

## COAST DOWN TEST – THEORETICAL AND EXPERIMENTAL APPROACH

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**ABSTRACT** – The paper presents considerations about the utility, suitability and limits of the coast-down experimental test in the automotive research. That test consists in launching of a motor vehicle from a certain speed with the engine disengaged and ascertaining of the current speed and distance covered during the free rolling, till vehicle stops.

First it is indicated a modality to determine by computation the vehicle kinematics. There are presented the vehicle resistances, the assumptions and the values influencing the analyzed process. Then, there are established the mathematical equations necessary to ascertain the speed and distance during cost-down. By the analysis of the different factors influences it can be determined the necessary precision of measurements, so that the results of the test might be conclusive.

Reciprocally, when experimental data exists, mathematical methods based on regression are presented to estimate the vehicle movement resistances.

In the paper are presented data obtained using coast-down method with different test vehicles and in various road conditions. The results include estimated values for the rolling resistance coefficient and aerodynamic drag coefficient. New ideas for additional valorisation of the method are also indicated.

### INTRODUCTION

Coast down is one of the most frequent tests for motor vehicles and consists in vehicle launch from a certain speed with the engine unengaged, simultaneously recording the speed and travelled distance until vehicle stops. This can be done for different reasons, mainly targeting to obtain valuable information about the general condition of the vehicle and about its interaction with the environment.

One main aim of this test is to evaluate the values of the resistant forces acting on the vehicle at certain speed and road conditions, so that to have the possibility to reproduce them on proving stands (dynamometers with or without rollers) for the measuring of fuel consumption, chemical or noise pollution or for other purposes. Another aim is to determine some possible abnormalities in the functioning of some subassemblies of the motor vehicle, i.e. to establish if the vehicle technical condition is acceptable in order to subject it to other, more complex, tests. The requirements to perform this kind of coast down tests are standardized [1], [2]. The value of coasting (free running) distance is then compared to values corresponding to some similar or closely related types of vehicles.

But these figures are not always available, especially when testing new models, which led to the idea of determining by computation the value of coasting distance and the vehicle's general dynamics. What follows is a briefing of the procedure proposed by the authors and that was used to realize a computer program. The results of simulation were confronted with experimental results, the model and the adopted hypothesis being satisfactory. The algorithm

is likely to be improved as well as more new ideas and experimental data will be accumulated.

### THEORY OF COASTING VEHICLE DYNAMICS

To obtain the movement equation of the vehicle, some simplifying hypotheses and a simple dynamic model were adopted [3]. The dynamic model was imagined based on the assumption that the motor vehicle consists of:

- mass  $m_t$  in translation motion;
- equivalent flywheel  $I_{nw}$ , corresponding to the non-driving wheels (kinematically unconnected to the transmission);
- equivalent flywheel  $I_{dw}$ , corresponding to the driving wheels and to all rotating parts kinematically connected to them (up to the shaft and toothed wheels of the gearbox).

For the coast down test will be assumed a road not too bumpy. Also, because the engine is disconnected, the wheels are not loaded with heavy moments. In these conditions, it can be assumed constant dynamic radius  $r_d$  for the vehicle wheels and very small tyre-ground slip. That means no energy loss will be generated by slip.

During moving, the vehicle interacts with the environment. As a result, some forces acts against it (figure 1): the vehicle gravitational force (the weight)  $W$ , inducing the reaction forces  $Z_1$  and  $Z_2$  normal to the road surface which, in turn, determines the rolling resistance forces  $F_{r1}$  and  $F_{r2}$  on the wheels of the front and rear axles, respectively; the air resistance force (aerodynamic drag)  $F_a$ ; the driving force at the wheel  $F_w$  that propel the vehicle.

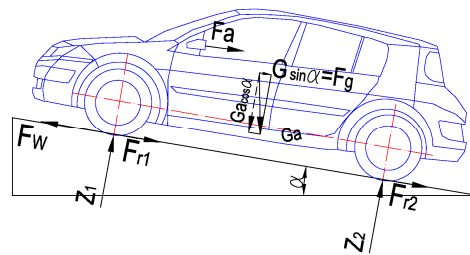


Fig. 1 – The external forces acting on a rolling vehicle

To obtain the equation of the vehicle motion, will be applied the principle of energy conservation, that states the derivative (the instantaneous change rate) of the vehicle’s total mechanical energy is equal with driving power  $P_w$  minus the total power losses:

$$\frac{dE_{tot}}{dt} = P_w - \sum P_{loss} \tag{1}$$

The vehicle’s mechanical energy consists on the kinetic energy  $E_k$  of the parts in translation and rotation and on potential energy  $E_p$ , determined by the vehicle altitude:

$$E_{tot} = E_k + E_p = E_{kt} + E_{kr} + E_p \tag{2}$$

The kinetic energy  $E_k$  include the one of the translational mass and those of any rotating part  $j$  kinematically connected with the wheels:

$$E_k = \frac{m_t v^2}{2} + \sum_j \frac{I_j \omega_j^2}{2} = \frac{m_t v^2}{2} + \frac{I_w \omega_w^2}{2} = \frac{m_{ap} v^2}{2} \tag{3}$$

where  $\omega_w = v r_d$  is the rotation speed of the wheels driving. The vehicle behaves as having a bigger mass, the apparent mass  $m_{ap}$ :

$$m_{ap} = m_t \left( 1 + \frac{I_w}{r_w^2} \right) = m_t \delta \quad (4)$$

where  $\delta = \left( 1 + \frac{I_w}{m_t r_w^2} \right)$  is the coefficient of the rotating parts influence and for cars is generally equal with 1.03...1.04.

The potential energy of the vehicle is:

$$E_p = m_t g h \quad (5)$$

where  $g = 9.81 \text{ m/s}^2$  represent the gravitational acceleration and  $h$  the altitude.

Introducing equations (2)...(5) in equation (1) obtains:

$$\frac{dE_{tot}}{dt} = m_{ap} v \frac{dv}{dt} + m_t g \frac{dh}{dt} = m_{ap} v a + m_t g v \sin\alpha = P_w - \sum P_{loss} \quad (6)$$

where  $\frac{dh}{dt} \approx \frac{ds}{dt} \sin\alpha = v \sin\alpha$  and  $a$  is the vehicle's instantaneous acceleration.

From the equation (6) results:

$$m_{ap} a = P_w/v - (\sum P_{loss})/v - m_t g \sin\alpha = F_w - (R_r + R_a) - R_g \quad (7)$$

that permits to calculate the vehicle's acceleration, in the well known form indicated in the literature (for example [4], [5]).

Because the engine is out of gear, at coasting vehicle the force  $F_w$  become negative due to the friction in axle mechanisms, drive line (if any) and transmission:

$$-F_w = R_f = M_f/r_d \quad (8)$$

where  $M_f$  represents an equivalent friction moment applied to the driving wheels.

Replacing equations (8) in equation (7) obtains:

$$-a = d = (R_r + R_g + R_a + R_f)/m_{ap} \quad (9)$$

which gives the vehicle deceleration  $d = -a$  during the coast down test.

In the equation (7)...(9),  $R_r$ ,  $R_g$  and  $R_a$  represent, respectively, the rolling resistance, the grade resistance and the air resistance, while  $R_f$  is the resistance due to the drivetrain frictions.

The formula that allows computing the air resistance (aerodynamic drag), will consider not only the vehicle-ground speed  $v$ , but also the wind velocity component along the running direction ([4], [5]), obtaining:

$$R_a = \frac{1}{2} \rho c_d A (v + w)^2 \quad (10)$$

where  $\rho$  is the air density;  $c_d$  – aerodynamic drag coefficient of the motor vehicle;  $A$  – frontal area of the vehicle;  $w$  – wind velocity in longitudinal direction.

Since the atmospheric conditions (pressure  $p$  and temperature  $t_C$ ) will be recorded for the test, the density of the air is determined according to them:

$$\rho = \rho_0 \frac{p}{p_0} \frac{t_{C0} + 273}{t_C + 273} \quad (11)$$

where  $\rho_0 = 1.225 \text{ kg/m}^3$  is the density of the air under standard conditions ( $t_0 = 15^\circ\text{C}$  and  $p_0 = 760 \text{ mm Hg} = 1 \text{ bar} = 101325 \text{ Pa}$ ).

The grade resistance, which is in fact the gravity's component parallel to the ground, is computed by the well known relation:

$$R_g = W \sin\alpha = m_t g \sin\alpha \quad (12)$$

where:  $W = m_t g$  is the vehicle weight and  $\alpha$  – the longitudinal slope angle of the road.

The rolling resistance is a mainly a consequence of the hysteresis (loss of energy, transformed partly in heat) manifested under deflection by the pair tyre-ground and by the suspension dampers and bushing. Its value can be computed with the equation:

$$R_r = W f \cos\alpha = m_t g f \cos\alpha \quad (13)$$

where:  $W \cos\alpha$  – is the vehicle's weight component perpendicular to the ground surface (pressing the tyres) and  $f$  is the (dimensionless) coefficient of rolling resistance. Experiments and finite element method simulations for tyre dynamic behaviour indicate an increase of the  $f$  value if the road is bumpier or the vehicle speed is higher [4], [5], [6]. For the moment, it isn't a generally accepted formula to compute the rolling resistance coefficient as function of vehicle speed, but practically all the references indicate a linear increase with speed for small and medium speeds (reported to the maximal admitted tyre working speed).

As the driving speed has moderate values during cost down running, the rolling resistance coefficient will be considered as a linear function of vehicle speed [4], [5], [6], [7]:

$$f = f_0 + f_1 v \quad (14)$$

where  $f_0$  and  $f_1$  are considered constants. The procedure [6] recommends the value  $f_1 = 0.2 \text{ s/m}$  in the case of high inflation-pressure truck tyres, while  $f_0$  varies with respect to the road condition, having an approximate minimum value of 0.0076 for quality roads.

In the same procedure [6], the influence of the suspension movements, generating supplementary losses mainly in the shock absorbers, rubber bushing, joints and tyres, are taken into account as an equivalent slope to overcome, greater if the road is more uneven, but unchanged with the vehicle speed. Due to the rapid and un-linear increase of the dampers forces with the speed, this hypothesis seems to simplify to much the true phenomenon and must be considered with prudence. Also, during tests, this suspension's energy loss is likely to appear as a supplementary rolling resistance. In this case, the equation (14) must be replaced with another, more complicated,  $f(v)$  function.

The last resistance to be considered is the one generated by the drivetrain frictions. This sum the frictional effects considered for the driving and non-driving wheels:

$$R_f = \frac{M_{fd}}{r_d} + \frac{M_{fn}}{r_n} = \frac{M_f}{r_w} \quad (15)$$

where  $M_f$  represents an equivalent friction moment applied to the driving wheels.

In the paper [3] were presented in more details the main causes of the drivetrain frictional resistance: the moment of friction losses in the wheel bearings; the moment of friction in the transmission, drive line and drive axle's mechanisms (final drive and differential), which sum the frictions in the bearings, gearings and sealing elements, but also the effects of energy consumed for oil bubbling.

The frictional torques on the driving and non-driving wheels,  $M_{fd}$  and  $M_{fn}$  can be measured if a dynamometer without rollers is available. These torques are dependent (in a complex way), on the rotational speeds of the wheels (related directly with the vehicle speed). If another

dynamometer, with rollers in this case, is also disposable, the sum of the drivetrain friction torque and rolling resistance torque can be measured. This summated torque depends on vehicle speed, but has a predominant linear component [7]. Aware of the extreme approximation, the drivetrain frictional resistance will be assumed as:

$$R_f = R_{f0} + R_{f1} v \quad (16)$$

where  $R_{f0}$  is the drivetrain resistance at very low speeds and  $R_{f1}$  is the increasing rate with speed  $v$  of these resistance.

Now, introducing in equation (9) the equations (10), (12), (13), (14) and (16), the expression of the vehicle deceleration during coast-down can be presented as a final function of speed:

$$\begin{aligned} d &= [m_t g (f_0 + f_1 v) \cos\alpha + m_t g \sin\alpha + \frac{1}{2} \rho c_d A (v + w)^2 + R_{f0} + R_{f1} v]/m_{ap} = \\ &= C_0 + C_1 v + C_2 v^2 \end{aligned} \quad (17)$$

where:

$$\begin{aligned} C_0 &= [m_t g (f_0 \cos\alpha + \sin\alpha) + \frac{1}{2} \rho c_d A w^2 + R_{f0}]/m_{ap} \\ C_1 &= (m_t g f_1 \cos\alpha + \frac{1}{2} \rho c_d A w + R_{f1})/m_{ap} \\ C_2 &= (\frac{1}{2} \rho c_d A)/m_{ap} \end{aligned} \quad (18)$$

For the first two equations (18) it observes that the unconsidered wind speed will modify apparently the rolling resistance coefficient, as well the drivetrain frictions. A similar unwanted effect can have a road which isn't quite horizontal.

If assumes a flat road ( $\alpha = 0$ ) and no wind ( $w = 0$ ), the first two equations (18) simplify:

$$\begin{aligned} C_0 &= f_0 g/\delta + R_{f0}/m_{ap} \\ C_1 &= f_1 g/\delta + R_{f1}/m_{ap} \end{aligned} \quad (19)$$

## COAST-DOWN MATHEMATICS

If precise and sufficient experimental data exists about the vehicle deceleration, equation (17) can be used to find the unknown constants  $C_0$ ,  $C_1$ ,  $C_2$  and then, based on these values, to determine important vehicle dynamics figures as coefficients of rolling-, aerodynamic- and drivetrain drag. Before the coast-down tests, the vehicle weight, frontal area and wheels radius must be measured and the equivalent moments of inertia must be estimated (using experimental methods, CAD programs or a combination of these). That will permit to calculate the coefficient  $\delta$  of the rotary masses influence and the apparent mass  $m_{ap}$ . Also, the atmospheric data (pressure, temperature, wind speed) must be recorded.

Pulling slowly on a horizontal road (at least two times) the vehicle at different weight levels  $W=m_t g$  and measuring the necessary forces  $F_p$ , the coefficient of the rolling resistance at low speed  $f_0$  and the drivetrain friction force at low speed  $R_{f0}$  can be calculated. Indeed, from the equation (7) results:

$$m_{ap} a = 0 = F_p - F_f - R_r = F_p - R_{f0} - m_t g f_0 \quad (20)$$

The values  $f_1$  and  $R_{f1}$  can be obtained repeating the procedure at a higher speed or, better, measuring torques on dynamometers. Lastly, the aerodynamic drag coefficient can be obtained if calculates first the  $C_2$  coefficient of the equation (18).

To find the coefficients  $C_0, C_1, C_2$ , there are necessary three different pairs of experimental values  $(V_i, D_i)$ , which means to find the coefficients of the interpolation polynomial of second degree:

$$d_i(v) = C_0 + C_1 v + C_2 v^2 \quad (21)$$

passing through these points  $i$ . Generalising, the interpolation will be adopted to find the coefficients of a function with known or presumed shape, considering a number of experimental points equal with the number of unknown parameters of the function.

The “method Bosch” [8] bases on that, assuming  $d_i(v) = C_0 + C_2 v^2$ , i.e. a parabolic dependence of  $d$  on  $v$ , with minimum deceleration for very small speed (it assumes constant value for the coefficient of rolling resistance, no drivetrain drag and negligible rotary inertia).

Because the experimental data are affected by errors, a better solution consists in replacing interpolation with regression (approximation) functions, because this way will be used the information contained by all measurement data. Considering equation (17), that means to find the polynomial of second degree

$$d_a(v) = C_0 + C_1 v + C_2 v^2 \quad (22)$$

that is the best fit of number of experimental data points  $(V_i, D_i)$ , which is bigger than the number of function’s unknown parameters (three in this case,  $C_0, C_1, C_2$ ). There are many possibilities to realise the “fitting” (best approximations) of the presumed function with the experimental data, each based on different assumptions.

The authors adopted the most used method – the best fit in the least-squares sense. That method bases on the next suppositions: the argument (the first value of the pair,  $v$  in this case) is not affected by errors; all the errors pertain to the function (the second value of the pair,  $d$  in this case); the distribution of the errors is a normal (Gaussian) one. Normally, these conditions are not fulfilled when records kinematic data, and may be another cause for some less precise or erratic results.

Most of the commercial mathematical computer-programs have implemented this kind of Gaussian regression as basic procedures, at least for the most common mathematical functions, as is the  $n^{\text{th}}$  order polynomial.

To generalise more the application of the cost-down method, the authors developed an algorithm which can be used, with only few modifications, for any function form with any number of unknown parameters. Starting from coarse estimations of these parameters, an iterative process, based on Levenberg-Marquardt method, will find estimations (“fitted” guesses) of the parameters. In other words, the procedure will change the “shape” of the function  $d_a(v)$ , minimizing the sum of squared errors:

$$S(C_0, C_1, \dots) = \sum_i [d_a(V_i, C_0, C_1, \dots) - D_i]^2 \quad (23)$$

until the change of that sum from one step to other will be smaller as an imposed tolerance.

## COAST-DOWN EXPERIMENTS

To obtain data about the vehicle kinematic behaviour during coast-down, many acquisition systems can be used, as accelerometers or time-speed-distance recorders. Each can be more or less suitable for that test. For coast-down acquisition, the authors have used different systems:

- three “fifth-wheel” type time-speed-distance recorders (one Peiseler and two build “in house”) [3], [9];

- three GPS-based acquisition devices (Garmin GPSmap 60CSx – 1 Hz, Racelogic Vbox III – 100 Hz professional device and, also a system build “in house” [10], DS-5 with sensor Garmin GPS 18x-5Hz) [10], [11], [12];
- one CORRSYS-DATRON's non-contact Correvit system with optical sensor;
- one on-board acquisition systems used for the vehicle’s ABS, with information available at the CAN bus and accessed by the OBD-II interface [10], [12].

The sample rate of these systems varies from 1 Hz to 100 Hz. The data available are the time, the speed and the distance (excepting the vehicle’s computer). The Correvit system offers also the acceleration. The filtering methods, excepting the “in house” systems, are less- or unknown. Practically, all systems used in these work measure the time and the distance. Then, by numerical derivation, it calculates the speed. Some GPS-based systems (as Racelogic Vbox) use also the Doppler effect to diminish the speed errors. Excepting the Correvit system, that repeats the derivation of the speed to obtain the acceleration, the use of all other presented systems needs numerical methods to calculate the acceleration.

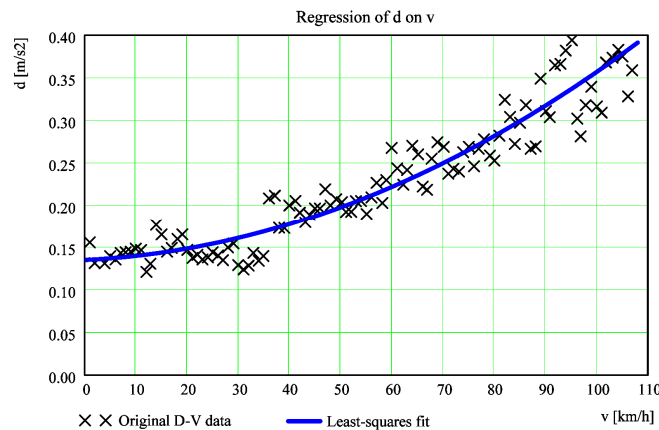


Fig. 2 – Least-squares approximation of experimental data (107 V-D points) with a second degree polynomial:  $d(v)= 0.135+0.00104 v + 0.00025 v^2$  ( $v$  in m/s,  $c_d=0.342$ )

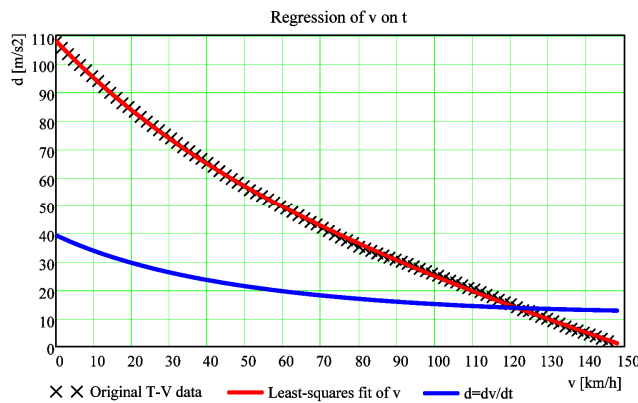


Fig. 3 – Least-squares approximation of experimental data (91 T-V points) with a tangent function having the derivative:  $d(v)= 0.133+0.00140 v + 0.00024 v^2$  ( $v$  in m/s,  $c_d=0.342$ )

In this work were tried (with almost similar results) two formulas for numerical derivative, needing respectively three and five neighbouring points ( $T_i, V_i$ ) [13]:

$$D_i = -\frac{V_{i+1} - V_{i-1}}{T_{i+1} - T_{i-1}} \quad D_i = -\frac{-V_{i-2} + 8V_{i-1} - 8V_{i+1} + V_{i+2}}{6(T_{i+1} - T_{i-1})} \quad (24)$$

Figure 2 shows how a second degree polynomial approximate, in the least-squares sense, the “cloud” of experimental points ( $V_i, D_i$ ).

In the paper [7] it is indicated the function:

$$v = -b/(2a) - r \operatorname{tg}(u t + w) \quad (25)$$

as a solution of the equation (9). The results plotted in figure 3 were obtained using this “fitting” function and the described algorithm, that are practically similar with the results of “Takagi” method [7] and with the ones of second-degree polynomial regression. The only observed need is to have records with (almost) constant step on time for the tangent function and on speed for the polynomial one, to be closer to the requirements of least-squares method.

## CONCLUSIONS

The paper presents theoretical and practical considerations regarding the modality to valorise the potential of the coast-down test to obtain valuable vehicle characteristics, in an easy way and with accessible means. The use of this experimental method is facilitated by the progresses in data acquisition systems, mainly GPS based, which ensures high accuracy at an affordable price. The algorithm presented is applicable for any type of function describing the vehicle movement. The acquisition errors and the change of wind speed and road slope are the main items affecting the coast-down test accuracy.

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