The 3rd International Conference on "Computational Mechanics and Virtual Engineering"
COMEC 2009
29 – 30 OCTOBER 2009, Brasov, Romania

THE EXISTENCE AND SINGLENESS OF THE MECHANICAL SYSTEMS.APPROXIMATION METHODS

OPRIȚESCU Cristina[1], PETCOVICIU Oana[2], TOADER Mihai Ilie [3]
[1] *ICECON Bucureşti, Departamentul de Cercetare Timişoara* opritescucristina@yahoo.com
[2] *Grup Scolar Industrial „Aurel Vlaicu" Arad,* oanapetcoviciu@yahoo.com
[3] *ICECON Bucureşti, Departamentul de Cercetare Timişoara* toader@mec.upt.ro

Abstract: *The impacts' analysis of elastic bodies is current and encountered in many applications, practical and theoretical, too. The finite elements method (MEF) is far from being perfect, but it is the best method currently available for a wide variety of calculation types, in all areas of engineering activities. The method and the programs based on it are fundamental components of the systems of computer-aided designe (CAD) and are indispensable in all the situations in which competitiveness of engineering activities is required. Therefore, in the presented paper, the situation of elastic collisions is put in evidence by the simulation using ANSYS code.*
Keywords*: contact/impact, FEM, simulation, ANSYS.*

1. INTRODUCERE

There follows a technique of obtaining the uniqueness of a solution when the material point is in contact with Columbian friction, when the external force is an analytical function of time.

It is taken into the account a mechanical system with a degree of freedom made of a mass forced to move along a half-line, the mass being compelled to hit a rigid wall using a mechanical impact, as in Figure 1.

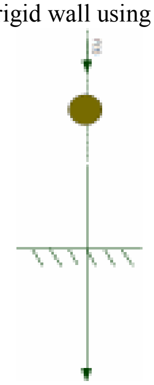

Figure 1: the particle moving along a straight line

$F(t)$ is the external force applied to the material point, in contact with the obstacle at the initial moment; the problem of elastic collision becomes:

P_1: Determine $U \in S([0,T];\Re)$ şi $N \in S([0,T];\Re)$ so that: $U(0)=0, \dot{U}^{+}(0)=0$

$$\ddot{U} = F + N \qquad (1)$$

$$U \le 0,\ N \le 0,\ UN = 0$$

$U(t)=0 \Rightarrow \dot{U}^{+}(t) = -R\dot{U}^{-}(t)$; where $R \in [0,1]$ is the restitution coefficient .

It is taken into the account the case when the external force F, is supposed to be of C^{∞} class and positive (it constantly retains the particle to the obstacle). It is verified that P1 problem accepts non – existing answer.

$$U \equiv 0$$

$$N(t) = -\int_0^t \int_0^s F(\sigma)\, d\sigma\, ds$$

which corresponds to the balance state of the material point. The objective wanted is to build a particular case of positive F function, for which the problem receives another solution then the usual one.

The lapse of time $[0,T)$ is divided into $[a_{n+1}, a_n)$, where $(a_n)_{n \in N}$ is a decreasing number towards zero, which will be presented farther on.

It is considered $F(0) = 0$ and:

$$F(t) = \begin{cases} 0 \quad \text{dacă} \quad t \in [a_{n+1}, a_{n+1} + d_n] \\ \dfrac{F_n}{2} \rho\left(\dfrac{t - a_{n+1} - d_n}{a_n - a_{n+1} - d_n}\right) \quad \text{dacă} \quad t \in [a_{n+1} + d_n, a_n] \end{cases} \tag{2}$$

$F(t)$ function will have the diagram:

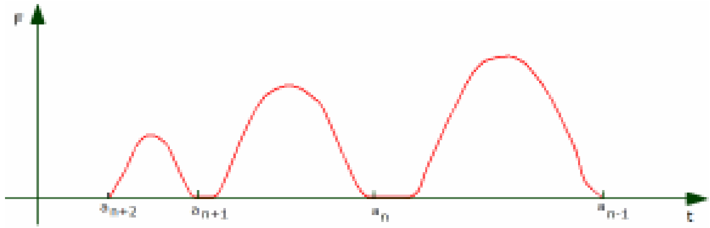

Figure 2: The variation shape of F for the law of elastic collision

For these functions of C^∞ class to t=0, it has to be known that the amplitude decreases fast enough, while t tends to zero, for this result it could be considered that:

$$F_n = \frac{1}{n!}$$

Using the considerations:

$$a_n = \sum_{i=1}^{\infty} \frac{(i+5)^2}{(i+1)(i+2)(i+3)(i+4)}$$

$$d_n = \frac{n+5}{(n+1)(n+2)(n+4)} = \frac{n+3}{n+5}(a_n - a_{n+1})$$

$$T = \sum_{n=0}^{\infty} \frac{(n+5)^2}{(n+1)(n+2)(n+3)(n+4)}$$

selected

$$V(t) = \begin{cases} -\dfrac{1}{(n+4)!} \quad \text{dacă} \quad t \in [a_{n+1}, a_{n+1} + d_n) \\ -\dfrac{1}{(n+4)!} + \dfrac{F_n}{2} \displaystyle\int_{a_{n+1}+d_n}^{t} \left(\dfrac{s - a_{n+1} - d_n}{a_n - a_{n+1} - d_n}\right) ds \quad \text{dacă} \quad t \in [a_{n+1} + d_n, a_n) \end{cases}$$ and than

$U(t) = \int_0^t V(s) ds$ results that the function $U(t)$ will have the shape:

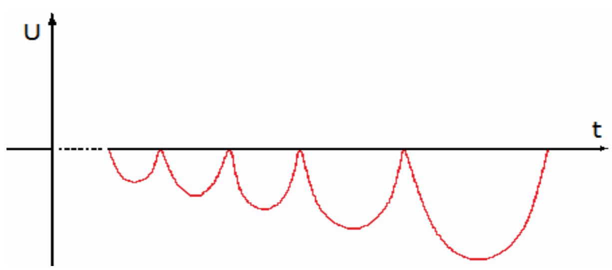

Figure 3: The shape of a solution for the law of elastic collision

It is easy to verify that $V(t)$ function has limited variation on $[0,T]$ and it results $U \in S([0,T]; \Re)$.

P1 problem is considered for the law of elastic collision, where R =0. If F function is always considered positive then P1 problem receives $U \equiv 0$ function as single solution. It results that there is a t' so that $U(t') < 0$ if a $t'' = \inf\{t, \forall s \in (t, t'], U(s) < 0\}$ is taken, the solution will be $U(t'') = \dot{U}^+(t'') = 0$ and $U(t') = \int_{t''}^{t'} \int_{t''}^{s} F(\sigma) d\sigma ds > 0$, which is irrational. It results, that in this case of elastic collision, the problem of lack of singleness is true.

Subdividing the period $[0,T)$ into $\left[\frac{1}{n+1},\frac{1}{n}\right)$, function $F(0)=0$ and

$$F(t)=\begin{cases} -F_{1,n}\rho\left(\dfrac{t-\frac{1}{n+1}}{d_{1,n}}\right) & \text{pentru} \quad t\in\left[\frac{1}{n+1},\frac{1}{n+1}+d_{1,n}\right) \\ 0 \quad \text{pentru} & t\in\left[\frac{1}{n+1}+d_{1,n},\frac{1}{n}-d_{2,n}\right) \\ F_{2,n}\rho\left(\dfrac{t-\frac{1}{n}+d_{2,n}}{d_{2,n}}\right) & \text{pentru} \quad t\in\left[\frac{1}{n}-d_{2,n},\frac{1}{n}\right) \end{cases} \tag{3}$$

where $n\in N^*$, $(F_{1,n})_{n\in N^*}$, $(F_{2,n})_{n\in N^*}$, $(d_{2,n})_{n\in N^*}$ are positive numbers which will be presented further on.
The diagram for the function is:

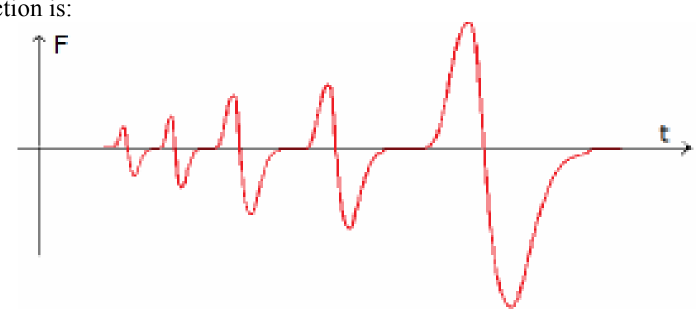

Figure 4: The variation shape of F function for elastic collision

The numbers $(d_{1,n})_{n>0}$ and $(d_{2,n})_{n>0}$ have to satisfy:

$$d_{1,n}\le\frac{1}{2}\left(\frac{1}{n}-\frac{1}{n+1}\right), \qquad d_{2,n}\le\frac{1}{2}\left(\frac{1}{n}-\frac{1}{n+1}\right) \tag{.4}$$

The numbers $(F_{1,n})_{n\in N^*}$, $(F_{2,n})_{n\in N^*}$, $(d_{1,n})_{n\in N^*}$, $(d_{2,n})_{n\in N^*}$ have to be created so that the problem P1 to receive two solutions U_1 and U_2. Functions U_1 and U_2 have to satisfy:

- if it is even

$$\begin{cases} U_1\left(\frac{1}{n}\right)=0 \\ \dot{U}_1^+\left(\frac{1}{n}\right)=0 \end{cases} \qquad \begin{cases} U_2\left(\frac{1}{n}\right)=-U_n \\ \dot{U}_2^+\left(\frac{1}{n}\right)=V_n \end{cases} \tag{5}$$

- if it is odd

$$\begin{cases} U_1\left(\frac{1}{n}\right)=-U_n \\ \dot{U}_1^+\left(\frac{1}{n}\right)=V_n \end{cases} \qquad \begin{cases} U_2\left(\frac{1}{n}\right)=0 \\ \dot{U}_2^+\left(\frac{1}{n}\right)=0 \end{cases} \tag{6}$$

where $(U_n)_{n\in N^*}$ and $(V_n)_{n\in N^*}$ are positive rows and will be defined further on. The fact that U_1 and U_2 satisfy (5) and (6) assures us that tey are different.

On the whole, it can't be obtained the singleness for the solution of the problem in the case of plastic collision $R=0$, even if $F\in C^\infty([0,T];\Re)$.

2. NUMERICAL SIMULATION USING ANSYS LS-DYNA CODE

For studious problem in this paper to select the code of the simulation ANSYS that is a medium whence integrate the activities of projection and analysis, in the scope of a realization products optimization. ANSYS LS is a program for the simulation problems of analyze nonlinear, dynamically speedster, with application in simulation of impact Figure 4.

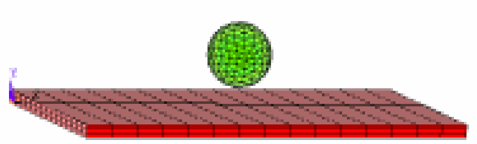

Figure 4 numerical simulation of the collision between wall and sphere

As it can be seen from the diagram there are successive collisions between wall and sphere

- at t=0.078[s] takes place first collision (displacement u4 and speed u5 of the wall are zero),
- at t=0.245[s] takes place the second collision
- at=0.432[s] takes place the third collision

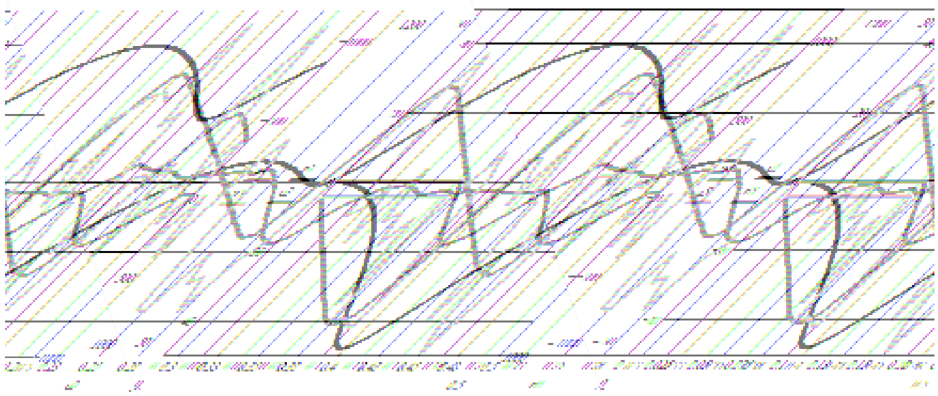

Figure 5 Diagram of the variation in movement and speed for the two bodies
u1 movement of the sphere, u4 the speed of the sphere; u2 movement of the wall , u5 speed of the wall

Because I haven't inserted amortization elements after the first and second collision, the wall vibrates freely. It can be seen from the diagram that amortization takes place during a long period of time, so the system wall- sphere can interact again. The impact of both wall and sphere causes distortions which will be analyzed in another work.

3. CONCLUSIONS

A number of approximations have been extracted, and from them it have been obtained a homogenous subnumber , which has given the solution for the studied problem. If the function isn't integrable, and is supposed to be analytical on some parts, the studied problem, receives a unique solution in the considered space.

The contact problems are very difficult problems from mechanical and in same time they are very importance in many engineering domains.

The efficiency of Finite Element Method in the numerical simulation, in general, substitutes the experiment, in many cases and at complex tests.

The choosing of ANSYS LS-DYNA program is realized because it comes very well to problems of contact/impact of elastic bodies, problems from which the problem of the considered ring makes part.

By processing of signal with the program MATHCAD, which is a reading of mathematical commands, with friendly interface and which displays data and expressions in a natural form, it was resulted and it is give to evidence the spectral of movements for both studied problems.

REFERENCES

[1] **N. Faur** 2002 ”*Elemente finite”*, Editura Politehnica, Timişoara

[2] **Cristina OPRIȚESCU, Oana PETCOVICIU, Mihai Ilie TOADER**: *Consideration on the simulation of contact with friction for rigid bodies,* „SIMEC” 2009, Bucureşti 27 martie 2009, ISSN 1842-8045, pg.194-197

[3] **M. SCHATZMAN** *A Class of Nonlinear Differential Equations of Second Order in Time Nonlinear,* Ana Theory Meth and Appli., 2, 1978, 355-373.

[4] **J.J. MOREAU.** *Une formulation du contact avec frottement sec, application au calcul numérique.* C.R.A.S. série II, 302 N^0 13, pages 799-801, 1986.