



## ABOUT MASONRY WALLS DUCTILITY CAPACITIES CALCULATION

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**Abstract:** *The development of capacity design principles in the 1970's (Park and Paulay, 1976) was an expression of the realization that the distribution of strength through a building was more important than the absolute value of the design base shear. It was recognized that a frame building would perform better under seismic action if it could be assured that plastic hinges would occur in beams rather than in columns (weak beam/strong column mechanism), and if the shear strength of members exceeded the shear corresponding to flexural strength. This can be identified as the true start to performance based seismic design, where the overall performance of the building is controlled as a function of the design process. One of the civil engineers goals is how to calculate more simplified the ductility capacities for the structural elements and in this paper just the problem of masonry walls was treated.*[2], [4]

**Keywords:** *masonry, walls, ductility, capacity, curves*

### 1. INTRODUCTION

Design for seismic resistance has been enduring a critical reconsideration in recent years, with the emphasis changing from "strength" to "performance". For most of the past 70 years – the period over which specific design analyses for seismic resistance have been required by codes – strength and performance have been considered to be synonymous.

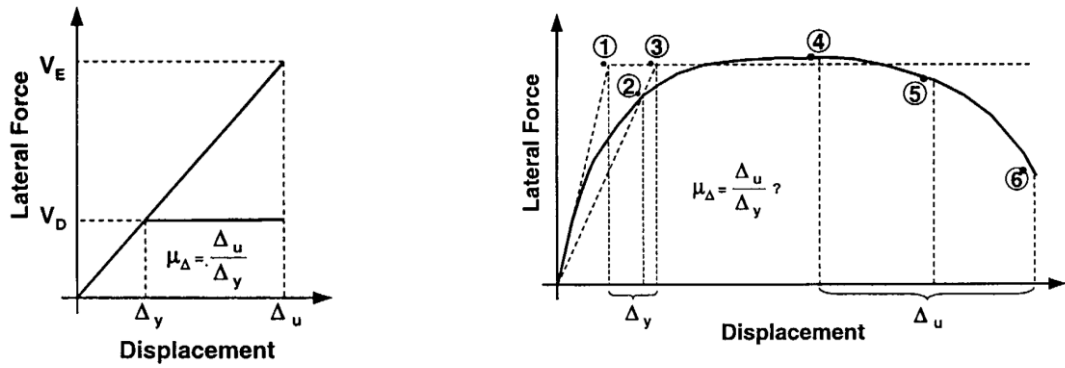
As an understanding developed in the 1960s and 1970s of the importance of inelastic structural response to large earthquakes, the research community became increasingly involved in attempts to quantify the inelastic deformation capacity of structural components. Generally this was expressed in terms of displacement ductility capacity,  $\mu_s$ , which was chosen as a useful indicator because of its apparent relationship to the force-reduction factor,  $R$ , commonly used to reduce expected elastic levels of base shear strength to acceptable design levels. As is seen in Fig. 1(a), the equal displacement approximation of seismic response implies that:

$$\mu_s = R \quad (1)$$

There have been problems with this approach, in that it has long been realized that the equal displacement approximation is inappropriate for both very short and very long period structures, and is also of doubtful validity for medium period structures when the hysteretic character of the inelastic system deviates significantly from elasto-plastic. Further, there has been difficulty in reaching consensus within the research community as to the appropriate definition of yield and ultimate displacements. With reference to Fig. 1(b), the yield displacement has been variously defined as the intersection of the initial tangent stiffness with the nominal strength, the intersection of the secant stiffness through first yield with nominal strength, and the displacement at first yield, amongst other possibilities. Displacement capacity, or ultimate displacement, has also had a number of definitions, including displacement at peak strength, displacement corresponding to 20% or 50% degradation from peak (or nominal) strength, and displacement at initial fracture of transverse reinforcement. Clearly with such a wide choice of limit displacements, there has been considerable variation in the assessed displacement ductility capacity of structures.

Implicit in the force-reduction factor approach to determination of required strength is the assumption that particular structural systems can be allocated characteristic ductility capacities, and hence characteristic force reduction factors. It has, however, become apparent over the past 15 years, that is an unacceptable approximation. Ductility capacity of concrete and masonry structures depends on a wide range of factors, including axial load ratio, reinforcement ratio, and structural geometry. Foundation compliance also can

significantly affect the displacement ductility capacity. Moehle (1992) later suggested a similar approach to that of Priestley and Park (1985), for building structures.



a) Equal displacement approximation      b) Definition of yield and ultimate displacement

Figure 1: Problems with definition of ductility capacity

These approaches recognize some of the imperfections of a pure force-based design, by requiring calculation of the ductility capacity of structures, and checking this against estimates of the ductility demand corresponding to the design level of seismicity and force reduction factor adopted for design. In New Zealand and Europe this is still considered to be force-based design, while in the US the addition of the displacement check, possibly with modification of the design strength as a consequence of the displacement check, has come to be known as displacement-based design, or performance-based design.

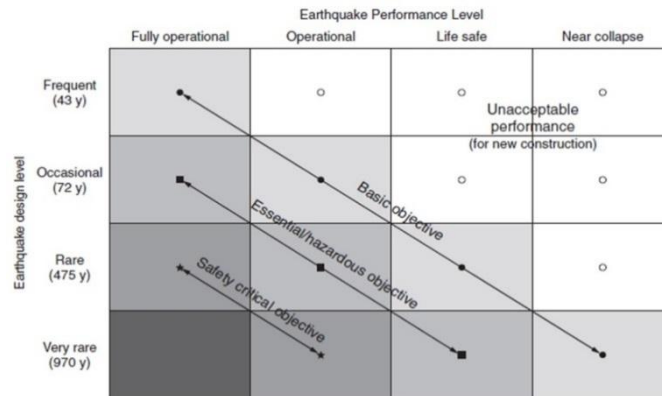


Figure 2: Seismic performance objectives for buildings (SEAOC, 1996), showing increasingly undesirable performance characteristics from left to right on the horizontal axis and increasing level of ground motion from top to bottom on the vertical axis. Performance objectives for three categories of structures are shown by the diagonal lines (Hall et al, 1995)

Taking into consideration the relations and simplifications exposed by Priestley in his works, and different lengths of potential plastic zones presented by various authors we tried to determine the approximate simplified relations to determine the ductility capacity of masonry walls.

All the relations to determine the ductility capacity are more complicated in reality. The following presented relationships are obtained only for a single case of masonry – solid bricks with  $f_b=15 \text{ N/mm}^2$  and  $f_k=6.6 \text{ N/mm}^2$  (mortar M15) and  $\epsilon_{mu} = 3.5 \text{ ‰}$ . For all the other types of masonry with different characteristics other simplified relationships must be determined (but having this experience it seems that will not be difficult). [2], [4]

## 2. CALCULATION OF ROTATIONAL DUCTILITY CAPACITY

$$\mu_\theta = \frac{\theta_u}{\theta_y} = \frac{\theta_y + \theta_p}{\theta_y} = 1 + \frac{\theta_p}{\theta_y} \quad (2) \text{ and } \theta_y \cong 0.6\epsilon_y \frac{h_w}{l_w} \cong 0.0014 \frac{h_w}{l_w} \quad (3), \text{ so } \theta_p \cong 0.0169 \frac{l_p}{l_w} \quad (4)$$

Having the following values for plastic hinges lengths the following relationships were obtained:

Nr.crt.	$l_p$	Author	Relation for $\mu_\theta$
1	$0.5l_w+0.05h_w$	Mattock	$\mu_\theta = 1.6 + 6.04 \frac{l_w}{h_w}$
2	$0.4l_w+0.05h_w$	Paulay and Uzumeri	$\mu_\theta = 1.6 + 4.83 \frac{l_w}{h_w}$
3	$0.5l_w$	Priestley	$\mu_\theta = 1 + 6.04 \frac{l_w}{h_w}$
4	$0.1l_w+0.04h_w$	Priestley	$\mu_\theta = 1.5 + 1.20 \frac{l_w}{h_w}$
<b>m</b>	<b>Average</b>	<b>Stoica</b>	<b><math>\mu_\theta = 1.5 + 5 \frac{l_w}{h_w}</math></b>

### 3. CALCULATION OF DISPLACEMENT DUCTILITY CAPACITY

$$\Delta_p = 0.70h_w\theta_p \text{ (5), so } \Delta_y = \frac{1}{200l_w} \frac{h_w^2}{3} = \frac{1}{600} \frac{h_w^2}{l_w} \text{ (6) and } \mu_\Delta = \frac{\Delta_u}{\Delta_y} = \frac{\Delta_y + \Delta_p}{\Delta_y} = 1 + \frac{\Delta_p}{\Delta_y} \text{ (7)}$$

Having the following values for plastic hinges lengths the following relationships were obtained:

Nr.crt.	$l_p$	Author	Relation for $\mu_\Delta$
1	$0.5l_w+0.05h_w$	Mattock	$\mu_\Delta = 1.36 + 3.55 \frac{l_w}{h_w}$
2	$0.4l_w+0.05h_w$	Paulay and Uzumeri	$\mu_\Delta = 1.36 + 2.84 \frac{l_w}{h_w}$
3	$0.5l_w$	Priestley	$\mu_\Delta = 1.00 + 3.55 \frac{l_w}{h_w}$
4	$0.1l_w+0.04h_w$	Priestley	$\mu_\Delta = 1.30 + 0.70 \frac{l_w}{h_w}$
<b>m</b>	<b>Average</b>	<b>Stoica</b>	<b><math>\mu_\Delta = 1.3 + 3 \frac{l_w}{h_w}</math></b>

### 4. CALCULATION OF CURVATURE DUCTILITY CAPACITY

$$\mu_\phi = \frac{\phi_u}{\phi_y} = \frac{(\mu_\Delta - 1)h_w^2}{3l_p(h_w - 0.50l_p)} = 1 + \frac{\mu_\Delta - 1}{1.5 \frac{l_w}{h_w} (1 - 0.25 \frac{l_w}{h_w})} \quad (8)$$

Having the following values for plastic hinges lengths the following relationships were obtained:

Nr.crt.	$l_p$	Author	Relation for $\mu_\phi$
1	$0.5l_w+0.05h_w$	Mattock	$\mu_\phi = 1 + \frac{0.24}{\frac{l_w}{h_w} (1 - 0.25 \frac{l_w}{h_w})} + \frac{2.37}{(1 - 0.25 \frac{l_w}{h_w})}$
2	$0.4l_w+0.05h_w$	Paulay and Uzumeri	$\mu_\phi = 1 + \frac{0.23}{\frac{l_w}{h_w} (1 - 0.25 \frac{l_w}{h_w})} + \frac{1.89}{(1 - 0.25 \frac{l_w}{h_w})}$
3	$0.5l_w$	Priestley	$\mu_\phi = 1 + \frac{2.37}{(1 - 0.25 \frac{l_w}{h_w})}$
4	$0.1l_w+0.04h_w$	Priestley	$\mu_\phi = 1 + \frac{0.20}{\frac{l_w}{h_w} (1 - 0.25 \frac{l_w}{h_w})} + \frac{0.47}{(1 - 0.25 \frac{l_w}{h_w})}$

<b>m</b>	<b>Average</b>	<b>Stoica</b>	$\mu_\phi = 1 + \frac{0.2}{\frac{l_w}{h_w} \left(1 - 0.25 \frac{l_w}{h_w}\right)} + \frac{2}{\left(1 - 0.25 \frac{l_w}{h_w}\right)}$
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Therefore, in the end can generally present the following table:

<b>Nr. crt</b>	<b><math>l_p</math></b>	<b>Relation for <math>\mu_\theta</math></b>	<b>Relation for <math>\mu_\Delta</math></b>	<b>Relation for <math>\mu_\phi</math></b>
1	$0.5l_w + 0.05h_w$	$\mu_\theta = 1.6 + 6.04 \frac{l_w}{h_w}$	$\mu_\Delta = 1.36 + 3.55 \frac{l_w}{h_w}$	$\mu_\phi = 1 + \frac{0.24}{\frac{l_w}{h_w} \left(1 - 0.25 \frac{l_w}{h_w}\right)} + \frac{2.37}{\left(1 - 0.25 \frac{l_w}{h_w}\right)}$
2	$0.4l_w + 0.05h_w$	$\mu_\theta = 1.6 + 4.83 \frac{l_w}{h_w}$	$\mu_\Delta = 1.36 + 2.84 \frac{l_w}{h_w}$	$\mu_\phi = 1 + \frac{0.23}{\frac{l_w}{h_w} \left(1 - 0.25 \frac{l_w}{h_w}\right)} + \frac{1.89}{\left(1 - 0.25 \frac{l_w}{h_w}\right)}$
3	$0.5l_w$	$\mu_\theta = 1 + 6.04 \frac{l_w}{h_w}$	$\mu_\Delta = 1.00 + 3.55 \frac{l_w}{h_w}$	$\mu_\phi = 1 + \frac{2.37}{\left(1 - 0.25 \frac{l_w}{h_w}\right)}$
4	$0.1l_w + 0.04h_w$	$\mu_\theta = 1.5 + 1.20 \frac{l_w}{h_w}$	$\mu_\Delta = 1.30 + 0.70 \frac{l_w}{h_w}$	$\mu_\phi = 1 + \frac{0.20}{\frac{l_w}{h_w} \left(1 - 0.25 \frac{l_w}{h_w}\right)} + \frac{0.47}{\left(1 - 0.25 \frac{l_w}{h_w}\right)}$
<b>m</b>	<b>Average Stoica</b>	$\mu_\theta = 1.5 + 5 \frac{l_w}{h_w}$	$\mu_\Delta = 1.3 + 3 \frac{l_w}{h_w}$	$\mu_\phi = 1 + \frac{0.2}{\frac{l_w}{h_w} \left(1 - 0.25 \frac{l_w}{h_w}\right)} + \frac{2}{\left(1 - 0.25 \frac{l_w}{h_w}\right)}$

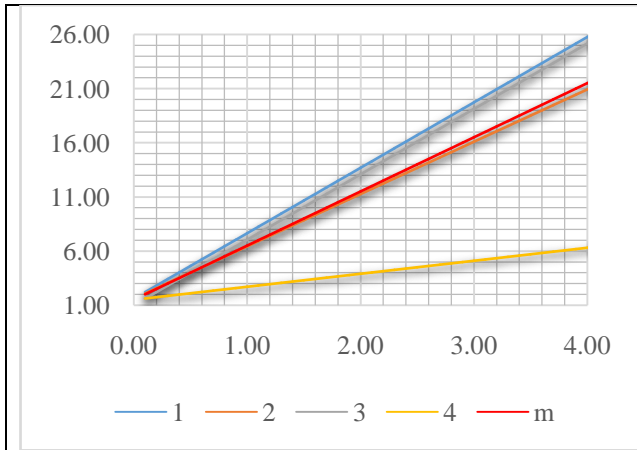


Figure 3: Approximately ductility capacities  $\mu_\theta$  curves for the ratio  $l_w/h_w$

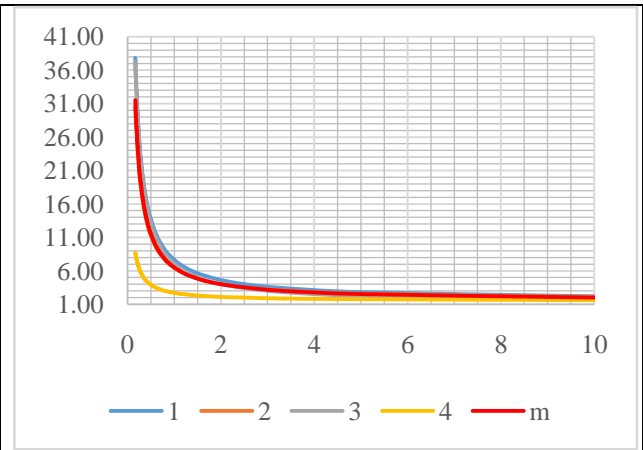


Figure 4: Approximately ductility capacities  $\mu_\theta$  curves for the ratio  $h_w/l_w$

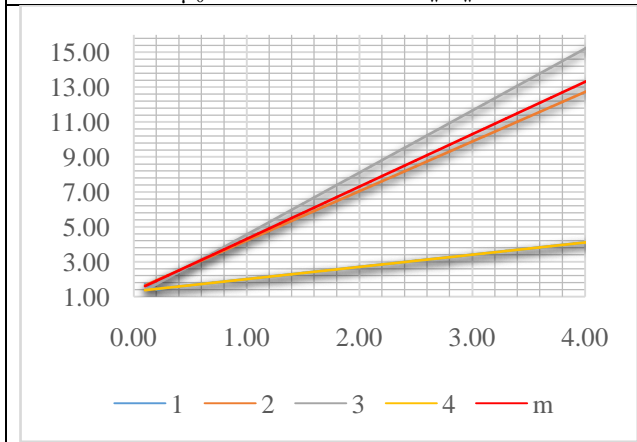


Figure 5: Approximately ductility capacities  $\mu_\Delta$  curves for the ratio  $l_w/h_w$

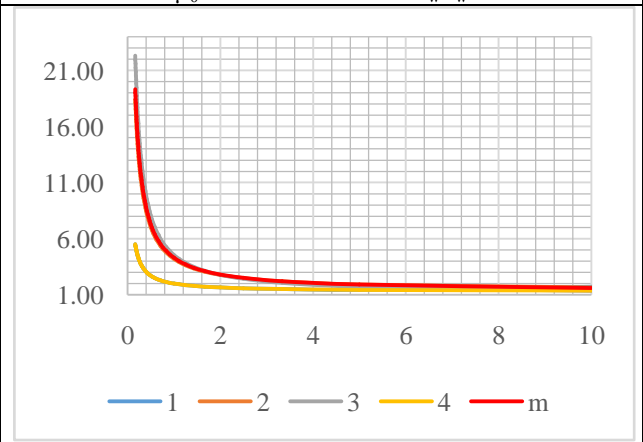


Figure 6: Approximately ductility capacities  $\mu_\Delta$  curves for the ratio  $h_w/l_w$

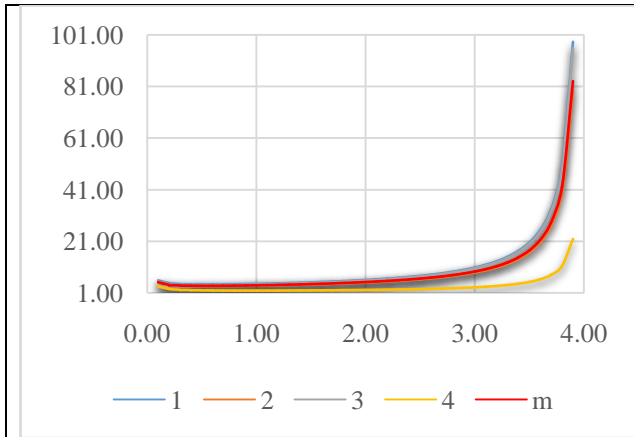


Figure 7: Approximately ductility capacities  $\mu_{\Phi}$  curves for the ratio  $l_w/h_w$

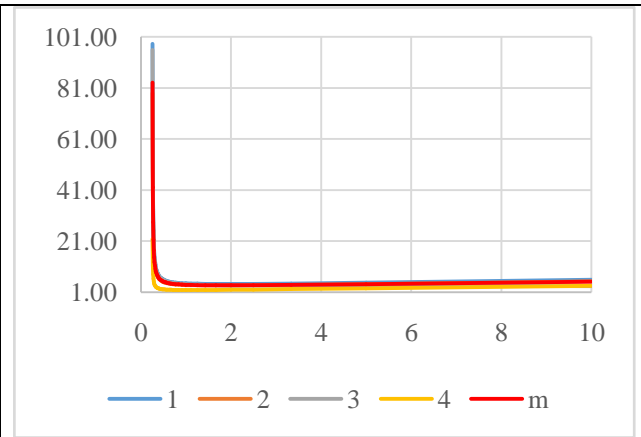


Figure 8: Approximately ductility capacities  $\mu_{\Phi}$  curves for the ratio  $h_w/l_w$

## 5. CONCLUSIONS

For six masonry wall types, models for ETABS and SAP were carried out to obtain the structural responses in order to compare somehow with the previous chapters presented above. [1], [2], [3], [4], [5]

According to **Priestley**, the masonry walls can be classify like:

- $\frac{h_w}{l_w} > 2.0$  – Ductile walls (P.D.);
- $1.0 \leq \frac{h_w}{l_w} \leq 2.0$  – Intermediary walls (P.I.);
- $\frac{h_w}{l_w} < 1.0$  – Squat walls (P.S.).

Another form of **Priestley** classification can be:

- $l_w < 0.5 h_w$  – Ductile walls (P.D.);
- $0.5 \leq l_w \leq h_w$  – Intermediary walls (P.I.);
- $l_w > 1.0 h_w$  – Squat walls (P.S.).

Another simplified similar classification appear from **Emilian Țîțaru**:

- $\frac{l_w}{h_w} < 0.6$  ( $\theta = 6\%$ ) – Short/Squat walls (P.S.);
- $0.6 \leq \frac{l_w}{h_w} \leq 1.5$  ( $\theta = 4\%$ ) – Intermediary walls (P.I.);
- $\frac{l_w}{h_w} > 1.5$  ( $\theta = 2\%$ ) – Long walls (P.L.).

Easily it can observe that more or less these relations are similarly even were proposed by different specialists at different historical periods.

In the paper „EN 1998: EUROCODE 8 Design of Structures for Earthquake Resistance” M.N. Fardish highlight that:

- $\mu_{\Delta} = q$  if  $T > T_c$  or
- $\mu_{\Delta} = 1 + (q-1) * (T_c/T)$  if  $T \leq T_c$  and/or the followings:
- $\mu_{\Phi} = 2q-1$  if  $T > T_c$  or
- $\mu_{\Phi} = 1 + 2(q-1) * (T_c/T)$  if  $T \leq T_c$ .

For building masonry structure, we expect to have the following values, depending on the type of masonry or of storeys:

Masonry types	URM (ZNA)	RM (ZC)	RM+HR (ZC+AR)
$\mu_{\Delta} = q$ according to P100/1-2013	<b>1.65-1.93</b>	<b>2.50-2.82</b>	<b>2.81-3.13</b>
Average 1	1.79	2.66	2.97
$\mu_{\Delta} = 1 + (q-1) * (T_c/T)$	<b>5.95-35.75</b>	<b>8.82-53.00</b>	<b>9.88-59.38</b>
Average 2	20.85	30.91	34.63
$\mu_{\Phi} = 2q-1$	<b>2.30-2.85</b>	<b>4.00-4.63</b>	<b>4.63-5.25</b>
Average 1	2.58	4.32	4.94
$\mu_{\Phi} = 1 + 2(q-1) * (T_c/T)$	<b>6.24-32.50</b>	<b>11.99-67.00</b>	<b>14.11-79.76</b>
Average 2	19.37	39.50	46.94

Average T	11.15	19.35	22.37
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- As the height of the structure is lower (the fundamental period of vibration results smaller than  $T_c$ ) as the structure is more ductile;
- Generally speaking almost in all the cases the  $T \leq T_c$  for masonry structures;
- Both the calculations performed by SAP2000 V17 and ETABS V13.1.5 shows ductility capacities in concordance with the results of approximate simplified calculations, presented above;
- After all the calculations made and from simplified displacement ductility relations obtained, may consider that they are close to  $q$  (median 2) and curvature ductility capacities are close to  $2q-1$  (mean 4), regardless the building height and/or the  $T_c/T$  ratio (relations specified also by M.Fardis).

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