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A MICROMECHANICAL BASED BOUNDING AND ELASTIC PROPERTIES ESTIMATION OF MULTIPHASE POLYMERIC COMPOSITE MATERIALS

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Abstract: The paper herein approaches a two step homogenization concept based on well known micromechanical theoretical models in order to predict the effective elastic properties of a new class of composite materials, namely the multiphase ones. The overall elastic properties for the particle-fibre particular case approached herein are bounded within inferior and superior limits, the later being retrieved using few theoretical models from the literature. Based on author's experience, in case of particle reinforced composite materials, the Hashin-Shtrikman and Milton models encompass all the information on the geometry and distribution of the inclusions and are close related with the corresponding values retrieved experimentally. In the paper the fibres were considered randomly distributed leading to the use of the Halpin-Tsai theoretical model as the best option in the second step of the effective elastic properties estimation.

Keywords: *multiphase, micromechanics, polymer, composite, elastic coefficients*

1. INTRODUCTION

The emerging new materials requires special attention especially when are designed for applications in the domain of aerospace, automotive or other areas of engineering. The concept of the multiphase composite materials is not quite a new approach, the literature knowing this class of materials as hybrid materials [2], [3]. The multiphase composite materials such is the particle-fibre combination approached herein offer the potential to tailor its elastic properties, mechanical response and reduce stress concentrations around inclusions and discontinuities.

In several papers of the same author were presented the experimental research done in order to retrieve the static, dynamic or thermal properties of samples manufactured by using the *particle-fibre* concept, or the *particle-particle* concept for which the electrical properties were supplementary measured [6-8]. The complementary phases were selected to have different volume fraction and were embedded into a polyester resin.

Effective application of multiphase polymeric composite materials involves analysis of their stiffness in structural systems with uniform or variable property distribution. Tailoring the volume fraction of one phase may represent an approach for characterizing this class of advanced materials. In such circumstances, the herein paper attempts to present some theoretical models, such as the three-point bounding technique developed by Milton and the Hashin-Shtrikman bounds (without demonstration), including information on the phases, starting from the volume fraction, to the geometrical parameters (e.g. particle diameter, fibre length and diameter, etc.). The homogenization technique applied herein was a two step approach: firstly, bounds and estimates are employed for the polymer matrix and the random, small amount of particles leading to the so called equivalent matrix, followed by the second step for the equivalent matrix developed previously and the random, long fibres. The approach was based on the following assumptions: all phases are isotropic and linear elastic; the fibres have circular section and all inclusions affect the strains in the matrix.

The phases were considered as being Fe particles, E-glass fibres embedded into a polyester resin in different volume fraction. The constitutive were selected based on existing samples having the combination as was mentioned and for which the experimental data, such as their elastic and thermal properties were presented in other papers [3-5].

2. THEORETICAL MODELS AND SIMULATION

The two step homogenization concept was applied firstly to the particle inclusions embedded into the polymeric matrix, in different volume fractions, leading to the so called *equivalent matrix*, and secondly to the long, random fibres embedded into the matrix ”generated” in the first step. With respect to the first step of the procedure, due to the different volume fraction of the inclusions considered, the elastic coefficients – Young, bulk and shear moduli were bounded within the superior and inferior limits.

The literature provide several theoretical models, like Reuss, Voigt, Hashin-Shtrikman, Milton, etc., the latter two of them being used herein because they prove to relate closely with the experimental data (see [1], [3]). The expressions of the models considered were limited to the Milton bounds, as an improvement made on a three-point bounds models due to the fact that are unknown or seldom employed into the literature.

The Milton three-point bounds on the equivalent shear moduli is given by the following:

$$G_m V_m + G_p V_p - \frac{V_m V_p (G_p - G_m)^2}{G_p V_m + G_m V_p + \zeta} \le G_{em} \le G_m V_m + G_p V_p - \frac{V_m V_p (G_p - G_m)^2}{G_p V_m + G_m V_p + \psi} \tag{1}$$

where:

$$\zeta = \frac{\left(\frac{128}{K_m}+\frac{99}{G_m}\right)(1-0.211V_p)+0.211V_p\left(\frac{128}{K_p}+\frac{99}{G_p}\right)+45\left[\frac{1}{G_m}(1-0.483V_p)+\frac{0.483V_p}{G_p}\right]}{30\left(\frac{1-0.211V_p}{G_m}+\frac{0.211V_p}{G_p}\right)\left[\left(\frac{6}{K_m}-\frac{1}{G_m}\right)(1-0.211V_p)+0.211V_p\left(\frac{6}{K_p}-\frac{1}{G_p}\right)\right]+6\left(\frac{1-0.483V_p}{G_m}+\frac{0.483V_p}{G_p}\right)\left[\left(\frac{2}{K_m}+\frac{21}{G_m}\right)(1-0.211V_p)+0.211V_p\left(\frac{2}{K_p}+\frac{21}{G_p}\right)\right]} \tag{2}$$

and

$$\psi = \frac{3[(1-0.483V_p)G_m+0.483V_pG_p][(6K_m+7G_m)(1-0.211V_p)+0.211(6K_p+7G_p)V_p]-5[(1-0.211V_p)G_m+0.211V_pG_p]^2}{6[(2K_m-G_m)(1-0.211V_p)+0.211V_p(2K_p-G_p)]+30[(1-0.483V_p)G_m+0.483V_pG_p]} \tag{3}$$

In case of the bulk modulus of an isotropic two-phase composite material the Milton's three point bounds are given by:

$$K_m V_m + K_p V_p - \frac{V_p V_m (K_p - K_m)^2}{K_p V_m + K_m V_p + \frac{4}{3}\left(\frac{\varrho_1}{G_m}+\frac{\varrho_2}{G_p}\right)^{-1}} \le K_{em} \le K_m V_m + K_p V_p - \frac{V_p V_m (K_p - K_m)^2}{K_p V_m + K_m V_p + \frac{4}{3}(\varrho_1 G_m + \varrho_2 G_p)} \tag{4}$$

where $\varrho_1 = 1 - \varrho_2$, and in case of random dispersed inclusions $\varrho_2 \approx 0.211 V_p$ [1].

The Young modulus of the equivalent matrix can be expressed in terms of shear and bulk moduli, having the corresponding bounds, by the aid of the expression:

$$E_{em} = \frac{9K_{em}G_{em}}{3K_{em}+G_{em}} \tag{5}$$

In figure 1 is being shown the normalized values of the equivalent shear modulus function of the iron particles volume fraction embedded into a polyester resin according to the Milton bounds and Hashin-Shtrikman superior limit, where as in figures 2 and 3 the corresponding normalized bulk and Young equivalent moduli bounds and estimate for the same combination.

The second step of the homogenization approach involves the equivalent matrix from the previous step and the long, random fibres, case in which the elastic coefficients will be retrieved by using one of the most used theoretical models known as Halpin-Tsai expressions:

$$E_c = \frac{3}{8}E_L + \frac{5}{8}E_T \tag{6}$$

$$G_c = \frac{1}{8}E_L + \frac{1}{4}E_T \tag{7}$$

$$\nu_c = \frac{E_c}{2G_c} - 1 \tag{8}$$

where E_c, G_c and ν_c are the Young, shear and Poisson ratio, respectively, of the effective multiphase composite material. Supplementary, the E_L and E_T are the longitudinal and transversal, respectively, elastic moduli of the overall composite structure. The following expressions can be used, with a relatively high precision, to estimate these elastic moduli:

$$\frac{E_L}{E_{em}} = \frac{1+2\frac{l_f}{d_f}\eta_L V_f}{1-\eta_L V_f} \tag{9}$$

$$\frac{E_T}{E_{em}} = \frac{1+2\frac{l_f}{d_f}\eta_T V_f}{1-\eta_T V_f} \tag{10}$$

where:

$$\eta_L = \frac{\frac{E_f^L}{E_{em}}-1}{\frac{E_f^L}{E_{em}}+2\frac{l_f}{d_f}} \tag{11}$$

respectively

$$\eta_T = \frac{\frac{E_f^T}{E_{em}}-1}{\frac{E_f^T}{E_{em}}+2\frac{l_f}{d_f}} \tag{12}$$

Previously, l_f stands for fibers length, d_f for fiber diameter and V_f for fibers volume fraction.

In figures 4 and 5 are presented the effective elastic modulus of the multi-phase composite material for different volume fraction of the fibers (from 0% to 60%) and the effective Poisson ratio, respectively. The plotted data corresponds to different particle volume fraction (0%, 10%, 20%, 50%), the latter being bounded by the limit mentioned by Torquato, at 63%, as the limit of dense packaging in case of identical spherical particles and by the mixing law [1].

With respect to the Hashin-Shtrikman bounds that were not presented herein, they are known in the literatures as the 2-points bounds. The expressions involves the elastic moduli of the individual phases and a two step homogenization technique was employed as previous in order to retrieve the variation of the effective elastic properties of the multiphase composite materials with the phases volume fraction.

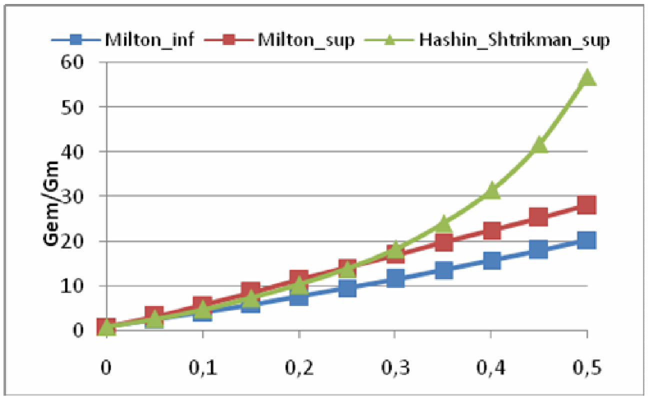

Figure 1: Normalized equivalent shear moduli bounds of iron particles embedded into different volume fraction into a polyester resin – 1st homogenization step

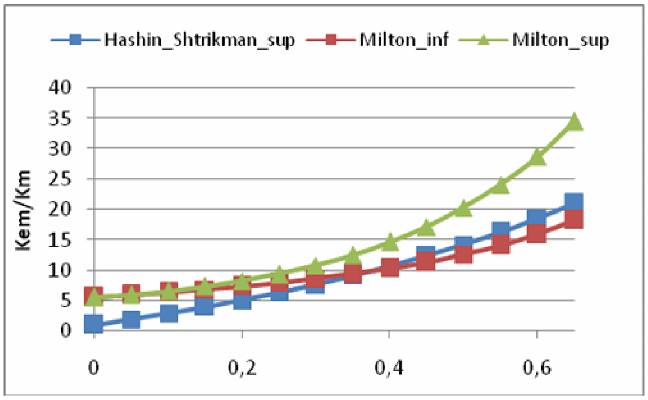

Figure 2: Normalized equivalent bulk moduli bounds of iron particles embedded into different volume fraction into a polyester resin – 1st homogenization step

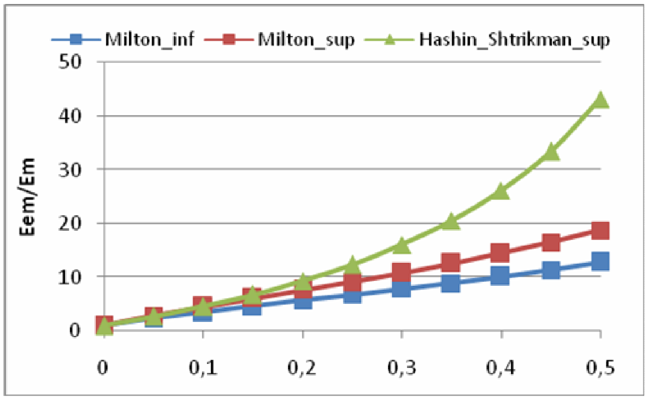

Figure 3: Normalized equivalent Young moduli bounds of iron particles embedded into different volume fraction into a polyester resin – 1st homogenization step

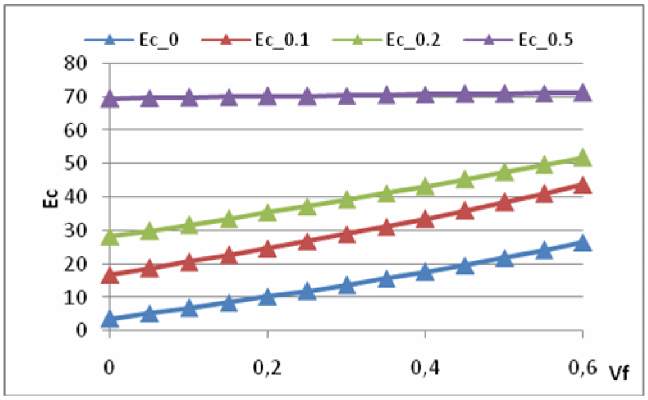

Figure 4: Effective multiphase composite modulus for different particle volume fraction (0%, 10%, 20% and 50%) – 2nd homogenization step

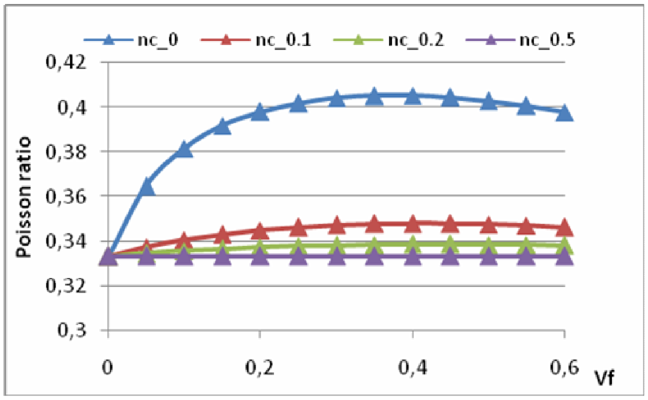

Figure 5: Effective Poisson ratio of the multiphase composite material for different particle volume fraction (0%, 10%, 20% and 50%) – 2nd homogenization step

3. CONCLUSION

Multiphase composite materials can be tailored such as to satisfy various requirements ranging from mechanical/thermal/electrical properties to various combinations of constitutive to be able to spread the application area. Light by strong materials have been every engineer's dream.

Acknowledging the limits imposed on desired property (herein, on the elastic moduli) this class of advanced materials can lead to structures having improved and optimized performance.

The paper herein presented a two step homogenization concept developed on micromechanical theoretical bounds, a 2 and a 3 point, respectively. The plotted data revealed the regions within the elastic property of the multiphase material has to be bounded and the predictable effective mechanical property for different combinations of the individual phases. The theoretical models include information on the geometry and distribution of phases and represent the most encompassing theoretical models from the literature.

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