behaviour of the isolators taking into account both the material characteristic, and the internal interactions between structural components of the isolators.
Continuous shifting of dynamic characteristic implies different outputs corresponding to the same input in respect with the time parameter.
In addition, the behaviour changes have affected by certain random character, which comes to complicate the entire evaluation of the isolation performances for the *BASE-STRUCTURE ↔ ISOLATOR ↔ SUPER-STRUCTURE* system. Based on the experimental tests developed by the author using rubber-based elements with different structural configurations it was framed the idea according with the previous factors have a great impact onto the global behaviour of the vibration isolation devices.
In the frame of these remarks it was compiled a set of nonlinear models which had been able to simulate an appropriate behaviour of vibration isolation systems comparative with experimental results. First approaches suppose only the nonlinear characteristics of the basic materials. Computational analysis and instrumental validations was performing based on these hypotheses.
Next step of computer analysis starting from the previous results and taking into account the latest instrumental tests developed on experimental laboratory setups supposes an additional thermal component into the computational assessments. Hereby, the nonlinear differential equations acquire additional terms that can modify continuously the instant characteristics of basic materials in respect with conservative and/or dissipative components.
Depending on the elastomer-type composites, used for the passive vibration isolation element, one or both of the stiffness and damping components can get thermal influences thus that an overall simulation, evaluation and analysis become a *Complex Multiple Inputs - Multiple Outputs* (C-MIMO) computational system.
In this paper it will be presented exclusive the thermo-mechanical assessments regarding the dynamics of passive vibration rubber-based isolators. Different structural configurations were adopting and different external dynamic loads were supposing such external disturbing actions. Computational techniques were combining with instrumental analysis for constitutive equations parametrical terms approaching and tuning.

2. ANALITICAL APPROACHES OF THERMO-MECHANICAL INFLUENCES

The analytical approach of this analysis has developed based on a single degree of freedom (SDoF) dynamic system schematization. The study supposes a reference-SDoF system given by a temperature-independent internal resistant force. The current model used for typical analysis was also an SDoF system, which has provided both dissipative and conservative terms temperature-dependent. It had to be note that damping in respect with *Rayleigh-type* law leads to a simplified analysis thus that the author uses a generally loss factor-type damping. Differential equation of this system in terms of independent variable named δ, with both dissipative and conservative components respect the follow formulation

$$m\ddot{\delta} + c_1\dot{\delta} + c_2\delta = Q_{ext}(t) \tag{1}$$

where m denotes system mass, c_i denotes damping (i=1), respectively stiffness (i=2), and Q_{ext} represents external dynamic perturbation. Reference system acquired c_{io} values for both elastic and dissipative parameters. The temperature-dependent system respect the following law for c_i parameters

$$\frac{c_i}{c_{io}} = 1 + \xi_i\ e^{\beta_i(\theta_o - \theta)},\ i=1,2 \tag{2}$$

where index i respect the previous convention and parameters ξ_i and β_i denote the temperature influence coefficients. Null values for both coefficients imply the reference system behaviour with $c_i = c_{io}$.

The analysis of the SDoF system evolution based on Eqn. (1) was performing with output parameter as absolute differences between instantaneous displacements of reference and, respectively, of temperature-dependent systems. This analysis was following up the imminence of resonance phenomenon taking into account the shifting of natural frequency and the continuous changes of damping of the basic SDoF system due to temperature influences. Both steady and unsteady states were evaluating.

The ratio between excitation and natural frequencies denoted by R was supposing an appropriate parameter for monitoring the evolution of relative displacement. Diagrams in Fig. 1 depicts six discrete cases in respect with R parameter, including the resonance case (c) with R = 100%. According the Fig. 1 legend cases (a) and (b) denotes pre-resonance status. In addition, cases (d...f) denote post-resonance status.

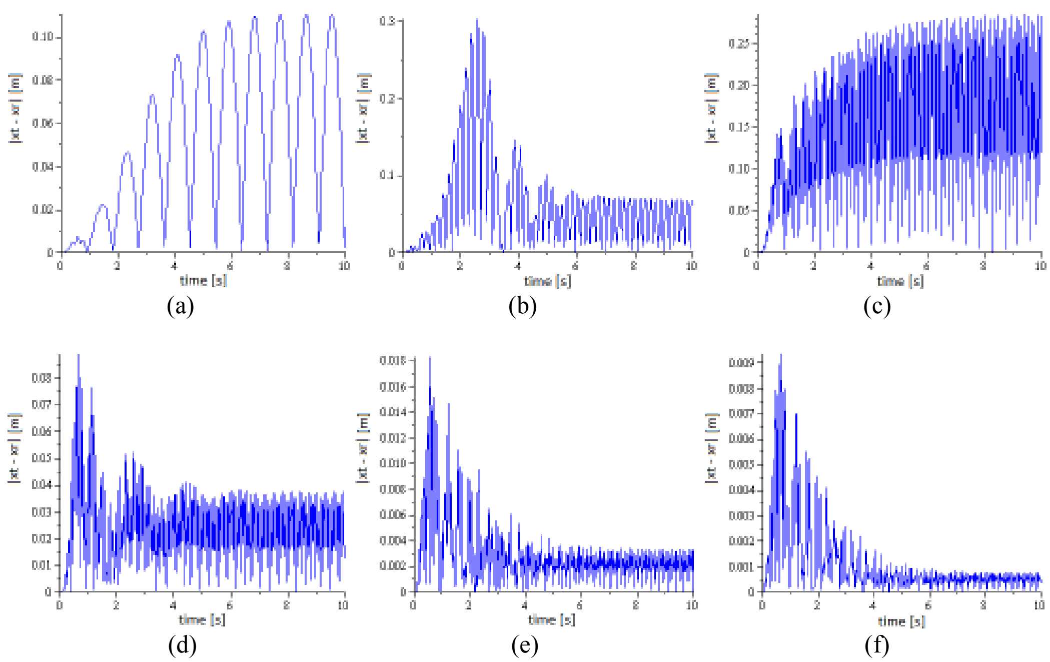


Figure 1: Instant absolute differences between evolution of theoretic and realistic displacements during the first 10 seconds of transitory state for an SDoF dynamic system. The cases has follows signification (a) R = 10%; (b) R = 50%; (c) R = 100%; (d) R = 110%; (e) R = 150%; (f) R = 200%.

Comparative analysis of diagrams in Fig. 1 shown that apart from (c) case, the others contain similar qualitative evolution, means resonance passing through for the first 2...3 seconds from the process started.

A quantitative analysis of these diagrams reveals that even the resonance-passing period has roughly the same value the relative displacement magnitudes have acquired great dangerous values for pre-resonance cases. The passing period is relative short but the maximum values of the relative displacement exceeds very much the bearing capacity of these elements thus that it is impossible passing the resonance period with structural and functional integrity maintaining. In the same time, for the post-resonance cases small values for relative displacement were acquiring. Hereby the resonance-passing period denotes only a regular transitory state with minimal influences on dynamic system in terms of structural integrity and functional conformity.

3. COMPUTATIONAL APPROACH AND RESULTS ANALYSIS

The computational approach of this analysis supposes a thermo-mechanical behaviour analysis of a rubber-based isolator during the regular exploitation time. The goals of this study consist by identification of heating sources inside the element due to damping and evaluation of internal temperature changing.
An appropriate simulation in this case has to includes heating equation for evaluation of the internal dissipation inside the isolator element

$$\rho\, c\, \frac{\partial \theta}{\partial t} + \nabla\left(-\boldsymbol{\lambda}\, \nabla \boldsymbol{\theta}\right) = \psi \qquad (3)$$

and also the equation of convection phenomenon on free boundaries including the loaded and constrained areas of the element as follows

$$-\boldsymbol{\lambda}\, \nabla \boldsymbol{\theta} = \boldsymbol{\alpha}\left(\theta - \theta_{ext}\right). \qquad (4)$$

In Eqns. (3) and (4) the term ρ denotes material density, c is the heat capacity, θ denotes temperature inside the element, θ_{ext} is the ambient temperature, $\boldsymbol{\lambda}$ is heat conductivity vector containing the values for (x,y,z) axis, $\boldsymbol{\alpha}$ denotes natural convection vector also containing the values for (x,y,z) axis.
The function ψ in Eqn. (3) denotes the internal heating sources inside the elastomeric element due to damping phenomenon. This volumetric dissipated power produced by the damping changes the internal temperature field inside the material. A continuously evaluation of damping parameter during the analysis can be made using 2D or 3D meshing according to the initial hypothesis.
A typical example of this kind of analysis has presented in Fig. 2. Thus, the diagram in Fig 2 shows the strain energy density (see color map in Fig.2) and the heat flux (see arrows grid in Fig.2) for sectional orthogonal planes within a parabolic cylinder shape isolator. For simulation it was used a simple geometrical configuration of two cones trunks with common lowest basis (as an internal interface).

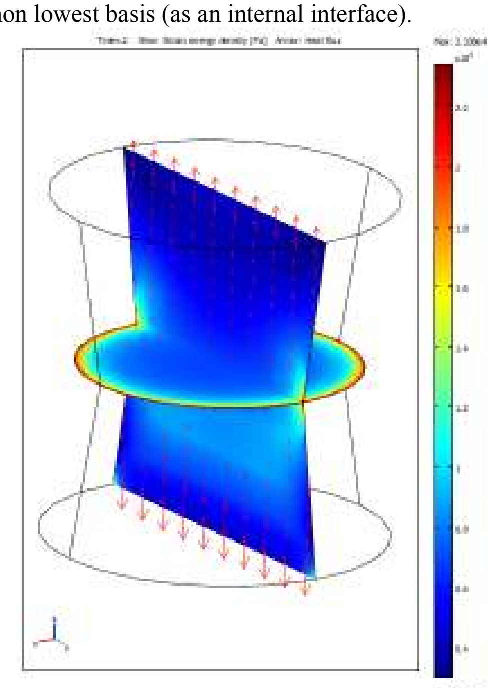
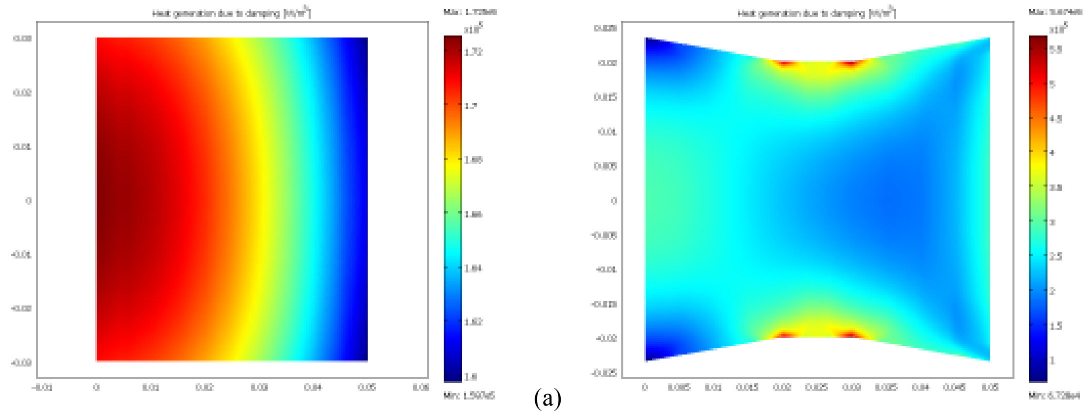


Figure 2: Distribution of strain energy density (color map) and heat flux (arrows field) within the orthogonal sections of a profiled cylinder isolator

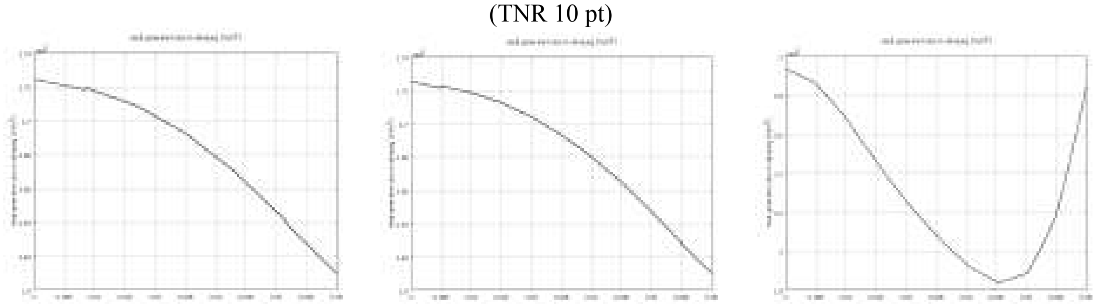
Based on previously presented hypotheses it has developed a comparative study of three types of synthetic isoprene rubber-based elements. The basic approach starts from a constant volume of material and various shapes such as rectangular prism, regular cylinder and parabolic (profiled) cylinder.
Basic element had a cubical shape with 0.05 m edge length. The other elements has maintained constant only height dimension at 0.05 m. The heating due to damping and the internal temperature field was the pair

parameters evaluated for each case. The entire analysis has performed for a short time period of 2 seconds with the integration step of 0.05 s.
Following the longitudinal section within the isolator in Fig. 3 has depicted the evolution of the heat generation due to damping for regular cylinder (see Fig.3.a) and parabolic cylinder (see Fig.3.b). In addition, the evolution of the same parameter along the longitudinal axis of each elastomeric element has depicted in Fig. 4. Note that the diagrams in Fig. 4 respect the symmetry axis along the correspondent sections in Fig. 3, where the regular prism case is missing because of the strength qualitative similitude with the regular cylinder case.



(a)
(b)

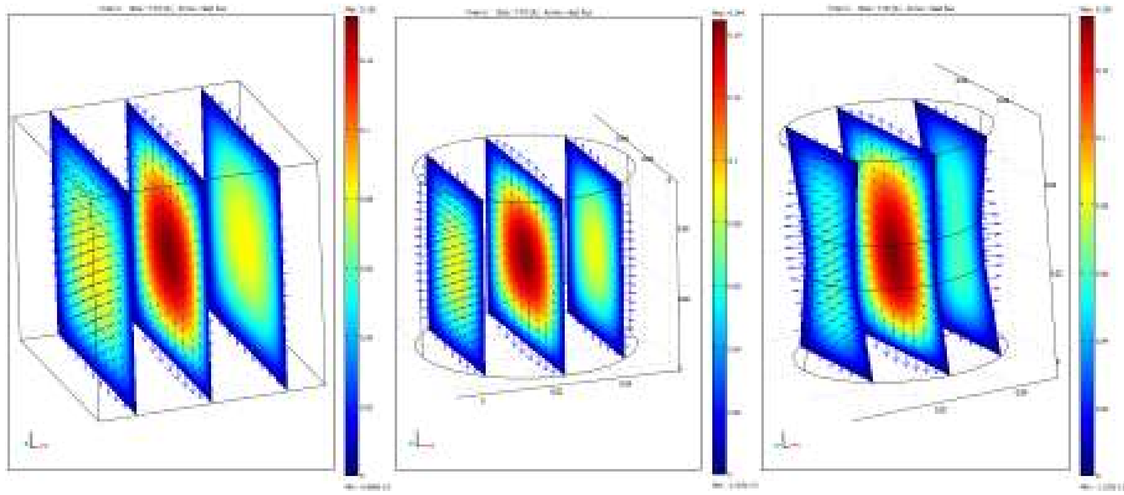
Figure 3: Distribution of the heat generation due to damping within the longitudinal section through a regular cylinder (a) and a profiled cylinder (b)
(TNR 10 pt)



(a) (b) (c)

Figure 4: Steady state evolutions of the heating due to damping parameter along the longitudinal axis of the elastomeric element. The cases have follows signification
(a) rectangular prism; (b) regular cylinder; (c) parabolic cylinder.

Following the diagrams in Fig. 3 and Fig. 4, it can be observe unusual evolution for the case corresponding to the profiled cylinder. For regular shapes of prism and cylinder results roughly similar evolutions of heat generation due to damping. The profiled cylinder situation provides different qualitative and quantitative evolutions comparative with the others. Hereby the heating internal sources have a minimum value inside the element and a maximum values on the boundaries.
An explanation for these simulation results consists by the existence of convergence points on the interface boundaries for the profiled cylinder and missing of these points for the regular shapes. In addition, the graphs in Fig. 4 reveal that the maximum values of heating due to damping for regular shapes are lower than the minimum value of the same parameter for profiled cylinder, and this fact sustains the previous explanation.
The temperature fields and the heat flux inside the proposed three elements has depicted in Fig. 5. It has supposed three vertical sections equally distributed with symmetrical middle. Comparative analysis of diagrams in Fig. 5 shows an identical behaviour for each case, with some differences about the extreme values for the monitored parameters. The maximum value of temperature changes for the cases presented in Fig. 5 reveals that regular cylinder shape induces higher modifications than the others do. However, taking into account that the differences have fewer than 10% and the parabolic cylinder shape provides some other functional advantages results that this last shape type is able to assure a suitable functional characteristic for the isolation system.



(a) (b) (c)

Figure 5: Inside temperature (see color maps) and heat flux (see arrows fields) distributions for a rectangular prism (a), regular cylinder (b) and parabolic cylinder (c) shapes.

4. CONCLUDING REMARKS

The observations in previous paragraphs reveal that regular shapes provides a uniform distribution of temperature gradient inside the material while the irregulars have convergence points both inside, and on the external boundaries of the element. The strain variation acquires various characteristics and implies variable damping along the orthogonal axis thus that result an inhomogeneous field of heating sources distribution inside the irregular shape elements.

Taking into account the instrumental laboratory tests related to the internal temperature field of viscous-elastic vibration isolators and the continuous changing of the dynamic characteristics for insulation systems including these elements, the main concluding remark dignifies both the necessity and the opportunity of presented analysis.

Correlative analysis between computational results and real behaviour data reveals a good correspondence of the models with the practice. Regarding the dissipative component influences on the entire dynamic evolution of the elastomeric-based elements, it had to be underlying the great importance of cold state, respectively hot state natural frequencies for the practical evaluations, integrated with both theoretical and experimental basics and validations. Previous idea assures the practical conditions in order to maintain the working state regime beyond the resonance domain and avoiding the system evolution to a strength amplification of transmissibility parameter.

Future developments in the area of this research must supply a comprehensive nonlinear law for conservative and dissipative characteristics with thermal influences. In addition, the structural shapes influences on the heating generated by damping phenomenon will be continuing in order to obtain a set of functional law with large serviceable application.

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