



ON RESPONSE OF RANDOM VIBRATION FOR NONLINEAR SYSTEMS

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Abstract: In his paper, a nonlinear system under external excitation of white noise was replaced by an equivalent nonhysteretic nonlinear system subject to the same excitation for the probability density of the stationary response was known. The basic idea of the method is proposed to obtain the approximate power spectral density for the stationary response. In general, and especially in random vibration analysis, it is difficult to obtain a closed form solution for dynamic response of a nonlinear system.

Keywords: Random oscillations, nonlinear damping, natural frequency, equivalent nonlinear system

1. INTRODUCTION

A nonlinear system under external excitation of white noise was replaced by an equivalent nonhysteretic nonlinear system subject to the same excitation for the probability density of the stationary response was known. Nonlinear dynamic systems subject to random excitations are frequently met in engineering practice. Random differential equations appear in several different applications: study of random evolution of system with a spatial extension, study of stochastic models where the state variable is infinite dimensional, for example, a curve or surface. In this paper, a technique is proposed in order to evaluate the probability density function of the solution, based on the combination of the probabilistic transformation methods.

2. SYSTEM MODEL

A nonlinear system will be considered with response $\eta(t)$ to an excitation $w(t)$ described by second differential equation

$$\ddot{\eta}(t) + 2\xi p \dot{\eta}(t) + \alpha p^2 \eta^5(t) = w(t), \quad (1)$$

where $\eta(t)$ is the displacement response of the system, c is the viscous damping coefficient, ξ is the critical damping factor, for the linear system, α is the nonlinear factor to control the type and degree of nonlinearity in the system.

The mechanical energy of the system is

$$H(\eta) = \int_0^{\eta} h(v) dv = \alpha \int_0^{\eta} p^2 \eta^5(v) dv = \alpha p^2 \eta^6, \quad (2)$$

The potential energy of the system [1,2,3] is

$$E_m(\eta, \dot{\eta}) = \frac{1}{2} \dot{\eta}^2 + \alpha p^2 \eta^6. \quad (3)$$

Obtain for the mechanical energy

$$2E_m = \dot{\eta}^2 + 2\alpha p^2 \eta^6, \quad (4)$$

$$\omega = \frac{d\varphi}{dt} = \frac{du_1}{d\eta} = \frac{1}{\sqrt{2H(\eta)}} H'(\eta) = \frac{\sqrt{2\alpha}}{2\alpha p\eta} \quad (5)$$

We have

$$\dot{u}_1 = \omega u_2 \quad (6)$$

and equation of motion becomes

$$\ddot{u}_2 + 2\xi p \dot{u}_2 + \omega u_1 = w(t). \quad (7)$$

Obtain

$$\ddot{u}_2 + 2\xi p \dot{u}_2 + \omega u_2 = w(t). \quad (8)$$

These two first order differential equation are conveniently combined into the matrix format

$$\frac{d}{dt} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} 0 & \omega \\ -\omega & 2\xi p \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} + \begin{bmatrix} 0 \\ w(t) \end{bmatrix}. \quad (9)$$

and this format is similar to that of a linear oscillator

The energy relation is written in the form

$$\frac{d\eta}{dt} = \sqrt{2(E_m - H(\eta))}, \quad (10)$$

and the period for the system is

$$T(E_m) = \int_0^T dt, \quad (11)$$

or

$$T(E_m) = 2 \int_{\eta_{\min}}^{\eta_{\max}} \frac{d\eta}{\sqrt{2(E_m - H(\eta))}}. \quad (12)$$

The integral vanishes in the lower limit and therefore differential with respect to the energy level gives

$$T(E_m) = 2 \int_{\eta_{\min}}^{\eta_{\max}} \frac{d\eta}{\sqrt{2\{E_m - \alpha p^2 \eta^6\}}}, \quad (13)$$

The function $h(\eta) = \alpha p^2 \eta^5$ is odd function, $h(-\eta) = -h(\eta)$, and the point $\eta = 0$ is an equilibrium position, so

$h(0) = 0$. We consider the initial conditions $t = 0, \eta = 0, \dot{\eta} = 0$.

If the expression (12) we do $\eta_{\min} = 0, \eta_{\max} = A$ is get the time to walk the distance MO and how the function $h(\eta)$ is symmetric, that this time period is a quarter. There was thus obtained between

$$T(E_m) = \frac{4\eta}{\sqrt{2E_m}} \eta \Big|_0^A + \frac{\omega_0^2 \sqrt{2E_m}}{3E_m^2} \eta^3 \Big|_0^A + \frac{\omega_0^2 \beta^2}{191E_m^4} \eta^4 \Big|_0^A, \quad (14)$$

relationship that writes

$$T(E_m) = \frac{4A}{\sqrt{2E_m}} + \frac{\omega_0^2 A^3 \sqrt{2E_m}}{3E_m^2} + \frac{\omega_0^2 A^4 \beta^2}{191E_m^4} \quad (15)$$

Therefore, if the period nonlinear vibration depends on the initial conditions.

If differentiating the polar representation [2,3] in the (4) and the (6) in relation to the time, we obtain the system of equations

$$\dot{u}_1 = \frac{\dot{E}_m}{\sqrt{2E_m}} \sin \varphi + \dot{\varphi} \sqrt{2E_m} \cos \varphi, \quad (16)$$

$$\dot{u}_2 = \frac{\dot{E}_m}{\sqrt{2E_m}} \cos \varphi - \dot{\varphi} \sqrt{2E_m} \sin \varphi, \quad (17)$$

Obtain

$$\frac{\dot{E}_m}{\sqrt{2E_m}} = \sin \varphi \dot{u}_1 + \cos \varphi \dot{u}_2, \quad (18)$$

$$\varphi \sqrt{2E_m} = \cos \varphi \dot{u}_1 - \sin \varphi \dot{u}_2, \quad (19)$$

or

$$\dot{E}_m = -4E_m \xi p \cos^2 \varphi + \sqrt{2E_m} w(t) \cos \varphi \quad (20)$$

$$\dot{\varphi} = \omega + 2\xi p \cos \varphi \sin \varphi - \frac{1}{\sqrt{2E_m}} w(t) \sin \varphi \quad (21)$$

The equations Fokker-Planck-Kolmogorov to determine the probability density [2,3] are

$$\frac{\partial p}{\partial t} = \frac{\partial}{\partial z_j} \left(-\alpha_j p + \frac{1}{2} \frac{\partial}{\partial z_k} (\beta_{jk} p) \right). \quad (22)$$

The differential equations (20) can be placed by the approximation to a set of diffusion equations [1,2,3] in the form:

$$\frac{d}{dt} Z_j(t) = a_j(Z_k, t) + b_j(Z_k, t) y(t), \quad j = 1, 2 \quad (23)$$

where

$$Z_1 = E_m \text{ and } Z_2 = \varphi. \quad (24)$$

For a broad-band excitation, we obtain Ito's equations [3,4] in the form

$$dZ_j = \alpha_j(Z_k, t) dt + \sigma_j(Z_k, t) d\tilde{W}, \quad (25)$$

The Fokker-Planck-Kolmogorov equations associated with equations (25) are

$$\begin{aligned} \frac{\partial}{\partial E} \left[-\alpha_{E_m} p_{E_m, \varphi} + \frac{1}{2} \frac{\partial}{\partial E_m} (\beta_{E_m, E_m} p_{E_m, \varphi}) + \frac{1}{2} \frac{\partial}{\partial \varphi} (\beta_{E_m, \varphi} p_{E_m, \varphi}) \right] + \\ + \frac{\partial}{\partial \varphi} \left[-\alpha_{\varphi} p_{E_m, \varphi} + \frac{1}{2} \frac{\partial}{\partial E_m} (\beta_{\varphi, E_m} p_{E_m, E_m}) + \frac{1}{2} \frac{\partial}{\partial \varphi} (\beta_{\varphi, \varphi} p_{E_m, \varphi}) \right] = 0. \end{aligned} \quad (26)$$

3. THE PROBABILITY DENSITY FUNCTION

The probability density function of simultaneous values of the response of a system of the form (1) to ideal white noise excitation satisfies a two dimensional Fokker-Planck-Kolmogorov equations, partial differential equation

$$p_{E_m}(E_m) = \frac{CT(E_m)}{\pi S_0} e^{-\int_0^{E_m} \frac{f_{eq}(r) dr}{\pi S_0}}, \quad (27)$$

where

$$f_{eq}(E_m) = \frac{\int_0^T 2\xi p dt}{\int_0^T \dot{\eta}^2 dt}. \quad (28)$$

Introducing equation (27) into (26), obtain the probability density of response is

$$p_{E_m}(E_m) = \frac{CT(E_m)}{\pi S_0} e^{-\frac{2\xi \omega_0}{\pi S_0} \left(E_m + \frac{3}{4} \alpha E_m^2 \right)}, \quad (29)$$

where C is the constant of normalization.

The probability density of response becomes

$$p_{E_m}(E_m) = \frac{C \left(\frac{4A}{\sqrt{2E_m}} + \frac{\omega_0^2 A^3 \sqrt{2E_m}}{3E_m^2} + \frac{\omega_0^2 A^4 \beta^2}{191E_m^4} \right)}{\pi S_0} e^{-\frac{2\xi\omega_0}{\pi S_0} \left(E_m + \frac{3}{4}\alpha E_m^2 \right)}. \quad (30)$$

4. THE POWER SPECTRAL DENSITY OF THE RESPONSE FOR THE SYSTEM

The operator $s(E_m)$ is determined [3,4,] by the expression

$$s(E_m) = \frac{\langle \dot{x} \rangle_t}{E_m} = \frac{1}{T(E_m)} \frac{1}{E_m} \int_0^{E_m} T(r) dr, \quad (31)$$

in which, if the period taken to introduce the expression (11)

$$s(E_m) = s_{E_m} = \frac{1}{E_m} \frac{4A\sqrt{2E_m} - \frac{2\sqrt{2}}{3} \omega_0^2 A^2 - \frac{\omega_0^2 A^4 \beta^2}{573E_m^3}}{2A\sqrt{2E_m} + \frac{\omega_0^2 A^3 \sqrt{2E_m}}{3E_m^2} + \frac{\omega_0^2 A^4 \beta^2}{191E_m^4}}. \quad (32)$$

The power spectral density of response [4,5,6] is

$$S_\eta(\omega) = \frac{1}{\pi} \int_0^\infty \frac{s_{E_m}^2 f_{E_m} E_m}{\left(\frac{s_{E_m}^2 f_{E_m}^2}{4\xi^2} - \omega^2 \right)^2 + s_{E_m}^2 f_{E_m}^2 \omega^2} p_{E_m}(E_m) dE_m, \quad (33)$$

or

$$S_\eta(\omega) = \frac{C}{\pi} \int_0^\infty \frac{s_{E_m}^2 f_{E_m} E_m T(E_m)}{\left(\frac{s_{E_m}^2 f_{E_m}^2}{4\xi^2} - \omega^2 \right)^2 + s_{E_m}^2 f_{E_m}^2 \omega^2} \frac{1}{\pi S_0} e^{-\frac{2\xi\omega_0}{\pi S_0} \left(E_m + \frac{3}{4}\alpha E_m^2 \right)} dE_m. \quad (34)$$

For the linear case we have

$$s(E_m) = \frac{\langle \dot{x} \rangle_t}{E_m} = \frac{1}{T(E_m)} \frac{1}{E_m} \int_0^{E_m} T(r) dr = \frac{p}{2\pi E_m} \int_0^{E_m} \frac{2\pi}{p} dr = 1. \quad (35)$$

Because nonlinearity factor is $\alpha=1$,

$$f_{eq}(E_m) = 2\xi p \quad (36)$$

and the probability density of response is

$$p_{E_m}(E_m) = \frac{2C}{p S_0} e^{-\frac{2\xi p}{\pi S_0} E_m}. \quad (37)$$

Introducing all these parameters in response spectral density expression given by (33.), we get

$$S_\eta(\omega) = \frac{S_0}{(p^2 - \omega^2)^2 + (2\xi p)^2 \omega^2}, \quad (38)$$

that formula known in the linear case.

5. NUMERICAL RESULTS

In this example, $m = 1kg$, $k = 36 \frac{N}{m}$, $c = 4 \frac{Ns}{m}$, $\alpha = 3m^{-2}$.

Obtain:

$$p = \sqrt{\frac{k}{m}} = 6s^{-1}, \quad \frac{c}{m} = 2\xi p \Rightarrow \xi = 0,33. \quad (39)$$

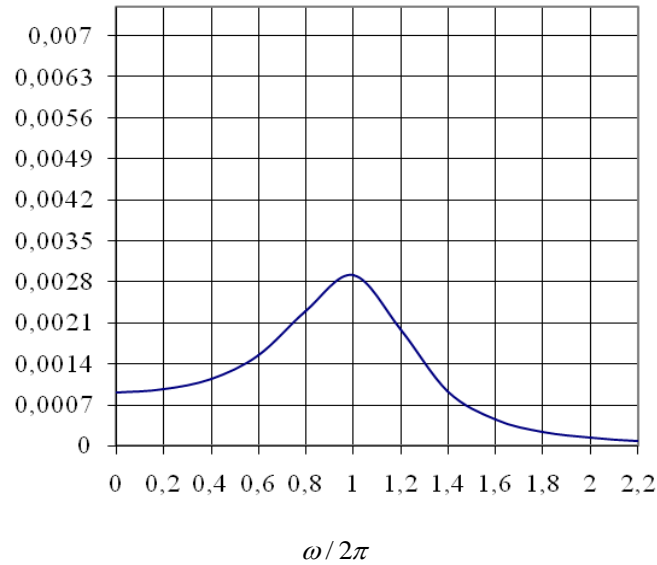


Figure 1. The power spectral density of the response $S_\eta(\omega)$ for $\xi = 0,33$, $\alpha = 3m^{-2}$.

6. CONCLUSION

In this article, we are analyzing a differential equation with random variable. Our new technique based on the combination of the transformation method with numerical method to evaluate the probability density function and the power spectral density of the response of the systems. The advantage of the method is that it yields the approximate probability density for the stationary response of nonlinear stochastic systems rather than just a few statistical moments and that is may be applicable to nonlinear stochastic systems.

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