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VARIABLE CONDUCTANCE HEAT PIPE MODEL FOR TEMPERATURE CONTROL OF PROCESSES

Ungureanu I. Virgil-Barbu[1]
[1] University Transilvania, Brasov, ROMANIA, virbung@unitbv.ro

Abstract: *The paper presents a model for an accurate and precise method of automatic control engineering for temperature control of processes. The method requires variable conductance heat pipe with non-condensable gas added. It is solved the system of equations who describes the heat transfer processes obtaining the transfer function of the regulating system. The transient behavior is analyzed for the system with conventional variable conductance heat pipe operating in open loop and feedback controlled variable heat pipe.*
Keywords: *heat pipe, variable conductance heat pipe, heat transfer equations, transfer function of automatic systems)*

1. INTRODUCTION

The basic heat pipe is a closed container which has evacuated all non-condensable gases and contains a capillary structure (wick) and a small amount of a vaporizable fluid [1]. The heat pipe employs a boiling-condensing cycle and the capillary pumps in order to return the liquid. Figure 1 shows schematically the basic heat pipe operating system. The essential components of a heat pipe are the sealed container, the wick and the suitable working fluid: liquid in equilibrium with its own vapour. Thus, the heat pipe is composed by three zones: evaporator, adiabatic and condenser. When heat is applied along the evaporator zone, the local temperature is raised slightly and part of the working fluid evaporates at the wick surface adjacent to this zone. Simultaneously, the condenser zone is slightly cold. Because of the saturation conditions this temperature difference causes a vapour pressure difference and a vapour flowing from evaporator zone to the condenser zone, through the adiabatic zone. The heat absorbed is commensurate by the heat flow rate of vaporization. During steady state operation, the first principle of thermodynamics requires that the amount of heat absorbed by the working fluid is identical with the heat released. The wick provides a flow part for the liquid return and is responsible for the pumping.

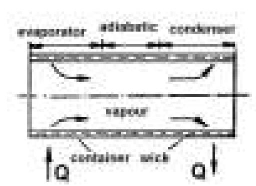

Figure 1: Heat pipe

The operating temperature ranges are cryogenic (0 to 150 K), low temperature (150 to 400 K) medium temperature (400 to 750 K) and high temperature (over 750 K). The working fluids are usually chemical elements or simple organic gases, polar molecules or halocarbons (with some restrictions), and liquid metals.
Then vapour temperature drop along the overall heat pipe is very small because the small pressure drops. Thus, the boiling-condensing cycle is essentially an isothermal process. Furthermore, the temperature losses between the heat source and the vapour on the one hand and between the vapour and the heat sink on the other hand can be very small by proper design. Therefore, the first feature of the heat pipe is that it can be designed to transport heat between two heat sources with different temperature with a very small temperature loss.
In the body forces field, the condense returns to the condenser section without the wick. But the wickless gravity assisted heat pipe contained the disadvantage that the condense returns against the vapour flow and establish an entrainment limit lower than in the case of the basic heat pipe.

Heat pipes are classified in two general types: conventional heat pipes and variable conductance heat pipes. The conventional heat pipe is a device with a very high thermal conductance with no fixed operating temperature. Its temperature varies according to variations in the heat source or heat sink. But the device can be designed in order to maintain a constant temperature of a volume. It was first realized by blocking a portion of the condenser with a non-condensable gas (figure 2). For a precise control of the temperature it is needed the addition of a reservoir downstream the condenser zone. A feedback controlled variable heat pipe can realize a quasi-absolute temperature control by heating/cooling the reservoir or gas mass control. A greater temperature control is obtained for space devices [2].

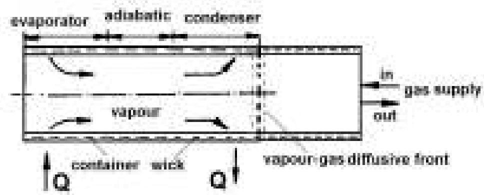

Figure 2: Variable conductance heat pipe

The most familiar variable conductance heat pipe systems include passive or active controlled system, both having the capability to control the source of heat (electronic devices, solar collectors, technologic processes, chemical reactors, etc.) at the evaporator end.

Sauciuc et al. [3] analyzed the operation of a variable conductance heat pipe for temperature control of solar collectors. The experimental results indicated that the starting point of variable conductance heat pipe is significant dependent of the amount of non-condensable gas and the superheat required for boiling.

Temperature control of the technologic processes is a slow process. The capacity of heat storage and the thermal resistances between some parts of the process are distributed along the entire heat flow.

2. MATHEMATICAL MODEL FOR THE TEMPERATURE CONTROL

A mathematical model for temperature control of the electronic components mounted in a satellite was presented in [4]. The temperature control was realized by temperature control of the reservoir, and so the non-condensable gas volume. For technologic processes, the model must to take account of the heat transfer process-heat pipe and the thermal inertia of materiel from the working space. The above physical model have not a reservoir. However, the temperature control of the technological processes in addition with a gas controlled variable conductance heat pipe was realized by two cocks: the first, by addition and the second by exhaust the gas. Thus, the gas buffer amount and subsequent the operating temperature of the heat pipe is controlled.

Supposing an exothermic process controlled by feedback variable conductance heat pipe(s). The condenser zone was in thermal contact with the cooling medium. Thus, the heat transfer is produced between the process and the cooling medium by the succession of three zones: the evaporator zone, working fluid vapour column and condenser zone of the heat pipe.

The heat flow rate transferred is:

$$\dot{Q} = G \cdot \left(T_p - T_s\right), \tag{1}$$

in which G is the global thermal conductance, T_p - the process temperature and T_s - the sink temperature.

3. DIFFERENTIAL EQUATIONS OF THE CONTROLLED SYSTEM

Thus, it can obtain four equations of thermal balance in the unsteady heat transfer regime for the process:

$$C_p \frac{\mathrm{d}T_p}{\mathrm{d}t} + G_{pe} T_p = Q_p + G_{pe} T_e, \tag{2}$$

evaporator zone:

$$C_e \frac{\mathrm{d}T_e}{\mathrm{d}t} + \left(G_{pe} + G_{ev} + G_{ec} + G_{es}\right) \cdot T_e = G_{pe} T_p + G_{ev} T_v + G_{ec} T_c + G_{es} T_s, \tag{3}$$

vapour column:

$$V_c \rho_v r_v \frac{\mathrm{d}l_a}{\mathrm{d}t} + \alpha_v \frac{\mathrm{d}T_v}{\mathrm{d}t} + \left(G_{ev} + G_{vc} l_a + G_{vg}\right) \cdot T_v = G_{ev} T_e + G_{vc} l_a T_c + G_{vg} T_s, \tag{4}$$

and condenser zone:

$$C_c T_c \frac{\mathrm{d}l_a}{\mathrm{d}t} + C_c l_a \frac{\mathrm{d}T_c}{\mathrm{d}t} + \left(G_{ec} + G_{vc} l_a + G_{cs} l_a\right) \cdot T_c = G_{ec} T_e + G_{vc} l_a T_v + G_{cs} l_a T_s , \tag{5}$$

adding an equation for mass balance of the gas inside the variable conductance heat pipe:

$$\frac{\mathrm{d}l_a}{\mathrm{d}t} = \beta_v \frac{\mathrm{d}T_v}{\mathrm{d}t} - \beta_s \frac{dT_s}{dt} - \beta_m \frac{\mathrm{d}m_g}{\mathrm{d}t} , \tag{6}$$

in which are notted:

$$\alpha_v = V_c\left(l_e + l_a\right)\left(r_v \frac{\mathrm{d}\rho_v}{\mathrm{d}T_v} + \rho_v \frac{\mathrm{d}r_v}{\mathrm{d}T_v}\right); \tag{7}$$

$$\beta_v = \frac{1 - l_a}{p_v - p_{vs}} \cdot \frac{\mathrm{d}p_v}{\mathrm{d}T_v} \cdot \frac{\mathrm{d}T_v}{\mathrm{d}t} ; \tag{8}$$

$$\beta_s = \frac{\mathrm{R} m_g}{V_c\left(p_v - p_{vs}\right)} + \frac{1 - l_a}{p_v - p_{vs}} \cdot \frac{dp_{vs}}{dT_s} ; \tag{9}$$

$$\beta_m = \frac{RT_s}{V_c\left(p_v - p_{vs}\right)} \cdot \frac{\mathrm{d}m_g}{\mathrm{d}t} . \tag{10}$$

In the transient heat transfer regime there are into account thermal capacities, C. The subsequent symbols are used too: T - temperatures, r_v - evaporation heat, ρ_v - vapour density, R - non-condensable gas constant, l_a - relative length of the active condenser region (active length and overall length of the condenser ratio), V_c - maximum volume of the condenser, p_v, p_{vs} - vapour pressure of the working fluid in the active and respectively blocked zone of the condenser, m_g - mass of gas. Subscripts are: p - process, e - evaporator zone, a - adiabatic zone, c - condenser zone, g - gas, s - sink.

The system of equations that describes this process is linear if it is considered that factors of main variables are constants having the values from the steady state regime.

Applying the Laplace transformation and considering null initial condition it is obtained a system with five linear equations having the unknown X_e, X_v, X_c, X_l, representing the Laplace's transform of the evaporator, vapour, condenser zones, and respectively the active length of the condenser. The regulated value of the temperature T_p is represented in the complex plane by X_p, the direct regulated magnitude (mass of gas) represented by X_m while disturbances are considered the heat flow of the exothermic process Q_p and the sink temperature T_s represented by X_Q and respectively X_s. The complex variable is s.

$$\left(C_p s + G_{pe}\right) \cdot X_p - X_Q = G_{pe} X_e , \tag{11}$$

$$\left(C_e s + G_{pe} + G_{ev} + G_{ec} + G_{es}\right) \cdot X_e = G_{pe} X_p + G_{ev} X_v + G_{ec} X_c + G_{es} X_s , \tag{12}$$

$$\left(V_c \rho_v r_v\right) \cdot s \cdot X_l + \left(\alpha_v \cdot s + G_{ev} + G_{vc} \cdot l_a + G_{ec} + G_{vg}\right) \cdot X_v = G_{pe} X_p + G_{ev} X_v + G_{ec} X_c + G_{es} X_s , \tag{13}$$

$$\left(C_c T_c\right) \cdot s \cdot X_l + \left(C_c l_a \cdot s + G_{ec} + G_{vc} \cdot l_a + G_{cs} l_a\right) \cdot X_c = G_{ec} X_e + G_{vc} l_a X_v + G_{cs} l_a X_s , \tag{14}$$

$$s \cdot X_l = \beta_v \cdot s \cdot X_v - \beta_s \cdot s \cdot X_s - \beta_m \cdot s \cdot X_m , \tag{15}$$

The coefficients of variables X_p, X_Q, X_e, X_v, X_c, X_s, X_m and X_l are transfer functions. The above equations can be arranged in the form:

$$X_p = H_{e1} X_e + H_{q1} X_q , \tag{16}$$

$$X_e = H_{p2} X_p + H_{v2} X_v + H_{c2} X_c + H_{s2} X_s , \tag{17}$$

$$X_v = H_{e3} X_e + H_{c3} X_c + H_{s3} X_s - H_{l3} X_l , \tag{18}$$

$$X_c = H_{e4} X_e + H_{v4} X_p + H_{s4} X_s - H_{l4} X_l , \tag{19}$$

$$X_l = H_{v5} X_v - H_{s5} X_s - H_{m5} X_m , \tag{20}$$

Eliminating the unknown X_l, X_e, X_v and X_c it obtain the characteristic equation of the process that operating in open circuit:

$$X_p = A \cdot X_m + B \cdot X_Q + C \cdot X_s , \tag{21}$$

in which coefficients are:

$$A = \frac{\left(m_1 s + m_0\right) \cdot s}{n_4 s^4 + n_3 s^3 + n_2 s^2 + n_1 s + n_0} ; \tag{22}$$

$$B = \frac{q_3 s^3 + q_2 s^2 + q_1 s + q_0}{n_4 s^4 + n_3 s^3 + n_2 s^2 + n_1 s + n_0}; \quad (23)$$

$$C = \frac{t_2 s^2 + t_1 s + t_0}{n_4 s^4 + n_3 s^3 + n_2 s^2 + n_1 s + n_0}. \quad (24)$$

The coefficients of polynomial are calculated with equation:

$$\begin{aligned} & n_4 = e_1 d_1;\ n_3 = e_2 d_1 + e_1 d_2 - e_{10} d_3;\ n_2 = e_3 \cdot d_1 + e_2 d_2 - e_{10} d_4 - e_{11} d_3; \\ & n_1 = e_4 d_1 + e_3 d_2 - e_{10} d_5 - e_{11} d_4;\ n_0 = e_4 d_2 - e_{11} d_5; \\ & m_1 = d_{10} e_{10} + d_1 e_{12};\ m_0 = d_{10} e_{11} + d_2 e_{12}; \\ & q_3 = d_1 e_5;\ q_2 = d_1 e_6 + d_2 e_5 + d_6 e_{10};\ q_1 = d_1 e_7 + d_2 e_6 + d_7 e_{10} + d_6 e_{11};\ q_0 = d_2 e_7 + d_7 e_{11}; \\ & t_2 = e_8 d_1 + e_{10} d_8;\ t_1 = e_9 d_1 + e_8 d_2 + e_{10} d_9 + e_{11} d_8;\ t_0 = e_9 d_2 + e_{11} d_9. \end{aligned} \quad (25)$$

The constants $d_1 ... d_{10}$ and $e_1 ... e_{12}$ are calculated with relations:

$$\begin{aligned} & d_1 = b_1;\ d_2 = b_2 + \frac{b_6 a_7}{a_8};\ d_3 = \frac{b_6 a_1}{a_8};\ d_4 = b_3 + \frac{b_6 a_2}{a_8};\ d_5 = b_4 + \frac{b_6 a_3}{a_8};\ d_6 = -\frac{b_6 a_4}{a_8}; \\ & d_7 = b_5 - \frac{b_6 a_5}{a_8};\ d_8 = b_7;\ d_9 = b_8 + \frac{b_6 a_6}{a_8};\ d_{10} = b_9; \end{aligned} \quad (26)$$

$$\begin{aligned} & e_1 = \frac{c_1 a_1}{a_8};\ e_2 = \frac{c_1 a_2 + c_2 a_1}{a_8};\ e_3 = \frac{c_1 a_3 + c_2 a_2}{a_8} - c_3;\ e_4 = \frac{c_2 a_3}{a_8} - c_4;\ e_5 = \frac{c_1 a_4}{a_8};\ e_6 = \frac{c_1 a_5 + c_2 a_4}{a_8}; \\ & e_7 = \frac{c_2 a_5}{a_8} + c_5;\ e_8 = \frac{c_1 a_6}{a_8} + c_8;\ e_9 = \frac{c_2 a_6}{a_8} + c_9;\ e_{10} = \frac{c_1 a_7}{a_8} + c_6;\ e_{11} = \frac{c_2 a_7}{a_8} + c_7;\ e_{12} = c_{10}, \end{aligned} \quad (27)$$

Constants $a_1 ... a_8$, $b_1 ... b_9$ and $c_1 ... c_{10}$ are calculated with relations:

$$\begin{aligned} & a_1 = \frac{C_e C_p}{G_{pe}};\ a_2 = C_e + \frac{C_p \left(G_{pe} + G_{ev} + G_{ec} + G_{es} \right)}{G_{pe}};\ a_3 = 2G_{pe} + G_{ev} + G_{ec} + G_{es}; \\ & a_4 = \frac{C_e}{G_{pe}} - c_4;\ a_5 = \frac{G_{pe} + G_{ev} + G_{es}}{G_{pe}};\ a_6 = G_{es};\ a_7 = G_{ev};\ a_8 = G_{ec}; \end{aligned} \quad (28)$$

$$\begin{aligned} & b_1 = \alpha_v + V_c \rho_v r_v \beta_v;\ b_2 = G_{ev} + G_{vc} l_a + G_{vg};\ b_3 = \frac{G_{ev} C_p}{G_{pe}};\ b_4 = G_{ev};\ b_5 = -\frac{G_{ev}}{G_{pe}}; \\ & b_6 = G_{vc} l_a;\ b_7 = V_c \rho_v r_v \beta_s;\ b_8 = G_{vg};\ b_9 = V_c \rho_v r_v \beta_m; \end{aligned} \quad (29)$$

$$\begin{aligned} & c_1 = C_c l_a;\ c_2 = G_{ec} + G_{vc} l_a + G_{cs} l_a;\ c_3 = \frac{G_{ec} C_p}{G_{pe}};\ c_4 = G_{ec};\ c_5 = -\frac{G_{ec}}{G_{pe}}; \\ & c_6 = -C_c T_c \beta_v;\ c_7 = G_{vc} l_a;\ c_8 = C_c T_c \beta_s;\ c_9 = G_{cs} l_a;\ c_{10} = C_c T_c \beta_m. \end{aligned} \quad (30)$$

It can observe that the system is represented only by three transfer functions.

The coefficients from these equations are real. Thus, these magnitudes are calculated with thermal balance equations in a steady state regime obtained from equations (2)…(10) if temporal derivatives are cancelled.

4. EQUATIONS OF THE CONTROL SYSTEM

Considering the case of an ideal proportional-integral-derivative controller device it can obtain the transient response in the complex plane chart for the reference signal, exothermic flow and sink temperature variations:

$$X_p = H_p \cdot X_p^r + H_Q \cdot X_Q + H_S \cdot X_S, \quad (31)$$

in which coefficients (transfer functions) are:

$$H_p = \frac{k_P k_E m_1 \left(s + \frac{m_0}{m_1} \right) \cdot \left(s + k_{0T} \right) \left(T_D s^2 + s + k_I \right)}{p_5 s^5 + p_4 s^4 + p_3 s^3 + p_2 s^2 + p_1 s + p_0}; \quad (32)$$

$$H_Q = \frac{\left(s + k_{0T} \right) \left(q_3 s^3 + q_2 s^2 + q_1 s + q_0 \right)}{p_5 s^5 + p_4 s^4 + p_3 s^3 + p_2 s^2 + p_1 s + p_0}; \quad (33)$$

$$H_S = \frac{(s+k_{0T})\cdot(t_2 s^2 + t_1 s + t_0)}{p_5 s^5 + p_4 s^4 + p_3 s^3 + p_2 s^2 + p_1 s + p_0}. \tag{34}$$

in which X_p^r is the Laplace's transform of the reference input. Coefficients of characteristic equation are:

$$\begin{aligned} &p_5 = n_4;\ p_4 = n_3 + k_{0T} n_4;\ p_3 = n_2 + k_{0T} n_3 + k_T k_P k_E m_1 T_D;\ p_2 = n_1 + k_{0T} n_2 + k_T k_P k_E (m_1 + m_0 T_D); \\ &p_1 = n_0 + k_{0T} n_1 + k_T k_P k_E (m_0 + m_1 k_I); \\ &p_0 = k_T k_P k_E m_0 k_I + k_{0T} n_0, \end{aligned} \tag{35}$$

where k_P is the proportional control factor (factor range ratio), k_I - integrating constant, T_D - derivative time of the control device, k_E - proportional constant of the actuator, k_T - proportional constant of the temperature transducer and k_{0T} - inverse of the time constant of the transducer.

It can observe that the tranzient response in the complex plane can be obtained by three transfer functions applyied on the reference signal X_p^r, disturbance created by the exotermic reaction heat flow X_Q, and respectively by the sink temperature variation X_S.

5. INVESTIGATION OF THE ANALYTIC CONTROL

The characteristic equation of the system is the denominator from equations (32)...(34), and poles of transfer functions are obtained by .

After some trials for real cases, the above equation can have two real roots and two complex conjugated roots, or all roots are real. The system stability is always assured because all roots of the characteristic equation are in the the left half-plane. Applying the theorem of limit in complex, the stationary deviation at an unitary step-load change is null (if it is neglected the disturbance) and is finite for each deviation.

It is proposed an example of the tranzient response of the system for an unitary step-load signal applyied to the reference signal and dispurbances, for example: k_r/s, k_q/s and k_s/s. It is obtained:

$$X_p = H_p \cdot \frac{k_r}{s} + H_Q \cdot \frac{k_Q}{s} + H_S \cdot \frac{k_S}{s}, \tag{36}$$

This equation is decomposed in simple fractions and then, applying the inverse Laplace's transformation it can obtain the transient response of the system:

$$\begin{aligned} T_p = &(c_{p0} + c_{Q0} + c_{S0}) + (c_{p1} + c_{Q1} + c_{S1})\cdot e^{x_{p1}t} + (c_{p2} + c_{Q2} + c_{S2})\cdot e^{x_{p2}t} + (c_{p3} + c_{Q3} + c_{S3}2)\cdot e^{x_{p3}t} + \\ &+ (c_{p4} + c_{Q4} + c_{S4})\cdot e^{x_{p4}t} + (c_{p5} + c_{Q5} + c_{S5})\cdot e^{x_{p5}t}, \end{aligned} \tag{37}$$

in which $x_{p1} \ldots x_{p5}$ are the roots of characteristic equations (if they are real), and coefficients are obtained by decompozition in partial fractions. If two from the three roots of the characteristic equation are complex conjugated the tranzient response in not aperiodic.

For the numerical analysis of the control is realized a computer program in order to solving the transfer function. The real roots of polynomial are obtained with Bierge-Viete algorithm.

Figure 3 presents the transient response of the system controlled with a variable conductance heat pipe in open loop. It can remark that the transient response is too slow.

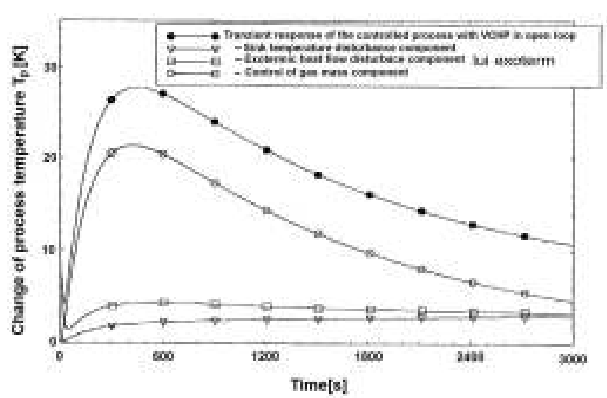

Figure 3: Transient response of the system with variable conductance heat pipe operating in open loop

Figure 4 presents a transient response of the system with a feedback controlled variable conductance heat pipe.

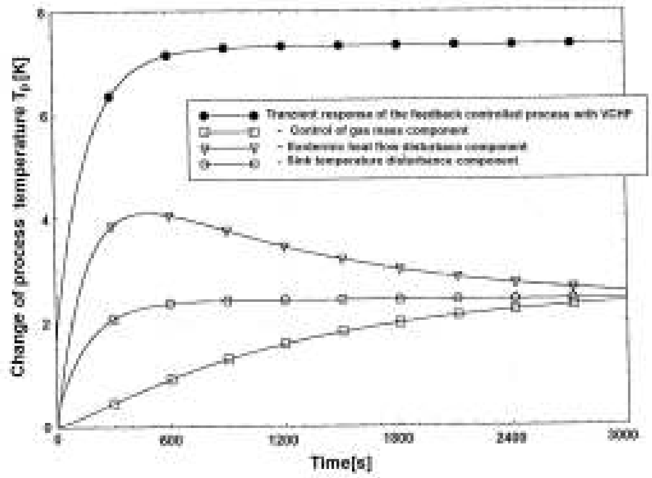

Figure 4: Transient response of the system with variable conductance heat pipe operating in open loop

If compare results from figures 3 and 4 it can observe improved performances of the system provided with variable conductance heat pipe. Thus, the system obtains high precision (from 30K to 8K) and reaction velocity (after 600s the regime is right established). Also, it can remarks an increase of the productivity because the control actuates on heat transfer surface and no affects the process speed progress.

3. CONCLUSION

Temperature control of the technologic processes is a slow process. The capacity of heat storage and the thermal resistances between some parts of the process are distributed along the entire heat flow. The temperature control of the technological processes in addition with a gas controlled variable conductance heat pipe was realized by two cocks: the first, by addition and the second by exhaust the gas.

It is obtained four equations of thermal balance in the unsteady heat transfer regime for the process. The system of equations that describes this process is linear if it is considered that factors of main variables are constants having the values from the steady state regime. Applying the Laplace transformation and considering null initial condition it is obtained a system with five linear equations and the system is represented only by three transfer functions. The coefficients from these equations are real. Considering the case of an ideal proportional-integral-derivative controller device it can obtain the transient response in the complex plane chart for the reference signal, exothermic flow and sink temperature variations.

It can observe that the tranzient response in the complex plane can be obtained by three transfer functions applyied on the reference signal, disturbance being created by the exotermic reaction heat flow, and respectively by the sink temperature variation.

This equation is decomposed in simple fractions and then, applying the inverse Laplace's transformation it can obtain the transient response of the system.

A greater temperature control is obtained using the feedback variable conductance heat pipes than the passive system.

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