CONSTRAINT FACTORS USED IN LIMIT ANALYSIS OF POLYETYLENE PIPES SUBMITTED TO INTERNAL PRESSURE

M. Mouwakeh*, G. Pluvinage**, S. Masri*

Department of Applied Mechanics, Faculty of Mechanical Engineering,

University of Aleppo*

E.N.I.M., Metz, France**

Corresponding author : pluvinage@cegetel.net

Abstract : By using codes and finite element (FE) analysis, limit load solutions of pipes containing surface cracks are determined. The study covers cracks with constant crack length and depth in pipes with different diameters D: 75, 90, 110, 125 and 160 mm . The crack configurations consist of semi-elliptical surface cracks with a/D =0.067 , 0.056 , 0.045 ,0.040 and 0.031.
The cracked pipes are subjected to internal critical pressure values fromby codes ASME B31G and Choi's formulas. Due to ductile behavior of polyethylene pipe, failure occurs plastic collapse i.e when the critical net stress reaches ultimate strength multiply by constraint factor. In this paper the constraint factor is evaluated and it=ts evolution with pipe diameter is analyzed. Three different definitions of the constraint factor based on global or local approaches are also compared

Key words: Limit Analysis, Constraint Factor,Crack, Ultimate Pressure, Finite Element.

1. INTRODUCTION

The theory of limit analysis has appeared in the late 30s of last century, it constitutes a branch of the theory of plasticity related to an elastic perfectly plastic behavior.
A lot of works has been conducted to obtain limit load solutions in pipes containing surface cracks subjected to internal pressure only or combined load (internal pressure and bending) [1,2,3,4,5,6,7,8,9].
Generally, pipes fail in a ductile manner due to the behaviour of the constitutive material. For these situations, failure prediction tools are based on limit analysis. This failure criterion assumes that failure occurs when critical net stress σ_N^c reach ultimate strength Rm. One notes that ductile failure is sensitive to net stress σ_N (load divided by the ligament cross section) whatever brittle fracture is sensitive to gross stress σ_g(load dividedby the entire section). The above mentioned criterion need to be modified to take into account, constraint, geometry and loading mode effects in the following manner:

$$\sigma_N^c = L.Rm \qquad (1)$$

where L is the so –called constraint factor.

Design codes for pipes such as code ASME B31G [10] and Choi's formulas [11]are based on limit analysis to calculate a critical internal pressure. They are based on limit analysis and incorporate safety factor through a lower bound of a plot of experimental results. The basic question is the values of the constraint factor which is incorporate in these codes and if this value is close to values generally obtained. In addition, it is interesting to know the evolution of constraint factor with ligament size, defect and pipe geometries.

$$L = L\left(\frac{a}{D}, \frac{a}{t}\right) \qquad (2)$$

where D is pipe diameter, t thickness and a defect depth.
Determination of critical pressure by codes ASME B31G and Choi's formulas has been made for 5 pipe diameters D, 5 thickness t and constant defect depth a. These critical pressures p_c lead to values of critical net stressσ_N^c and then to constraint factor L :

$$L = \sigma_N^c / Rm \qquad (3)$$

Calculation of stress distribution along the ligament has done using Finite Element method depending on pressure values obtained from ASME B31G and Choi's formulas and then average value of maximal principal stress σ_m is calculated. Determination of constraint factor deduced from finite element method can be expressed as following:

$$L^{*} = \sigma_m / R_m \qquad (4)$$

Stress distribution ahead of the crack tip led us to determine failure process zone using the Volumetric Method [12] which is is a local fracture criterion.Inside the fracture process, the effective stress σ_{ef} which is the average value of the stress distribution acting as a local fracture stress. Another definition of the constraint factor can be made:

$$L^{**} = \sigma_{eq} / R_m \qquad (5)$$

These three different constraint factors are compared in a discussion.

2. MATERIAL AND DEFECT GEOMETRY

The studied pipes are made of high density polyethylene which has the following characteristics:

Table 1 : Mechanical properties of polyethylene

σ_y(MPa)	ρ(kg/m^3)	E (MPa)	ν
23.00	960.00	400.00	0.45

whereσ_y, ρ, E and ν are respectively yield stress, volume mass, Young's modulus and Poison's ratio.

3. CRITICAL PRESSURE

Critical Pressure is calculated for different pipe diameters by using codeASME B31G and Choi'formulas as following:

ASMEB31G

For Parabolic defects ,the critical failure pressure is given by the following expression:

$$P_f = \frac{2(1.1\ \sigma_Y) \times t}{D} \left[\frac{1-(2/3)\ \times (a/t)}{1-(2/3)\ \times (a/t)/M} \right], \qquad (6)$$

$$\text{where} \quad M = \sqrt{1+0.8\left(\frac{L}{D}\right)^2 \left(\frac{D}{t}\right)}, \quad \text{for} \quad \sqrt{0.8\left(\frac{L}{D}\right)^2 \left(\frac{D}{t}\right)} \le 4$$

where, P_f , D , a , t , M , σ_Y and L are the failure pressure, outer diameter, maximum corrosion depth, wall thickness, bulging factor, yield stress and longitudinal corrosion defect length, respectively.

Choi'sformulas

$$P_f = \begin{cases} 0.9 \times \dfrac{2(\sigma_U) \times t}{D_i}\left[C_0 + C_1\left(\dfrac{L}{\sqrt{R}}\right) + C_2\left(\dfrac{L}{\sqrt{R}}\right)^2\right], & \dfrac{L}{\sqrt{R}} < 6 \\ 1 \times \dfrac{2(\sigma_U) \times t}{D_i}\left[C_3 + C_4\left(\dfrac{L}{\sqrt{R}}\right)\right], & \dfrac{L}{\sqrt{R}} \geq 6 \end{cases} \quad (7)$$

where P_f , σ_U , D_i , a , t and R are the failure pressure or maximum pressure, ultimate tensile strength, inside diameter, defect depth, wall thickness and average pipe radius, respectively.

In this study five pipe diameters with different wall thicknesses are chosen. The dimensions of pipes are as the following:

Table 2 : pipe diameter and thickness.

D (mm)	**t (mm)**
75.0	6.8
90.0	8.2
110.0	10.0
125.0	11.4
160.0	14.6

where D is the Diameter and t is the wall thickness.The crack geometry is assumed to be semi-elliptical where *2c* is the crack length and *a* the crack depth. The crack configuration is identical for all pipes. The crack dimensions are as the following: 2c = 100 mm , a= 5mm.

Evolution of computed critical pressure versus a/D ratio is shown in figure (1). One notes that code ASME B31G and Choi's formulas codes have the same attendance, i.e. the critical pressure decreases in linear manner with increasing of crack depth or a/D ratio. By comparison of the two methods, we notice that the maximum relative difference is about 30%.Curve fitting procedure in Matlab gives the following relationships:

ASME B31 G P_f= -37.77(a/D)+ 5.1869

(Choi) P_f= -63.244(a/D)+ 6.0994

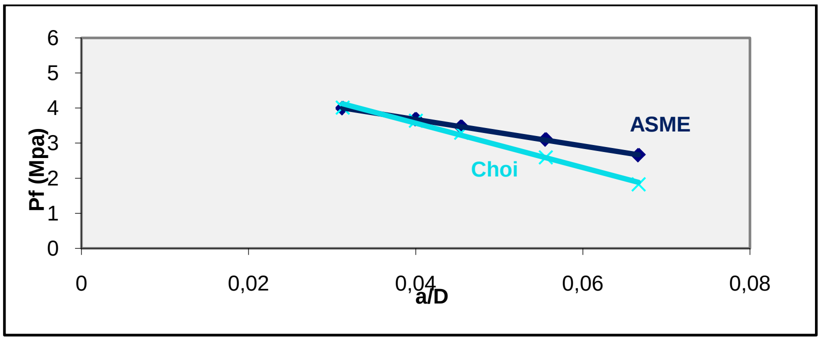

Figure 1: variation of critical pressure versus a/D ratio

4. CRITICAL NET STRESS

Critical net stress is calculated from critical pressure values for different diameters by using thefollowing formula:

$$\sigma_N^c = p_f . D/2t^* \qquad (8)$$

where$\sigma_N^c$is the critical netstress, p_f is the critical pressure and t* is the length of the ligament :

$$t^* = t - a \qquad (9)$$

wheret is the ligament thickness and a is the crack depth.From figure2 we notice that the critical stress increases with polynomial manner with the increasing of a/D ratio or with the decreasing of diameter.By comparison of the two methods, we notice that the maximum relative difference is about 30%.By using curve fitting procedure , we obtain :

(ASME B31G)

σ_N^c= 1E+07$(a/D)^4$ - 2E+06$(a/D)^3$ + 135403$(a/D)^2$ -3626.2(a/D)+66.558

(Choi)

σ_N^c=1E+07$(a/D)^4$ - 2E+06$(a/D)^3$ + 134817$(a/D)^2$ - 3188.4(a/D)+57.355

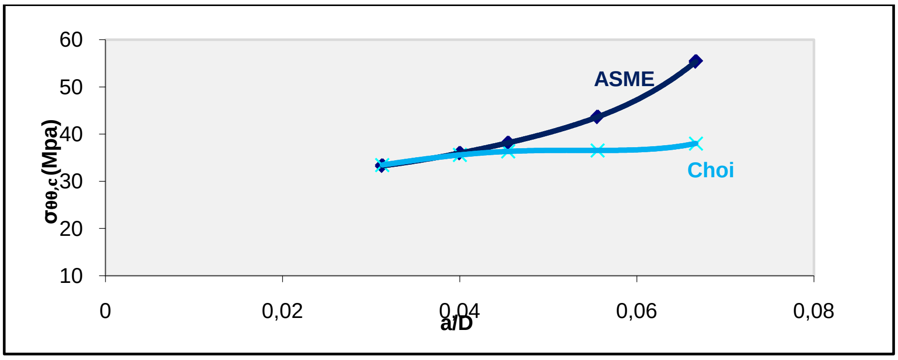

*Figure 2:*variation of critical net stress versus a/D ratio

5. CONSTRAINT FACTOR L

L is calculated for each different pipe diameters by using the formula :

$$L = \sigma_N^c / R_m \tag{10}$$

Where σ_N^c is critical net stress obtained from codes , R_m ultimate tensile strength.

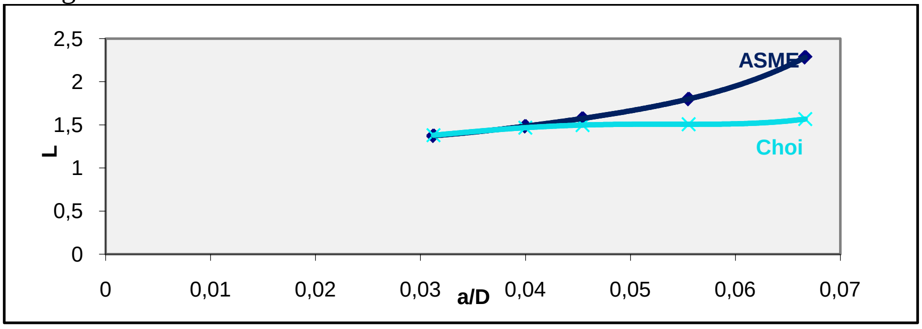

Figure 3: variation of constraint factor L versus a/D.

One note from figure 3 that the constraint factor L increases with the increasing of a/D ratio or decreasing diameter.The maximum relative difference between the two methods is about 30% according to :

(ASMEB31G)

$L = 531608(a/D)^4 - 87133(a/D)^3 + 5586.4(a/D)^2 - 149.61(a/D) + 2.746$

(Choi)

$L = 537902(a/D)^4 - 92389(a/D)^3 + 5562.2(a/D)^2 - 131.55(a/D) + 2.3663$

6. AVERAGE CRITICAL STRESS OVER LIGAMENT ; CONSTRAINT FACTOR L*

The Finite Element Method (FEM) program ABAQUS [13] was used for the computations of the stress distribution over ligament. The material is assumed to be completely elastic perfectly plastic obeying the Von-Mieses flow criterion. Critical pressure values obtained from codes are applied on the internal surface of the pipe. FE calculations are done for different diameters toobtain the maximal principal stress and σ_m stresswhich is the average of the distributed stress over the ligament.For this purpose 3D FE model is used for the analysis as shown in the figure 4.

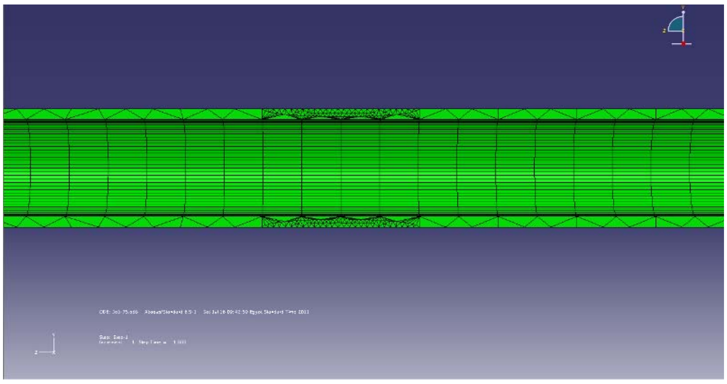

Figure 4: FE model of a surface crack in pipe

Mean stress increasesin polynomial manner with the increasing of a/D ratio until a/D=0.06 and then starts decreasing .We obtained polynomial function by using curve fitting as following :

(ASMEB31G)

σ_m = -2E+08$(a/D)^4$ + 3E+07$(a/D)^3$ - 2E+06$(a/D)^2$ + 56990(a/D)- 580.32

(Choi)

σ_m = -1E+08$(a/D)^4$ + 3E+07$(a/D)^3$ - 2E+06$(a/D)^2$ + 51002(a/D)- 520.79

L* is obtained from the formula :

$$L^* = \sigma_m / R_m \qquad (11)$$

where$\sigma_m$is the average of distributed stress in the ligament obtained from FE method and Rm is the ultimate tensile strength.Figure 5 shows the variation of constraint factor L* versus a/D ratio for both codes. We notice that constraint factor L* increase in polynomial manner with increasing of a/D ratio until a/D=0.06 and then starts decreasing.

7. DETERMINATION OF CONSTRAINT FACTOR THROUGH VOLUMETRIC METHOD

The volumetric method is a local fracture criterion, which assumed that the fracture process requires a certain volume. This volume is generally assumed as cylindrical with a diameter called effective distance X_{ef}. The physical meaning of this effective distance corresponds to the size of the high stressed region at defect tip. This effective distance is considered as the distance of the inflexion point on the stress distribution at defect tip. A graphical method based on the relative stress gradient χ associates the effective distance to the minimum of χ.The relative stress gradient is given by:

$$\chi(r) = \frac{1}{\sigma_{yy}(r)} \frac{\partial \sigma_{yy}(r)}{\partial r} \qquad (12)$$

$\sigma_{yy}(r)$ is the opening stress. The effective stress is defined as the average value of the stress distribution over the effective distance and weighted by the stress gradient.

$$\sigma_{ef} = \frac{1}{X_{ef}} \int_0^{X_{ef}} \sigma_{yy}(r) \times (1 - r \times \chi(r))\, dr \qquad (13)$$

r is distance. Figure 6 gives an example of stress distribution along ligament at defect tip for the pipe of 125 mm diameter submitted to critical pressure calculated by ASME B31 G code.

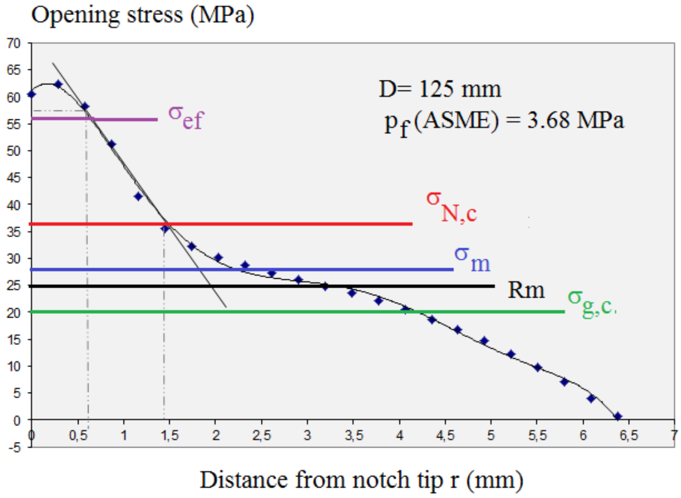

*Figure 6:*stress distribution along ligament at defect tip for the pipe of 125 mm diameter submitted to critical pressure calculated by ASME code.

Figure 6 gives also the relative values of these stresses compare to the ultimate strength Rm=24.38MPa.

$$\sigma_{ef} > \sigma_{N,c} > \sigma_m > R_m > \sigma_{g,c} \qquad (14)$$

8. DISCUSSION

Constraint factor gives an idea of the stress elevation due to constraint introduces by geometry, ligament size, thickness and gradient effect.

However several definition of the critical stress can be used as the critical net stress, the effective stress, the mean stress or the critical gross stress. They leads to different definition of constraint factor, L, L*, L**and L***:

$$L = \sigma_N^c / Rm \quad L^* = \sigma_m / R_m \quad L^{**} = \sigma_{eq} / R_m \quad L^{**} *= \sigma_{g,c} / R_m$$

L** cannot be considered as a constraint factor because plastic collapse is sensitive to net stress and not gross stress. It is given as indicative values.

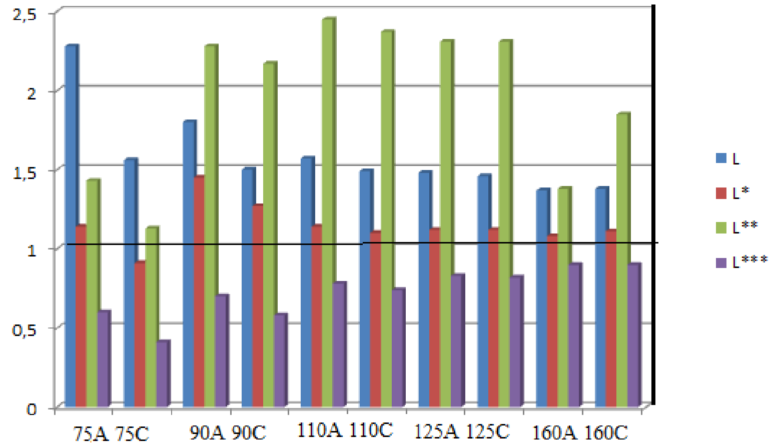

Figure 7: Chart of constraint factor values according to diameter and critical pressure method (A: ASME code ; C Choi's method).

Values of L*** $L^{**} *= \sigma_{g,c} / R_m$ are always less than unit. This point confirms that plastic collapse is not sensitive to gross stress and then L*** cannot be considers as a constraint factor.
$L^{**} = \sigma_{eq} / R_m$ is a priory the most realistic value of the constraint factor if we assumes that ductile failure needs a fracture process zone and that in this zone fracture occurs when then effective stress reach a critical value. Maximum value of L** is L** =2.45 which is less than the theoretical value of L**=3 for pure plane strain conditions and for a Poisson's ratio of ν=0.3. Except for small and large diameters L** has a value greater than 2 which seems acceptable.
$L = \sigma_N^c / Rm$values given by codes are less than 1.5 and conservative.

Values of L* $L^* = \sigma_m / R_m$ are less than values of L. Both definitions refer to an average value of the net stress over the ligament. σ_m is an average value over the ligament the longitudinal stress distribution and $\sigma_{N,c}$ is the gross stress of a pipe of a reduced thickness. This explains certainly this difference.

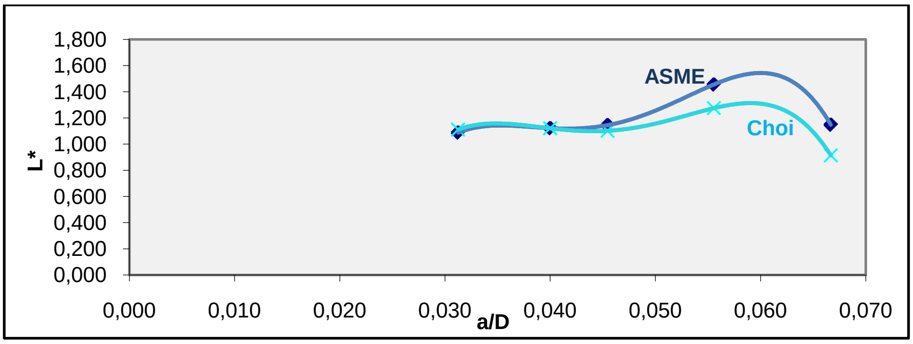

*Figure 8:*Variation of constraint factor L* versus a/D

(ASMEB31G)

$L^* = -7E+06(a/D)^4 + 1E+06(a/D)^3 - 81130(a/D)^2 + 2351.3(a/D) - 23.942$

(Choi)

$L^* = 6E+06(a/D)^4 + 1E+06(a/D)^3 - 71868(a/D)^2 + 2104.2(a/D) - 21.487$

9.CONCLUSION

Polyethylene pipe fail in ductile manner and failure criterion is plastic collapse predicted by Limit Analysis. In this criterion, failure occurs when net stress reaches the ultimate strength multiplied by the constraint factor L.

This constraint factor quantifies the increases of flow stress due to the plasticity preventing due to geometrical effect, scale and gradient effects.

In this study critical internal pressure has been obtained from codes ASME B31 and Choi's method.

Three definitions of the constraint factor has been proposed L; L*and L**. The last one L** is based on a local failure criterion called the Volumetric method and is certainly the more realistic if we assumes that ductile failure needs a fracture process zone and that in this zone fracture occurs when then effective stress reach a critical value . This definition leads to L** close to 2 but sensitive to pipe geometry which seems realistic. Value L obtained directly from critical pressure given by ASME code or Choi's method are more conservative.

Values of constraint factor are helpful for the choice of pipes material through the value of the ultimate strength.

REFERENCES

[1] DNV-RP-F101.Corroded Pipelines,*Det Norske Veritas*,(1999).

[2] Dowling, A.R., Towley, C.H.A., The effect of defect on structural failures: a two-criteria approach, *International Journal for Pressure Vessels and Piping*,Vol. 3, 2 : 77-107,(1975).

[3] Pluvinage,G.General approaches of pipeline defect assessment. Laboratoire de FiabilitéMécanique ENIM-UPV Metz (France),(2008).

[4] Staat, M.Local and global collapse pressure of longitudinally flawed pipes and cylindrical vessels, *International Journal for Pressure Vessels and Piping* ,82, (3) : 217–225,(2005).
[5] Staat, M. Vu Duc Khoi. Limit loads of circumferentially flawed pipes and cylindrical vessels under internal pressure. International Journal for Pressure Vessels and Piping,83, (3) :188–196,2006, .
[6] Sattari-Far,I.Dillstrom,P.Local limit load solutions for surface cracks in plates and cylinders using finite element analysis. *International Journal for Pressure Vessels and Piping*,81,(1) :57–66,(2004).
[7] Lei,Y.J-integral and limit load analysis of semi-elliptical surface cracks in plates under combined tension and bending. *International Journal for Pressure Vessels and Piping*, (2003).
[8] Yagawa,G.Kitajima,Y.Ueda,H.Three-dimensional fully plastic solutions for semi-elliptical surface cracks. *International Journal for Pressure Vessels and Piping*, 74 :77–80, (1997).
[9] Miller, A.G.Review of limit loads of structures containing defects. *International Journal for Pressure Vessels and Piping*, 32:197–327,(1988).
[10] ASME B31G, Manual for determining the remaining strength of corroded pipelines, The American Society of Mechanical Engineers, New York, USA1991,.
[11] Choi, J. B. Goo, B. K. J. Kim, C. Kim, Y. J. Kim, W. S. Development of limit load solutions for corroded gas pipelines, *International Journal of Pressure Vessel and Piping*, 80, (2) :121-128,(2003).
[12] Pluvinage, G. Fracture and Fatigue emanating from stress concentrators, *Editeur Kluwer*, (2003).
[13] ABAQUS, RI: Hibbitt, Karlsson and Sorensen,Inc V5.8, Providence, New York ,USA,(1998).