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ANALYTICAL AND EXPERIMENTAL DETERMINATION OF STRESSES IN THE PLANE COMPOSITE PLATES USED TO BUILD CRAFTS

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Abstract: *This work presents the mathematical method for solving the differential equations by means of which we can determine the stresses in the plane composite plates used to build crafts (the impregnation resin is NESTRAPOL 450). The results analytically determined are compared with the experimental ones.*
Keywords*: composite plates, stresses, mathematical method, build crafts.*

1. INTRODUCTION

The fundamental researches on the composite materials (with material orthotropy) are in process of development. By applying the elements of “The elastic theory” to the composite plates normally and in median plane stressed, the differential equations of strained median surfaces for deflected plate (*0,5<w<5h*) are:

$$E_x \frac{\partial^4 F}{\partial x^4} + 2E_{xy} \frac{\partial^4 F}{\partial x^2 \partial y^2} + E \frac{\partial^4 F}{\partial y^4} = E_x E_y \left[\left(\frac{\partial^2 w}{\partial x \partial y} \right)^2 - \frac{\partial^2 w}{\partial x^2} \frac{\partial^2 w}{\partial y^2} \right]$$
$$D_x \frac{\partial^4 w}{\partial x^4} + 2H \frac{\partial^4 w}{\partial x^2 \partial y^2} + D_y \frac{\partial^4 w}{\partial y^4} = h \left[\frac{p(x,y)}{h} + \frac{\partial^2 F}{\partial y^2} \frac{\partial^2 w}{\partial x^2} - 2 \frac{\partial^2 F}{\partial x \partial y} \frac{\partial^2 w}{\partial x \partial y} + \frac{\partial^2 F}{\partial x^2} \frac{\partial^2 w}{\partial y^2} \right] \quad (1)$$

By analyzing the equation system (1) and comparing it with the equation system for isotropic plate (steel or aluminum) we note the appearance of stiffness on the two directions which changes the structure of solutions.
The equation system (1) has as unknowns the stress function *F(x,y)* and the deflection *w(x,y),* which can be determined by means of the boundary conditions for various supporting ways (rigid fixing, simple or free side suspension). In the previous work, the special forms of equation system (1) have been presented, from which we are interested in particular, in the rigid plate with small deflection ($w \le 0.2h$) of the following form:

$$D_x \frac{\partial^4 w}{\partial x^4} + 2H \frac{\partial^4 w}{\partial x^2 \partial y^2} + D_y \frac{\partial^4 w}{\partial y^4} = p(x,y) \quad (2)$$

The presence of a small strain of the plate means that the ship’s shape changes very little due to the water action, the stream lines don’t change very much and the heading resistance doesn’t increase very much due to the hull ‘s strain.

The mathematical resolution of differential equation system (1) is possible only in particular cases. So, it is necessary a careful analysis of strength structure of ship and her skin. Taking into account only the local loading, the ship’s strength structure is formed both by keelsons, girders and lines alongside and floors, frames and beams athwart wise forming a network on which the ship’s skin is fixed. I consider the plate mesh, between the stiffening members, stressed by water pressure, being rigid with a small deflection ($w \le 0.2h$) where the sectional stresses N_x, N_y, N_{xy} don’t influence the bending. In this case, the equation system (1) under the form of (2) represents a linear differential equation with partial derivates and constant coefficients.
To determine the stress and strain conditions in the plate mesh resulted from the local loading, means to find a function *w* which to check the differential equation (2) and in the same time the boundary conditions depending on the supporting pattern.

2. THE ANALYTICAL RESOLUTION OF DIFFERENTIAL EQUATIONS

We consider the general case when the plate is of *a x b x h*, simply supported on the contour line, normally loaded with *p(x,y)* varying on both directions. The system of axes is like in Figure 1.

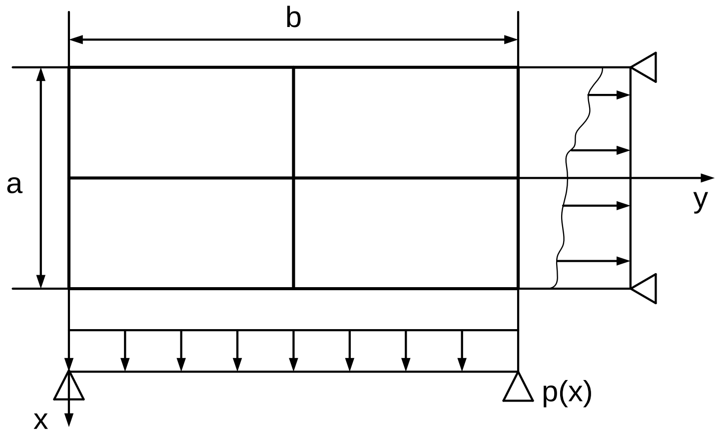

Figure 1. Plate loaded with a load distributed on the surface varying on both directions.

It is developed the normal load *p(x,y)* in double Fourier's series:

$$p(x,y)=\sum_m\sum_n p_{mn}\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b} \qquad \text{m, n = 1,2,3,} \tag{3}$$

The parameters p_{mn} are determined by Euler's method and become:

$$p_{mn}=\frac{4}{ab}\int_0^a\int_0^b p(x,y)\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b} \qquad \text{m, n = 1,3,5,...} \tag{4}$$

For the load uniformly distributed p_o the parameters p_{mn} become:

$$p_{mn}=\frac{16p_0}{\pi^2 mn} \qquad \text{m, n=1,3,5, ...} \tag{5}$$

The strain function (deflection) is also developed in Fourier's series under the form of:

$$w(x,y)=\sum_m\sum_n w_{mn}\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b} \tag{6}$$

The coefficients w_{mn} are determined from the condition that the expression (6) to satisfy the differential equation of the plate (2) for any values x, y and the boundary conditions on the contour line (for the plate simply supported $w = 0$ and $M_x = M_y = 0$) and it is obtained:

$$w_{mn}=\frac{16p_0}{\pi^6 mn}\frac{1}{D_x\frac{m^4}{a^4}+2H\frac{m^2n^2}{a^2b^2}+D_y\frac{n^4}{b^4}} \tag{7}$$

The expression of deflection (6) for the particular case when the load $p(x,y) = p_0$, the case of bottom plates of the ship, becomes:

$$w(x,y)=\frac{16p_0}{\pi^6}\sum_m\sum_n\frac{\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}}{mn\left(D_x\frac{m^4}{a^4}+2H\frac{m^2n^2}{a^2b^2}+D_y\frac{n^4}{b^4}\right)} \tag{8}$$

The expressions of sectional moments become:

$$M_x=\frac{16p_0}{\pi^4}\sum_m\sum_n\frac{\left(D_x\frac{m^2}{a^2}+D_1\frac{n^2}{b^2}\right)\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}}{mn\left(D_x\frac{m^4}{a^4}+2H\frac{m^2n^2}{a^2b^2}+D_y\frac{n^4}{b^4}\right)} \tag{9}$$

$$M_y = \frac{16p_0}{\pi^4}\sum_m\sum_n \frac{\left(D_y\frac{n^2}{b^2}+D_1\frac{m^2}{a^2}\right)\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}}{mn\left(D_x\frac{m^4}{a^4}+2H\frac{m^2n^2}{a^2b^2}+D_y\frac{n^4}{b^4}\right)} \qquad m, n = 1,3,5,\ldots \tag{10}$$

$$M_{xy} = -\frac{32p_0}{\pi^4 ab}\sum_m\sum_n \frac{D_{xy}\cos\frac{m\pi x}{a}\cos\frac{n\pi y}{b}}{\left(D_x\frac{m^4}{a^4}+2H\frac{m^2n^2}{a^2b^2}+D_y\frac{n^4}{b^4}\right)} \tag{11}$$

The expressions of shearing forces become:

$$T_x = \frac{16p_0}{\pi^3}\sum_m\sum_n \frac{\left[D_x\left(\frac{m}{a}\right)^3+H\frac{m}{a}\left(\frac{n}{b}\right)^3\right]\cos\frac{m\pi x}{a}\sin\frac{n\pi y}{b}}{mn\left(D_x\frac{m^4}{a^4}+2H\frac{m^2n^2}{a^2b^2}+D_y\frac{n^4}{b^4}\right)} \tag{12}$$

$$T_y = \frac{16p_0}{\pi^3}\sum_m\sum_n \frac{\left[D_y\left(\frac{n}{b}\right)^3+H\frac{n}{b}\left(\frac{m}{a}\right)^3\right]\sin\frac{m\pi x}{a}\cos\frac{n\pi y}{b}}{mn\left(D_x\frac{m^4}{a^4}+2H\frac{m^2n^2}{a^2b^2}+D_y\frac{n^4}{b^4}\right)} \tag{13}$$

The maximum deflection is produced at the middle of the plate, that is, in the coordinate point x = *a/2* and *y* = *b/2*, and in this case the relation becomes:

$$w_{\max} = \frac{16p_0}{\pi^6}\sum_m\sum_n \frac{(-1)^{\frac{m+n}{2}-1}}{mn\left(D_x\frac{m^4}{a^4}+2H\frac{m^2n^2}{a^2b^2}+D_y\frac{n^4}{b^4}\right)} \qquad m,n = 1,3,5,\ldots \tag{14}$$

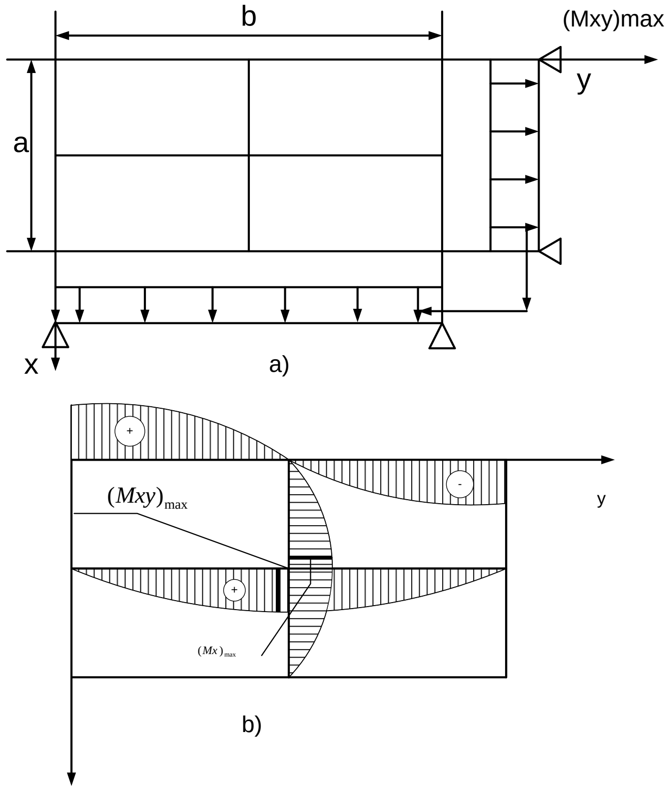

Figure 2. The rectangular orthotropic plate loaded with a normal load uniformly distributed:
a) The plate simply supported loaded with p*(x,y)* = p_0 = ct.
b) The diagram of stresses in the rectangular plate simply supported.

The numerical results are listed in Table 1. The calculus was performed for the first three terms.

3. EXPERIMENTAL RESULTS AND CONCLUSION

To check the value of maximum deflection obtained by the theoretical methods mentioned above, we built a device by means of which we measured the maximum deflection in the middle of the plate. The device is formed of two rigid angle bar frames by means of which we performed the fixing and with only one frame we made the support on the sides. The loading was made with fine, dry sand with a density of $\rho = 1.3$ kg/dm^2 . The thickness of sand layer was calculated from the condition of loading with a load uniformly distributed p = 3000N/m^2 . The deflection was measured in the middle of the plate by a comparator. The comparation between the calculated values and the measured ones for the five plied laminar is shown in Table 1.
We conclude that the methods of resolution can be divided into:

- approximative analytical methods (energetically) : when the unknown function, w, is approximated, from energetically reasons, satisfying both the system and the supporting conditions on the contour line. They are: the orthogonally method, Ritz, Rayleigh-Ritz, Bubnov-Galerkin, Treftz, etc.
- approximative numerical methods: the finite element method or the finite difference method.

Both methods offer the possibility of determining the efforts and good results with acceptable approximations.

Table 1. Experimental results.

Method of determination	Maximum analyzed values					
	w_{max} [mm]	M_x [mm/mm]	My[mm/mm]	M_{xy} [mm/mm]	σ_x[mm/mm]	Σ_y [mm/mm]
The plate simply supported						
Experimental	4,2	43,601	11,91	-6,91	14,15	3,86
Double trigonometric	4,383	-	-	-	-	-
Simple Trigonometric	1,413	44,157	16,31	-5,44	-	-
Ritz method	4,18	44,8	13,2	-8,31	14,57	4,278
MEF (COSMOS M program)	4,424	-	-	-	-	-
MEF (ALGOR program)	4,205	-	-	-	-	-
The fixed plate						
Experimental	1,19	-	-	-	-	-
MEF (COSMOS M program)	0,961	16	3,44	-	5,91	1,12

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