3rd International Conference
″Research & Innovation in Engineering ″
COMAT 2014
16-17 October 2014, Braşov, Romania

MATHEMATICAL MODEL OF A SPECIAL VEHICLE CLUTCH SERVOMECHANISM

Vilău Radu[1,A *], Alexa Octavian [1,B]

[1]Military Technical Academy, Faculty of Mechatronics and Integrated Systems for Armament, Dept. of Military Automotive Engineering and Transportation, Bd. George Cosbuc nr. 39-49, sector 5, Bucharest 050141, Romania
[a *]radu.vilau@yahoo.com, [b] alexa.octavian@gmail.com

Abstract. *The working characteristic features of a process can be synthetically expressed as a mathematical model. For the technical systems, this model can be obtained based on the mathematical expressions that describe the working way of the system's components. The mathematical model can be used to analyse the dynamic performances of the system or a proper way to improve them, as far as the actual evolution of the system is accurately enough described. The paper makes an analysis using modelling and simulating techniques of a special clutch servomechanism. The modern simulation software allows studying the complex automotive systems; at the same time allows the fast variation of the system's parameters and the study of all modifications that appear within the working system.Some of dynamic charts are presented and improvement of the dynamic performances methods are given.*
Keywords: *Mathematical model, simulation, servomechanism, dynamic performances.*

1. INTRODUCTION

This paper deals with the analysis of a servomechanism working on the main clutch of a tracked vehicle's transmission. The main clutch ensures a smooth and progressive coupling of the engine to the gearbox as well as a fast de-coupling of the previously mentioned components. It also works as a safety device [1], starting to slip whenever the drag at the vehicle's driving wheels generates a coasting torque that exceeds the engine's output. The clutch is placed on the input shaft of the gearbox and it is referred as "the main clutch". The main clutch is a multi-disc type (it consists of two driving discs and three driven ones). It is also a "dry clutch" and its clutching (engaging/disengaging) mechanism is either hydraulic or pneumatic (when the hydraulic system fails). The main components of this clutch can be seen in fig. 1. The main hydraulic distributor is depicted in fig. 2.

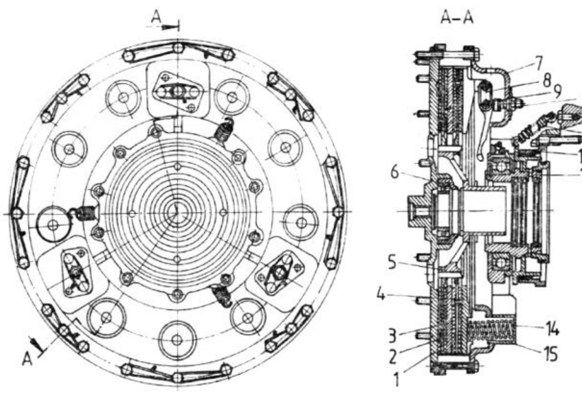

Figure 1 - The main clutch:
1-front plate; 2-friction discs; 3-intermediate plate; 4-plate; 5-friction discs' hub; 6-rolling bearing; 7-case; 8-clutching lever; 9-fork; 10-adjusting screw; 11-gearbox's case; 12-restoring spring; 13-piston; 14-springs; 15-cup; 16-cylinder.

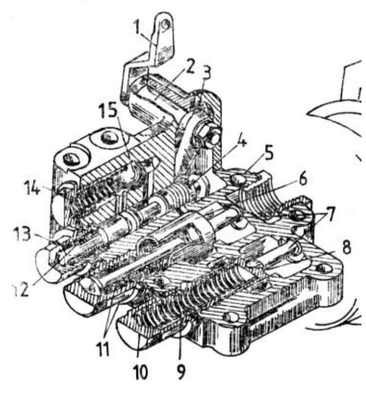

Figure 2 - The main distributor:
1–external command lever of the clutch; 2–shaft; 3–internal command lever; 4–spool valve; 5–body; 6–command system's valve; 7-guidance rod; 8–lubrication system's valve; 9–spring; 10–breaker; 11–gasket; 12–spring; 13–gasket; 14–spring; 15-ball;

The main distributor is in charge with distributing the working fluid, keeping the needed oil pressure within the hydraulic system, providing the power to clutching the main clutch, feeding the gear-shifting power system and providing a 0,2 MPa of oil pressure within the lubricating system of the gearbox. It is placed on the top of the gearbox's upper case.

To providing a smooth engagement of the main clutch, the driver should release the clutch pedal half-way of its whole travel and hold it in this position for about 4...5 seconds. In this situation, the plunger of the main distributor 2 reaches position II and allows the discharge of the oil pressure towards the oil tank via the smooth engaging valve. Hence, the oil pressure within the clutching system drops with a small gradient. When the clutch pedal is completely released, the plunger moves again, the distributor reaches position I, the oil is driven to the gearbox's case and the clutch is completely engaged under the springs' force.

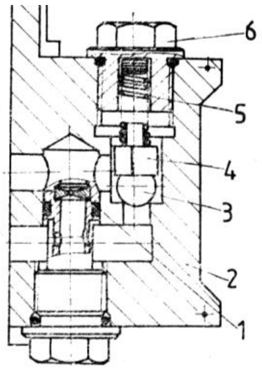

Figure 3 - Smooth engaging valve

The smooth engaging valve is depicted in figure 3, where: 2 - body, 1- throttle and ball valve, 4 – valve seat, 5 - spring. Within the oil circuit an additional flow-resistance is given by the throttle, which allows the command system's pressure smoothly drop.

2. MATHEMATICAL MODEL

The mathematical model of the hydraulic servomechanism has been issued taking into account the following equations

- the continuity equation, corresponding to the non-permanent motions within the acting hydraulic systems;
- the equation of the displacement - flow rate - pressure;
- the equation of the hydraulic engine piston's movement.

The hydraulic servomechanisms are mounted using assembling elements. The elasticity of these elements influences both the stability and the positioning accuracy of the servomechanism. If not considering the finite stiffness of these components, then the servomechanism is as good as ideal mounted (the stiffness of all the mounting elements to the supporting structure and to the actuated system is considered to be infinite). In real life, the stiffness is finite. Moreover, it slowly decreases due to the loads the mounting components have to bear.

Considering Q [2], [3], the flow through a stream tube, the continuity equation can be written as follows:

$$\frac{A\cdot\rho}{E}\cdot\frac{dp}{dt}+\rho\cdot\frac{\partial Q}{\partial s}=0 \tag{1}$$

If further processed, it leads to:

$$\begin{aligned}\frac{dp}{dt}&=-\frac{E}{A}\cdot\frac{\partial Q}{\partial s}\\ \frac{dp}{dt}&=\frac{E}{\vartheta}\cdot\left(Q_1-Q_2\right)\end{aligned} \tag{2}$$

where:

- Q_1 – liquid flow through first (inlet) section;
- Q_2 - liquid flow through second (outlet) section;
- A – flow cross-section;
- υ - liquid system's volume;
- s – displacement (travel);
- t- time;
- p – liquid system's pressure;
- E – elasticity modulus;
- ρ - oil density

Liquid's elasticity modulus ranges between 7000 and 14000 bar. It reaches 14000 bar when the oil is free of dissolved gases

The continuity equation is applied to the "liquid system" which consists of both variable volumes of the hydraulic distributor and the hydraulic motor. It also subsumes the connection pipes between the distributor and the motor.

The flow Q_1, delivered by the motor's distributor [2], [3], generates the motor's piston displacement. It also completes internal and external losses as well as the extra fluid needed to compensate the liquid compression.

$$Q_1 = c_{ip} \cdot (p_1 - p_2) + c_{ep} \cdot p_1 + A_p \cdot \frac{d(z+u)}{dt} + \frac{\vartheta_1}{E_e} \cdot \frac{dp_1}{dt} \quad 3)$$

In the same time, the flow Q_2 runs out the distributor:

$$Q_2 = A_p \cdot \frac{d(z+u)}{dt} + c_{ip} \cdot (p_1 - p_2) - c_{ep} \cdot p_2 - \frac{\vartheta_2}{E_e} \cdot \frac{dp_2}{dt} \quad 4)$$

where:

- c_{ip}– internal leakage motor's coefficient;
- c_{ep} – external leakage motor's coefficient
- p_1 – pressure of the intake motor's chamber;
- p_2 - pressure of the exhaust motor's chamber;
- Ap - effective cross-section of the motor's piston;
- z – piston's rod displacement;
- u – servomechanism's body displacement;
- υ_1, υ_2 – flows through the motor's chambers and the coupling pipes;
- E_e - equivalent modulus of elasticity.

For a symmetrical hydraulic distributor, if using the hydraulic stiffness of the hydraulic motor, R_h, one could write:

$$R_h = \frac{2E_e}{\vartheta_0} \cdot A_p^2 \quad (5)$$

where: $2\upsilon_0 = \upsilon_1 + \upsilon_2$ – the initial volume of the chambers. Hence, the following continuity equation emerges:

$$Q = K_l \cdot P + A_p \cdot \frac{d(z+u)}{dt} + \frac{A_p^2}{R_h} \cdot \frac{dP}{dt} \quad 6)$$

where K_l – total flow coefficient throughout the distributor;
P - pressure drop on the hydraulic motor.

Next coefficients are defined:

a) amplifier flow factor of the distributor:

$$K_{Qx} = \frac{\partial Q_m}{\partial x} \quad (7)$$

b) flow – pressure coefficient:

$$K_{Qp} = -\frac{\partial Q_m}{\partial P} \quad (8)$$

c) pressure – displacement coefficient:

$$K_{px} = \frac{\partial P}{\partial x} = \frac{K_{Qx}}{K_{Qp}} \quad (9)$$

where Q_m is the flow taken by the hydraulic motor

The transfer function describing the dynamic behavior of the distributor is given by:

$$H_A(s) = \frac{X(s)}{I(s)} = \frac{K_A}{\frac{s^2}{\omega_n^2} + \frac{2\xi}{\omega_n} s + 1} \quad (10)$$

where:

- $\omega_n = \sqrt{\frac{K_s}{M_s}}$ - free oscillations frequency;
- $\xi = \frac{1}{2} \cdot \frac{f_s}{\sqrt{K_s \cdot M_s}}$ - damping factor;
- K_A – amplifying factor;
- M_s – spool valve mass;
- K_s – spool valve spring's coefficient of elasticity;
- f_s – fluid friction factor.

The pressure acting on the servomechanism's piston moves it with a motion law described by:

$$F = m_p \cdot \ddot{z} + K_{fv} \cdot (\dot{z} + \dot{u}) + R_c \cdot (z - v) + D_c \cdot (\dot{z} - \dot{v}) \quad (11)$$

where

- R_c – the stiffness of the connecting mechanism between the servomechanism and the load (actuated

element)
- D_c – the damping coefficient of the connecting mechanism;
- z – displacement of the actuated element;
- K_{fv} – viscous friction coefficient;
- m_p – piston's mass.

3. SIMULATING THE SERVOMECHANISM'S WORK

The main distributor's work was simulated using a special simulating software, Simulink modulus from Matlab. The simulation diagram is depicted in fig. 4. The simulation results are given in the following pictures (fig. 5-9) that mainly analyze the time histories of the pressure within the hydraulic system [4], as well as the piston's displacement, speed and acceleration. The input, independent variable, was considered the driver's action upon the clutch pedal, on the second stage of the main distributor, when the oil flows via the hydraulic throttle.

When analyzing the distributor's working way, as well as the influence of various parameters on its functioning modes, useful conclusions emerge:

- The piston's speed and acceleration rapidly become steady. We can conclude that, after the first stage of clutch's engagement (the "smooth" stage) the friction discs slowly engage.
- Using throttling, the smaller flowing cross-section, the higher system's response time
- The higher the flowing cross-sections (as in second stage), the lower the system's response time; hence, the clutch's plate will quickly engage the discs and also the pressure gradient will increase.
- The check input signal was the ramp type (assuming the clutch pedal is steadily actuated, at constant speed)
- The higher the pressure coefficient (i.e. the pressure losses due the increased pressure within the distributor are higher) the larger the oscillations' amplitude of the piston's displacement while the system's response isn't quite affected. Nevertheless, high values of the flow coefficient lead to a destabilizing trend of the system (it is a common fact for the hydraulic system that need throttling to increase their stability)
- As the losses due to the pressure increase, the piston will have larger speed oscillations. This phenomenon leads to supplementary mechanical loads due to the inertia forces.
- A low hydraulic stiffness of the working oil leads to a pressure decrease within the system. On the other hand, we could notice an increase of the piston's travel during the first stage. It can be explained by a stronger compression of the oil; this phenomenon can also be met in terms of the speed variation.

On the contrary, a high hydraulic stiffness of the working oil leads to a pressure increase within the system. The piston's displacement during the first stage decreases, but the speed's oscillation frequency becomes higher.

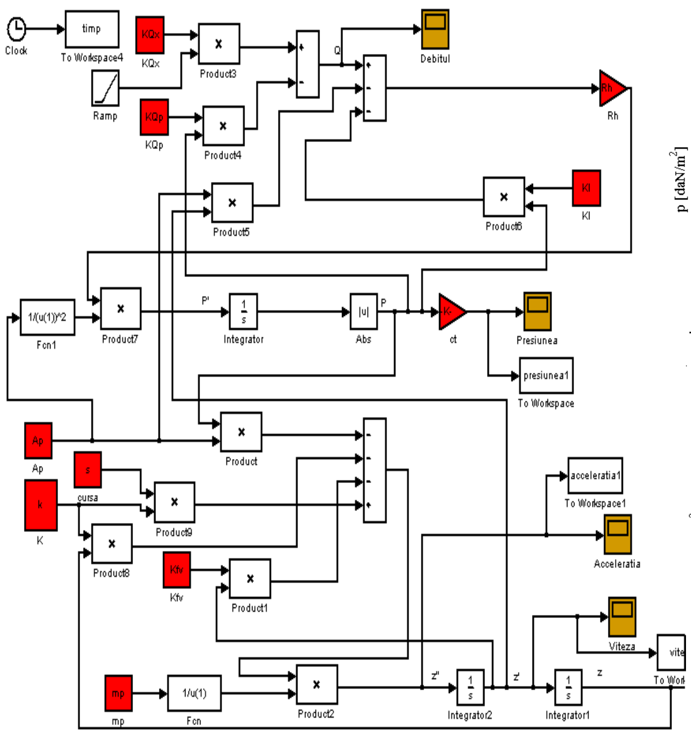

Figure 4 - Simulating chart

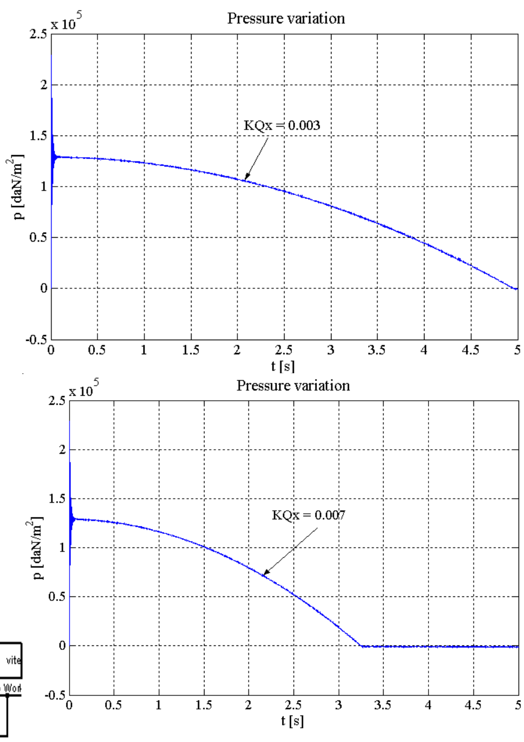

Figure 5 - The influence of the flow coefficient K_{Qx}, on the cylinder's pressure

- When the leaks on the piston are high, the pressure rapidly drops and the system' response time decreases; the systems behaves like taking advantage of a larger cross-section of the throttle.
- If the restoring springs were stiffer, the response time decreases. On the other hand, the displacement (travel) of the piston in the first stage increases due to the strong compression of the working fluid. This leads to an increased loading of the piston due to the inertia effect (both by higher frequency and amplitude of the oscillation).
- If the friction of the piston-cylinder coupling increased, the system pressure decreases. Also, the speed increase of the piston is smoother.
- Eventually, when the piston's mass is changed, the pressure waves amplitude and frequency also vary.

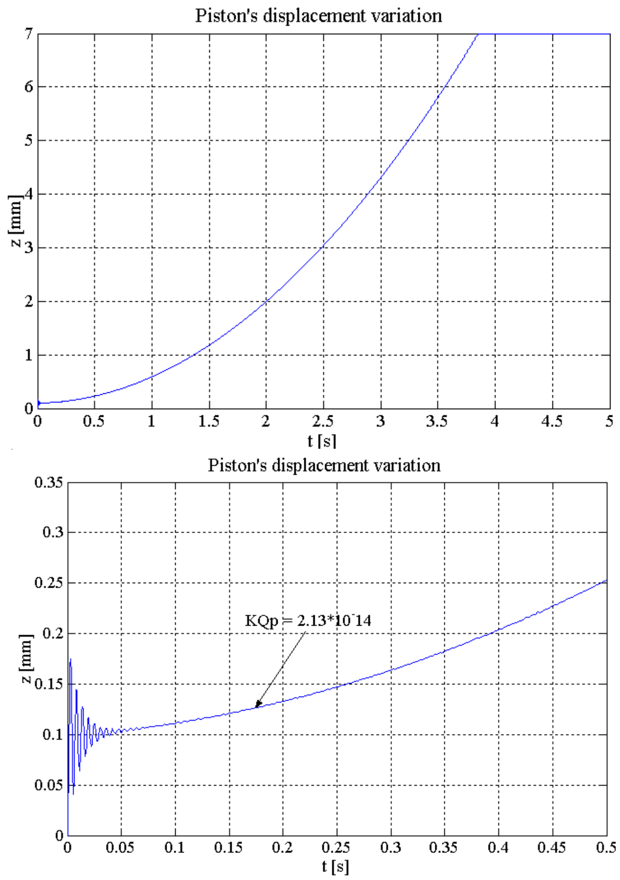

Figure 6 - The influence of the pressure coefficient K_{Qp}, on the cylinder's pressure over the

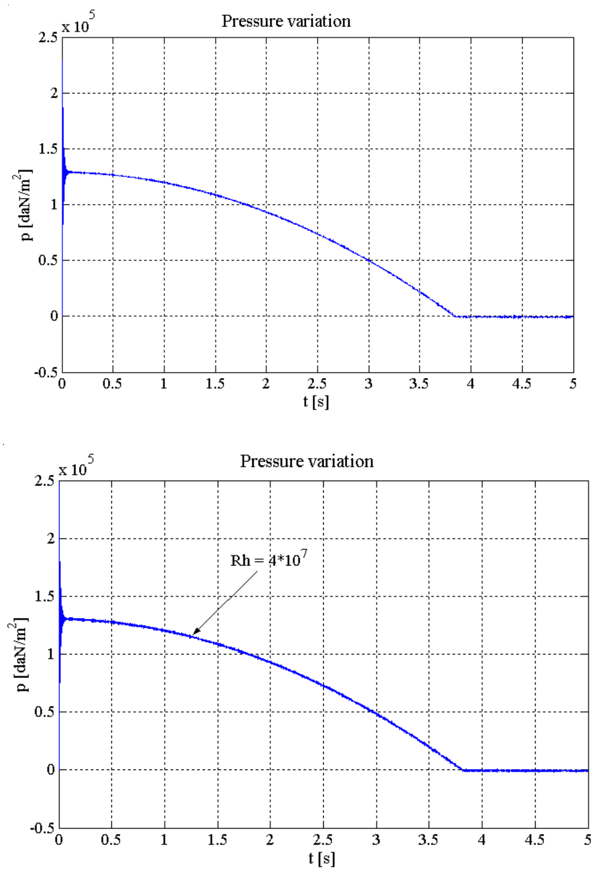

Figure 7 - The influence of the oil's hydraulic stiffness R_h over the system's pressure

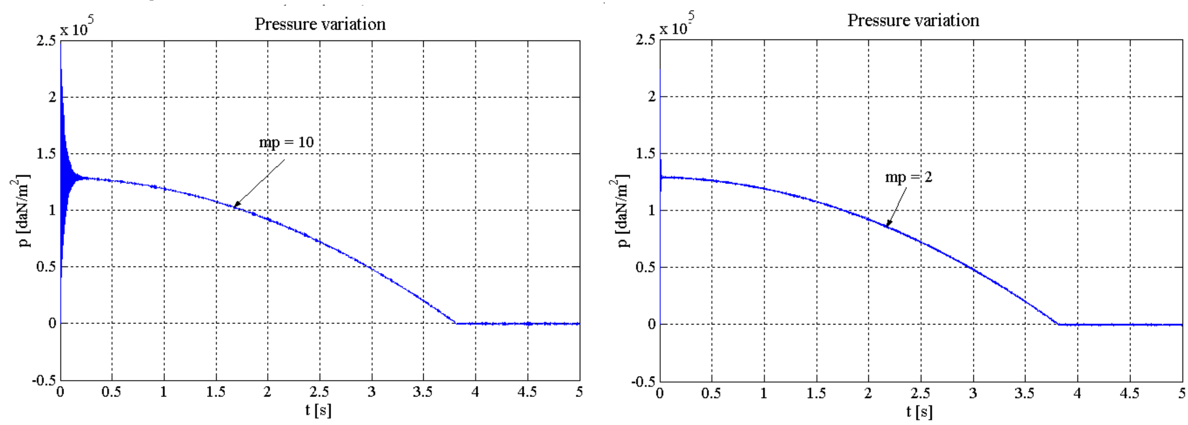

Figure 8 - The influence of the actuated load m_p over the system's pressure

From the main distributor's work analysis we could notice that, when opening the oil path via the throttle, due to the restoring springs of the clutch, the piston is strongly pushed backwards. There is a small leap of displacement and but it is due to it to assuring the necessary flow that fills the distributor's bulks. Eventually, it could lead, according to the clutch's adjustment or to a fast-developing stage, to sudden engagements of the friction discs. After that, the flow becomes steady then drops slower. High masses of the actuated elements lead to the occurrence of the pressure oscillations from the very beginning of the process. The displacement leap occurs due the commuting the flowing process through one port to another. To avoiding this drawback, a continuously variable cross-section throttle can be used. The speed and acceleration of the piston rapidly stabilize; hence, after the first stage is completed, the rest of the engagement is gradually achieved.

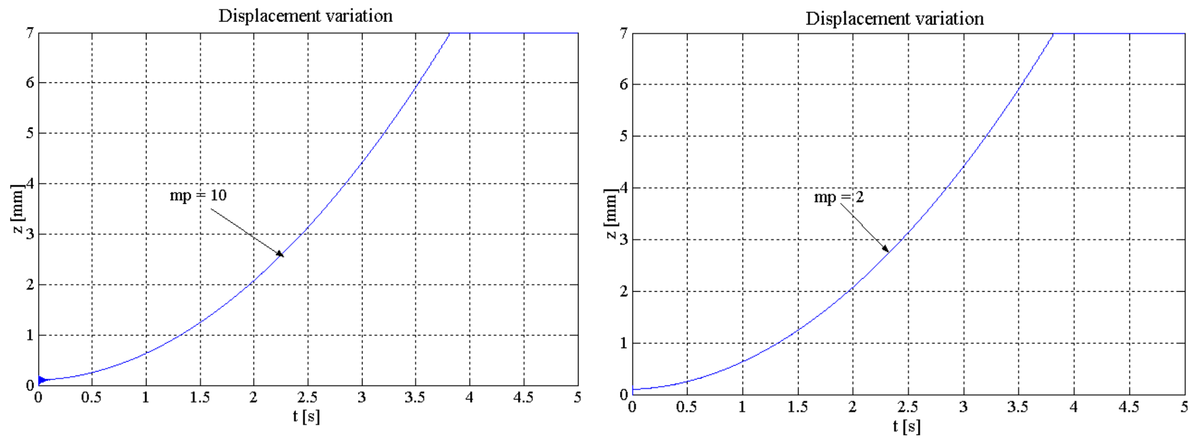

Figure 9 - The influence of the actuated load m_p over the system's r esponse

REFERENCES

[1] Costache, D. - Actionari hidraulice si pneumatice la autovehicule - Editura Academiei Militare, Bucuresti, 1985.
[2] Oprean, A. ş.a. – Acţionări şi automatizări hidraulice. Modelare, simulare, încercare, Editura Tehnică, Bucureşti, 1989;
[3] Călinoiu, C. ș.a. – Modelarea, simularea și identificarea experimentală a servomecanismelor hidraulice, Editura Tehnică, București, 1998
[4] Marinescu, M. ș.a. - Theoretical and data-based mathematical model of a special vehicle braking system, Advanced Materials Research Vol. 837 (2014) pp 428-433, ISSN:1662-8985
[5] *** - Matlab toolbox and documentation.

ACKNOWLEDGMENT

This paper has been financially supported within the project entitled ***"Horizon 2020 - Doctoral and Postdoctoral Studies: Promoting the National Interest through Excellence, Competitiveness and Responsibility in the Field of Romanian Fundamental and Applied Scientific Research ",*** contract number POSDRU/159/1.5/S/140106. This project is co-financed by European Social Fund through Sectoral Operational Programme for Human Resources Development 2007-2013. **Investing in people!**"