



## THE SYSTEM RESPONSE AT THE DIFFERENT NATURAL FREQUENCIES

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**Abstract:** At the application of a dynamic excitation for a structure (a. g. freely hung flat plate) the response of this can be measured as displacement, velocity or acceleration; in fact will measure the amplitude of the plate response when the excitation coincides with each clear resonant frequency for each peak from the FRF.

In general Modal Analysis and also Impact Modal Analysis are a wide range of application fields; all linear dynamic systems whose inputs and outputs can be registered, can be investigated by modal analysis.

**Keywords:** vibrations measurement, excitation type, peaks of diagram.

### 1. INTRODUCTION

The usual response in a modal analysis test of a mechanical structure is the motion of the object, expressed as displacement, velocity or acceleration; theoretically it is irrelevant which of the three motion parameters is measured:

-displacement measurement has importance on the low frequencies;

-acceleration measurement is for higher frequencies domain;

-velocity vibration (through root mean square value) is considered to be a measure of severity of the vibration.

To measure of the velocity may be an important reason for relate the vibration energy, but both displacement/ velocity transducers tend to be relatively heavy.

In opposite of these, the lower mass of an accelerometer will have a much smaller influence when are attached to the structure.

The measurement result will be more accurate, these signals may be easily and validly integrated electronically to obtain velocity and displacement.

### 2. THE MODE SHAPES OF A SIMPLE PLATE

If we place fifteen evenly distributed accelerometers on the plate and will measure the amplitude of the plate response when the excitation coincides with each of the four resonant frequencies at each of the peaks in the Frequency Response Function (FRF). In Fig.1 it shows the deformation patterns that results when the excitation coincides with one of the natural (eigen) frequencies of the system.

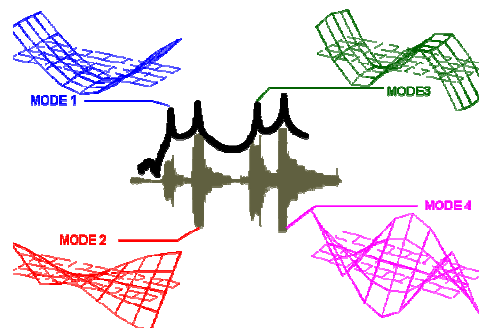


Figure 1: Response of the plate for each peaks on the FRF

The response of the structure is different at each of the different natural frequencies; these deformation pattern (responses) are called mode shapes.

In fact Modal Analysis is the study of the natural characteristics of the structures; both the natural frequency, which depends on the mass and stiffness distributions in each structure and mode shape are used to help and improve the structural system for noise and vibration applications.

### 3. THE DESCRIPTION OF THE IMPACT TESTING

Any vibration test system requires a device to subject the test object at some vibration motions, in this case is used non-fixed excitation, the best known example is the hammer excitation; this presents an important advantage, don't being attached on the structure.

As mentioned before, the aim of the exciting of the desired investigation structure is to generate a certain level of force over a specified frequency range; an impact input, e.g., using an impact hammer, generates a relatively smoothly evolving force level up to a specific frequency.

The hammer is instrumented with a force transducer; a hard hammer tip, a low hammer weight, a low force and a hard test object surface will generally cause a short contact time, between hammer and test object; the energy and the frequency span are determined by the force of the user, the weight/ hardness of the hammer tip and the compliance of the impact point on the structure.

The closer of the input force approximate a Dirac Function impulse (zero duration, infinite amplitude, unit energy content) able to see the Fig. 2.a., the wider of the excited base band frequency span will be.

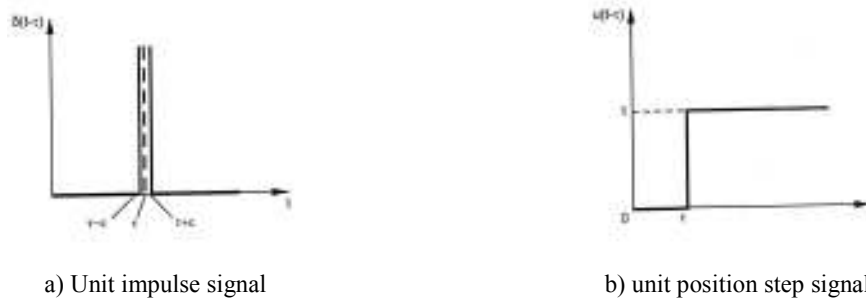


Figure 2: Excitation impulse for a mechanical system

The unit impulse (Dirac Function) applied at the  $t = \tau$  moment, is defined with relations:

$$\delta(t - \tau) = 0 \quad \text{for } t \neq \tau \quad \text{we have} \quad \lim_{\epsilon \rightarrow \infty} \int_{\tau - \epsilon}^{\tau + \epsilon} \delta(t - \tau) dt = 1 \quad (1)$$

In this case for the  $\zeta[\delta(t)] = 1$ , the transfer function of the desired system represents the response (in complex domain) of the excited system with one impulse function.

The response in time of the excited system at the moment  $t=0$  with an unit impulse signal is given with the inverse Laplace transform, with relation:

$$w(t) = \zeta^{-1}[W(p)] \quad (2)$$

where  $W(p)$  is the transfer function.

Since the unit impulse signal is difficult to obtain in physical form, for experimental test use the indicial function; the dynamic response of excited system through one unit position step signal is named the indicial function, see fig.2, b.

This signal, apply at the moment  $t = \tau$  is defined with :

$$\begin{aligned} u(t - \tau) &= 0 \quad \text{for } t < \tau \\ u(t - \tau) &= 1 \quad \text{for } t \geq \tau \end{aligned} \quad (3)$$

Heavy hammers with soft tips will cause a long contact time, especially the lower frequencies of structures will be excited; in extreme cases this excitation is used for very heavy structures, exhibiting very low resonance frequencies (buildings, foundations, trains, ships).

As see in Fig. 3, hammer excitation always generated a smoothly evolving force up to a specific frequency; on the other hand, since the total input energy is concentrated in a very short time.

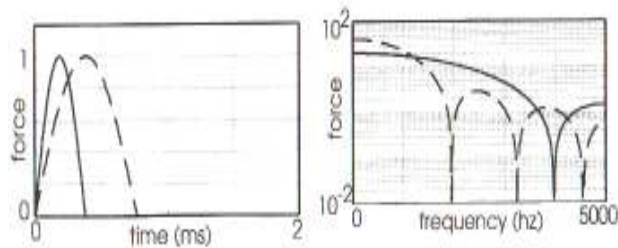


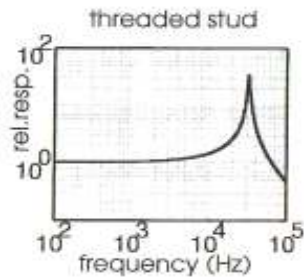
Figure3: Hammer signals and their frequency content

The problems arise for nonlinearities and local deformations; actual, some companies developed “automatic hammers” in order to improve the repeatability and level of control of the impact.

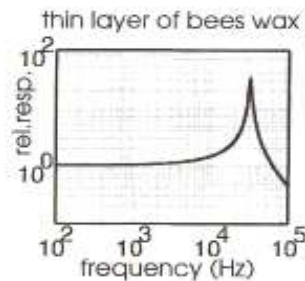
As motion transducers in the desired test will use accelerometers, simple mass-spring-damper system themselves; an accelerometer produces as an input value the acceleration in a frequency band well below its proper resonance.

In many types of construction the mechanical deformations (either in tension, compression or shear stress) of the piezoelectric element generates an electric charge proportional with the applied acceleration.

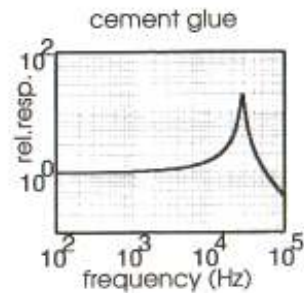
An important factor, limiting the useful frequency domain of an accelerometer is the type /location of mounting it on the structure, fig. 4.



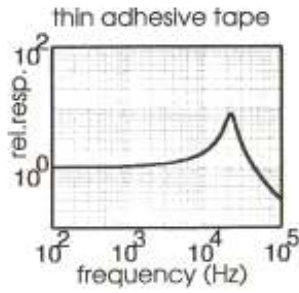
a) threaded stud



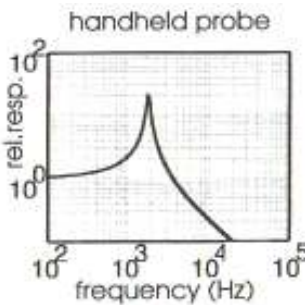
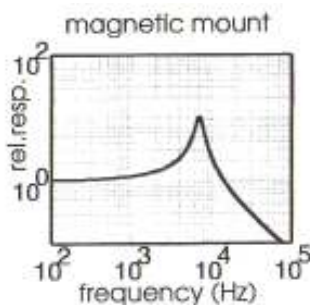
b) thin layer of bees wax



c) cement glue



d) magnetic mount



e) handheld probe

Figure 4: Response of structure at different types of accelerometer mounting

Each type of mounting acts as a spring and forms with the accelerometer a mass again a mass- spring system.

The usage of a threaded stud, fig. 4, a., gives the best results(resonance about 32 kHz) but demands threaded holes in the structure at all desired locations and a smooth flat surface.

A commonly usage is with a thin layer of bees wax(maximum temperature 40 C°, maximum acceleration 100m/s--)the layer should be kept as thin as possible in order to keep the high mounting resonance, 30kHz, fig.4, b.

For a ferromagnetic surface, easy to move around on structure is a permanent magnet, fig. 4, d.

At last the handheld probe, fig.2.4.e, is very convenient for quick look survey work, but the maximum frequency is about 0,7 kHz and the results are generally not very repeatable. Also is important the mounting location of accelerometers; the transducers must be located at the spot and in the direction where are the desired vibrations and if possible nothing else measured.

The last requirements, but not least important, of mounting the accelerometers on the structure is that the mass should have a minimum influence on vibration register of a structure.

These sometimes limiting the usage of high sensitivity accelerometers (most often heavy) on light weight structures; the multichannel measurements, the current tendency, increase the overall consistency and accuracy, but the mass loading effect of the larger number of accelerometers may become critical.

All these considerations make the accelerometers the most widely usage for motion transducer in Modal Analysis tests.

#### 4. CONCLUSIONS

In most test with standard piezoelectric transducers do an excellent job in measuring forces and accelerations; The measurement results in term of frequency domain, accuracy, numbers and locations of measured degrees of freedom always will be limited by the equipment and/or time.

Extreme conditions, e.g., very low or very high frequencies, very heavy or very light weight objects, environmental circumstances as humidity, magnetism....., will always require special dedicated equipment.

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