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MATHEMATICAL MODELS OF FORCE AND MOMENT IN MACHINING PRODUCTS MADE BY SANDWICH COMPOSITES POLYMERIC MATERIALS

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Abstract: *In industrial application of products made by sandwich composites polymeric materials it is important to determine mathematical models of force and torque, when machining this. Thus, it would be possible to know optimum process parameters values or to predict force and torque values, once process parameters values set. Machining, more specifically milling process, for products made by materials sandwich composites polymeric, the existing data published on this topic are very poor and, rather, quantitative ones. So, a study on products made by materials sandwich composites polymeric has been done and the results presented by this paper, mainly considering the wide industrial application of this material in Romanian industry.*
Keywords: *mathematical model, regressions, milling process, force, moment.*

1. INTRODUCTION

In the 50's the development was mainly concentrate on honeycomb materials. Honeycomb was mainly used as core material in the aircraft industry. However, it had some limitations, for example there big problems with corrosion, see figure 1. At the end of 50's and during the 60's and until this days different cellular plastics where produced, suitable as core materials [1, 2, 3].
In the beginning rather soft materials were used because of their insulation properties, for example polystyrene and polyurethane, see figure 2. Later it was possible to produced harder cellular plastics with higher densities and by that time sandwich became a very useful and flexible concept, see figure 3.
In this paper the sandwich composite polymeric products are made of fiber glass and the core is polystyrene extruded, see figure 3.

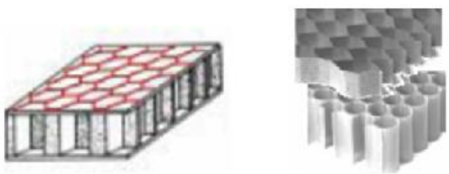

Figure 1: Example of honeycomb core

Figure 2: Example of polystyrene or polyurethane core

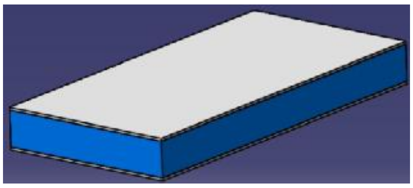

Figure 3: Example of polystyrene extruded core

The combination of two composite faces and a lightweight core allows obtaining a high flexural stiffness with a weak mass. The faces carry most of the tensile and compressive stress due to axial loading and bending whereas the core carries most of the shear stress. A priori, the weaving of the sandwich structures should constitute a non negligible value-added for the development of these structures. Every part has its specific function to make it work as a unit.

The aim is to use the material with a maximum efficiency. Two faces are placed at a distance from each other to increase the moment of inertia, and thereby the flexural rigidity, about the neutral axis of the structure [4,5,6].

2. RESEARCH APPLIED METHODS

Determining mathematical models for the relationship of parameters specific to a certain machining process, based on experimental results, involves some steps to be followed, such as: -the "definition" of both independent and dependent variables associated to this models [1]; -an appropriate experiments design type to be considered; -the regression analysis, if available; -the fitted mathematical model to be obtained.

The mathematical models for axis cutting force and torque for milling process are mentioned by most of the articles and books dealing with this problem, and are represented in relations (1) to (6).

- in milling process

$$F_x = f(v_c, v_f, a_e, a_p), \quad [N] \tag{1}$$

$$F_x = C_{Fx} \cdot v_c^{a1} \cdot v_f^{a2} \cdot a_e^{a3} \cdot a_p^{a4}, \quad [N] \tag{2}$$

$$F_y = f(v_c, v_f, a_e, a_p), \quad [N] \tag{3}$$

$$F_y = C_{Fy} \cdot v_c^{a1} \cdot v_f^{a2} \cdot a_e^{a3} \cdot a_p^{a4}, \quad [N] \tag{4}$$

$$M_z = f(v_c, v_f, a_e, a_p), \quad [Nm] \tag{5}$$

$$M_z = C_{Mz} \cdot v_c^{a1} \cdot v_f^{a2} \cdot a_e^{a3} \cdot a_p^{a4}, \quad [Nm] \tag{6}$$

where:

F_x, F_y – represent the axial cutting force (dependent variable);

M – the torque (dependent variable);

v_c – the cutting spead of the process [m/min] (independent variable);

v_f – the cutting feed of the process [mm/min];

a_e – axial depth of cutting process [mm] (independent variable);

a_p – redial depth of cutting process [mm] (independent variable);

a_1, a_2, a_3, a_4 – polytropic exponents;

C_F, C_M, - constants.

In fact, the aim of this paper, meaning, presenting the steps followed and, specially, the new mathematical models obtained for the axial force and torque in milling process.

For obtaining the constants and polytropic exponents' values, relations (2), (4), and (6) must be of linear type and, so, by logarithm they will "turn" into relations (7) to (9), as follows:

- in milling:

$$\lg F_x = \lg C_{Fx} \cdot \lg v_c^{a1} \cdot \lg v_f^{a2} \cdot \lg a_e^{a3} \cdot \lg a_p^{a4}, \; [N] \tag{7}$$

$$\lg F_y = \lg C_{Fy} \cdot \lg v_c^{a1} \cdot \lg v_f^{a2} \cdot \lg a_e^{a3} \cdot \lg a_p^{a4}, [N] \tag{8}$$

$$\lg M_z = \lg C_{Mz} \cdot \lg v_c^{a1} \cdot \lg v_f^{a2} \cdot \lg a_e^{a3} \cdot \lg a_p^{a4} \; [Nm] \tag{9}$$

So, it can be mentioned that the first method, REGS, applied was that of solving a four / five linear equations system – as there were four / five constants to be determined (C, a_1, a_2, a_3 and a_4), five constants for the milling process. The second method applied dealt with experiments design and regression analysis – done with a special software, DOE KISS. [5] Due to limited license rights (as the authors have only the "student version"), there could only be determined regression models with three independent variables, each of them with two "levels'.

Table 1: The experiments design (Central Composite Design, CCD)

Central Composite Design (CCD)	Experiments Design			
	Run	**A**	**B**	**C**
	1	-1	-1	-1
	2	-1	-1	+1
	3	-1	+1	-1
	4	-1	+1	+1
	5	+1	-1	-1
	6	+1	-1	+1
	7	+1	+1	-1
	8	+1	+1	+1

So, based on the results obtained by solving relations (7) to (9), for the regression analysis there have been considered the three most "significant" variables – for axial force and torque values. The three most important variables are: v_c, v_f and a_e [9, 10]. The experiments design (Central Composite Design, CCD) is evidenced in table 1.

Regression analysis performed by the software resulted in models like the ones mentioned by relations (10) to (12):

- in milling:

$$Fx = a_0 + a_1 \cdot v_c + a_2 \cdot v_f + a_3 \cdot a_e + a_{12} \cdot v_c \cdot v_f + a_{13} \cdot v_c \cdot a_e + a_{23} \cdot a_e \cdot v_f + a_{123} \cdot v_c \cdot v_f \cdot a_e \text{ ,[N]} \quad (10)$$

$$Fy = a_0 + a_1 \cdot v_c + a_2 \cdot v_f + a_3 \cdot a_e + a_{12} \cdot v_c \cdot v_f + a_{13} \cdot v_c \cdot a_e + a_{23} \cdot a_e \cdot v_f + a_{123} \cdot v_c \cdot v_f \cdot a_e \text{ ,[N]} \quad (11)$$

$$Mz = a_0 + a_1 \cdot v_c + a_2 \cdot v_f + a_3 \cdot a_e + a_{12} \cdot v_c \cdot v_f + a_{13} \cdot v_c \cdot a_e + a_{23} \cdot a_e \cdot v_f + a_{123} \cdot v_c \cdot v_f \cdot a_e \text{ ,[Nm]} \quad (12)$$

The relationship of "coded" variables, and "natural" ones, z_j (z_j=v_c, v_f, a_e, a_p) is (13):

$$x_j = \frac{z_j - \frac{z_{min} - z_{max}}{2}}{\frac{z_{max} - z_{min}}{2}} \quad (13)$$

where:
z_{min} - is the minimum experimental value;
z_{max} - the maximum experimental value.

Figure 4: Experimental stand – force and torque study for machining milling process

Figure 5: Special dynamometric device

Equipment and materials:

a) Computer Aided Process, special software;

b) Amplifier analog-digital;

c) Special dynamometric, type 9257B, device for 6 components, Fx, Fy, Fz, Mx, My, Mz. The dynamometer has a great rigidity and consequently a hight natural frequency. Its hight resolution enables the smallest dynamic changes in large forces to be measured.

d) Studied material: Products made by materials sandwich composites products with two faces of fiber glass and the core made by polyester extruaded, some plate characteristcs are presented in table 2 and some core characteristics are presented in table 3.

Table 2: Plates characteristics

Material code	Plate dimensions, [mm]	Thickness, [mm]	Density, [kg/m^3]	Tension resistance, [MPa]	Elastic Modulus, [N/mm^2]
EC12-2400-P207	100x200	1.5/2/4	2,54x10^{-3}	3450	5000

Table 3: Core characteristics

Material code	Thickness [mm]	Density, [kg/m^3]	Compression Resistance de 10% [N/mm^2]	Conductivity, [W/m.K]
AplaXfoamBT	20	28	0.30	0.018

Experimental values obtained for the forces Fx and Fy and torque Mz measurement are shown in the tables 4, and table 5 as follows: - for the first method, REGS considered, that of solving the four / five linear equations systems – see table 4; - for the second method studied, we use just 3 variables, because the program used was DOE KISS with 3 variables and 2 levels that of experiments design (CCD) and regression analysis – see table 5.

Table 4: Experimental values obtained for the forces Fx and Fy and torque Mz

Cutting force and torque - F_x, F_y şi M_z[N]										
Factorial program P2.1				v_c [m/min]	v_f [mm/min]	a_e [mm]	a_p [mm]	F_x [N]	F_y[N]	M_z[Nm]
-1	-1	-1	-1	62.8	480	1	12	8.48	11.87	0.59
+1	-1	-1	+1	219.8	480	1	23	7.83	9.54	0.59
-1	+1	-1	+1	62,8	1680	1	23	24.65	23.31	1.68
+1	+1	-1	-1	219,8	1680	1	12	20.44	17.18	1.56
-1	-1	+1	+1	62.8	480	5	23	19.53	21.36	1.32
+1	-1	+1	-1	219.8	480	5	12	16.19	15.74	1.23
-1	+1	+1	-1	62.8	1680	5	12	51.02	38.45	3.48
+1	+1	+1	+1	219.8	1680	5	23	47.06	30.91	3.49
0	0	0	0	117,48	897,9	2,3	16.61	20.25	19.34	1.45
0	0	0	0	117,48	897,9	2,3	16.61	20.46	19.21	1.36
0	0	0	0	117,48	897,9	2,3	16.61	20.85	19.45	1.49
0	0	0	0	117,48	897,9	2,3	16.61	20.20	19.36	1.29

Table 5: Experiments design (CCD) and regression analysis

Cutting force and torque - F_x, F_y şi M_z[N]								
Factorial program P 1:2			v_c[m/min]	v_f[mm/min]	a_e[mm]	F_x	F_y	M_z
-1	-1	-1	62.8	480	1	4.95	9.73	0.37
-1	-1	1	62.8	480	5	12.95	19.75	1.54
-1	1	-1	62.8	1680	1	20.57	21.37	1.32
-1	1	1	62.8	1680	5	37.67	30.99	3.59
1	-1	-1	219.8	480	1	5.05	6.75	0.38
1	-1	1	219.8	480	5	13.95	22.98	1.98
1	1	-1	219.8	1680	1	32.91	32.85	1.97
1	1	1	219.8	1680	5	4.95	54.99	4.37

Observations: for each experience, replicates number equaled five.

3. MATHEMATICAL MODEL

A. For the first method considered, REGS, obtained experimental results were further "processed", in order to solve the four / five linear equations systems, required for models' constants and polytropic exponents values determination (C, a_1, a_2, a_3 and a_4), [5, 6].

Knowing that initial dependence relationships were exponential, – relations (7) to (9), there were obtained the mathematical models of force and torque, for machining milling process for products made by materials sandwich composites polymeric[7, 8].

So, in milling process, there are the final equations - see relations (13) to (15):

$$Fx = 0.073 \cdot v_c^{-0.107} \cdot v_f^{0.809} \cdot a_e^{0.485} \cdot a_p^{0.082} \quad ,[N] \qquad (13)$$

$$Fy = 1.063 \cdot v_c^{-0.209} \cdot v_f^{0.504} \cdot a_e^{0.338} \cdot a_p^{0.067} \quad ,\text{[N]} \tag{14}$$

$$M_z = 0.004 \cdot v_c^{-0.028} \cdot v_f^{0.805} \cdot a_e^{0.477} \cdot a_p^{0.056} \quad ,\text{[Nm]} \tag{15}$$

Some graphical representations of the obtained mathematical models can be noticed in figure 6a, b, c (for the milling process for REGS- Fx, Fy and Mz) and in figure 7 (for the milling process for DOE KISS).

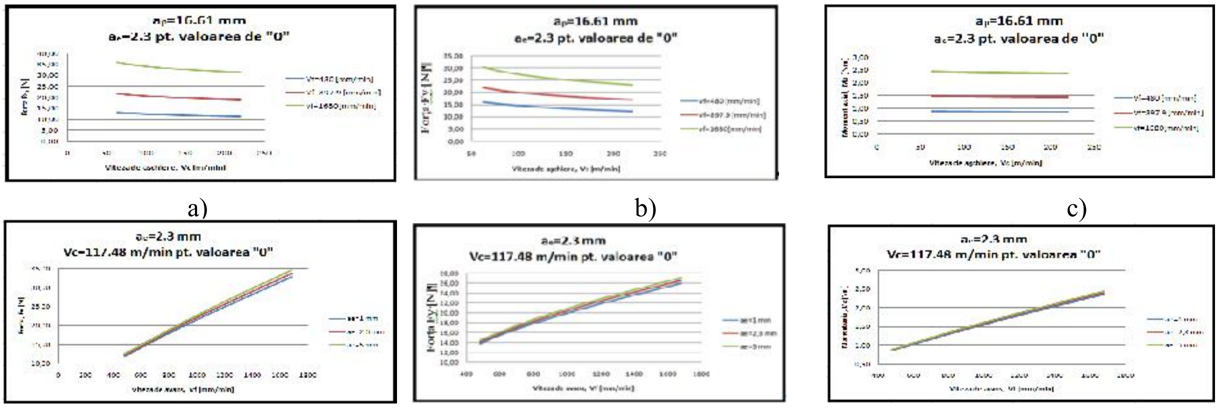

Figure 6: Graphical representation of Fx-a, Fy-b and Mz-c with REGS models - relations (13), (14), (15).

As observation, there should be mentioned the fact that these graphs were plotted for the variables with higher influence on force and torque values.

B. One can notice the fact that relations (13), (14) and (15) were obtained by solving classical linear equations system.

It means, four / five unknown parameters and, consequently, the need for four / five linear equations systems to be solved [10]. So, an "improvement" of the method to obtain mathematical models was considered to be right [9, 10].

So, for the new sets of experiments, as mentioned before, there have been considered, both the Central Composite Design of experiments and the regression analysis, performed with the special software, DOE KISS.

The only three independent variables studied were the ones that proved (by previously obtained mathematical models) to strongly influence the dependent variable (axial force or torque).. All of these are evidenced by table 6.

Table 6: Independent variables values

	Real, yz			Coded, x		
	min		max	min		max
			Milling			
Cutting speed, v_c[m/min]	62,8		219,8	-1		+1
Cutting feed, v_f[mm/rot]	480		1680	-1		+1
Axial depth of cutting process, a_e [mm]	1		5	-1		+1

Examples of the DOE KISS software results, are shown in figure 7. So, as result of this method, the obtained mathematical models of force and torque, for machining milling process are the ones mentioned above – see relations (15), (16) and (17):

$$Fx = -2.857 - 0.0152 \cdot v_c + 0.01009 \cdot v_f + 1.883 \cdot a_e + 0.00005 \cdot v_c \cdot v_f + 0.00097 \cdot v_f \cdot a_e + 0.036 \cdot v_c \cdot a_e + 0.0000034 \cdot v_c \cdot v_f \cdot a_e \quad ,\text{[N]} \tag{15}$$

$$Fy = 3.064 - 0.058 \cdot v_c + 0.031 \cdot v_f + 2.556 \cdot a_e + 0.000065 \cdot v_c \cdot v_f + 0.004 \cdot v_c \cdot a_e - 0.00093 \cdot a_e \cdot v_f + 0.00001 \cdot v_c \cdot v_f \cdot a_e \quad ,\text{[N]} \tag{16}$$

$$Mz = 1.549 - 0.002 \cdot v_c + 0.0033 \cdot v_f + 0.207 \cdot a_e + 0.00003 \cdot v_c \cdot v_f + 0.006 \cdot v_c \cdot a_e + 0.00016 \cdot a_e \cdot v_f \quad ,\text{[Nm]} \tag{17}$$

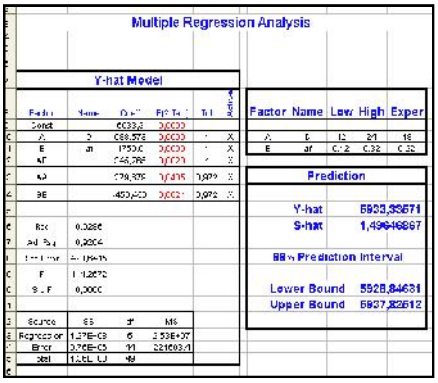

Figure 7: Examples of the DOE KISS software results

Some graphical representation for Fx , Fy and M_z models - relations (15) , (16), (17) – see figure 8.

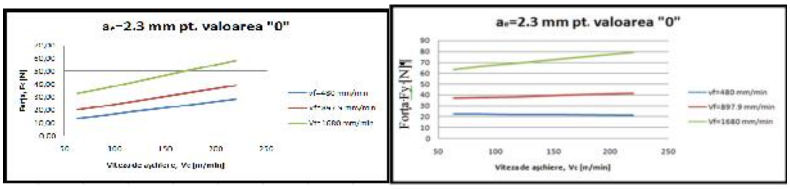

Figure 8: Graphical representation for Fx , Fy and M_z models - relations (15) , (16), (17)

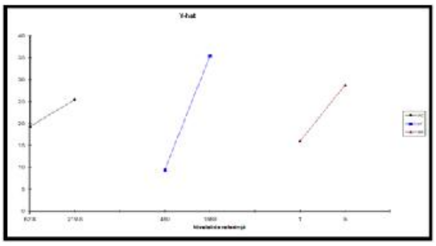

Figure 9: Examples of DOE KISS software - marginal means plots

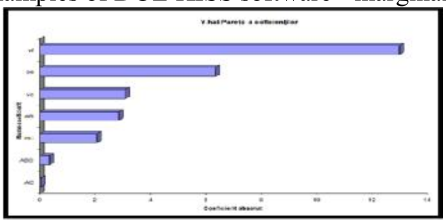

Figure 10: Examples of DOE KISS software – Pareto chart of coefficients

4. CONCLUSION

This study is about two methods REGS and DOE KISS, used for determining new adequate mathematical models of the forces Fx and Fy and torque Mz in machining, milling process of products made by materials sandwich composite polymeric. The machining procedures considered were milling process, while the material studied is one of great importance for Romanian industry.

The first method, consisted in solving the four / five linear equations systems, required for models' constants and polytrophic exponents values determination (C, a_1, a_2, a_3 and a_4).

The second method, involved design of experiments (CCD) and regression analysis performed with a special software.

There were obtained two independent variables polynomial type models and by plotting graphs, it was evidenced their influence, as well as their interaction, on the forces and torque values.

All the steps carried in order to obtain the new mathematical models can be considered as parts of a "procedure" to be followed / applied for any materials type and different machining procedures, whenever dependence relations of machining parameters are worth to be determined.

Further research involves other process parameters and materials to be studied, as well as the application of obtained models on real time control of the machining milling process.

REFERENCES

[1]Opran C., Research concerning polymeric laminate composite materials products under impact conditions, PROCEEDINGS CNC TEHNOLOGIES 2011, Fifth Edition, ISSN: 2068-2093, Politehnica Press, Bucharest, May 12-13, pp.115-120, 2011.

[2]Tsai S.W., Strength & life of composites, Editor Aeronautics & Astronautics Stanford University, SUA, 2011.

[3]Bivolaru C., Opran C., Murar D., Milling of polymeric sandwich composites products, 22nd DAAM INTERNATIONAL, World Symposium, Uno City-Austria, Vienna, pp.1295-1296, 2011.

[4] Hsu-Hwa C., Chih-Hsien C., Use of esponse Surface Methodology and Exponential Desirability Functions to Paper Feeder Design, WSEAS TRANSACTIONS on Applied and Theoretical Mechanics Issue 9, Volume2, ISSN: 1991-8747, 2007.

[5] Marcos M., Gomez- Lopez A., Batista P., Roughness based study of milled composite surfaces, 22nd DAAM INTERNATIONAL, World Symposium, Uno City-Austria, Vienna, pp.0153-0154, 2011.

[6]Fong T. J., Composites failure criteria and estimation of associated A-and B –basis design allowable, JEC COMPOSITES MAGAZINE, N0 71, 2012.

[7] Camanho P., Turon A., Costa J., Structural integrity of thin –ply laminates; JEC COMPOSITES MAGAZINE, N0 71, 2012.

[8] Bivolaru C., Opran C., Murar D., Influence of temperature on polymeric sandwich composite products during milling processing; PROCEEDINGS CNC TEHNOLOGIES 2011; Fifth Edition; ISSN: 2068-2093; Politehnica Press; Bucharest, pp. 89-93, 2011

[9] Lupeanu M.E., Cercetări privind aplicarea analizei funcţionale tehnice la dezvoltarea de noi produse realizate prin tehnologii neconvenţionale de fabricare aditivă, Teză de doctorat, Coordonator: Neagu C., Bucureşti, 2012.

[10] Neagu C., Lupeanu M.E, Rennie A.E.W., Geometry optimization of tools used in milling of spherical surfaces, Proceedings of The 1st Sustainable Intelligent Manufacturing Conference, IST PRESS, ISBN 978-989-8481-03-0, Leiria, Portugalia, pg. 87-94, 2011.