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AUTOMOTIVE SUSPENSION ANALYSYS USING APPROXIMATE MODELS

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Abstract: *A high order multi-degree-of-freedom model involving many suspension parameters is typically required in order to analyze the influence of suspension design on all the performance functions.This paper utilized approximate decoupling to obtain simple single degree of freedom models from a high order automotive suspension model. Each simple model involved a small number of parameters and enabled easy analysis of the performance of some suspension functions. The most commonly used indexes for ride comfort and road handling are analyzed for judge the effectiveness of the suspension.*
Keywords: *suspension, control, ride quality, road holding*

1. INTRODUCTION

Two important suspension performance metrics considered in the literature are passenger comfort and suspension deflection, i.e., the relative displacement between the car body and wheel assembly. It is widely accepted that lower vertical acceleration levels correspond to increased comfort. Structural features of a vehicle place a hard limit on the amount of suspension deflection available to reduce the car body acceleration. Hence, the goal in designing vehicle suspensions is to minimize car body acceleration, subject to the hard constraint on available suspension deflection.
In general, ride comfort, road handling, and stability are the most important factors in evaluating suspension performance. The main concern in suspension design and control is the fact that currently, achieving improvement in these three objectives poses a challenge because these objectives will likely conflict with each other in the vehicle operating domain [1], [3], [4].
A high order multi-degree-of-freedom model involving many suspension parameters is typically required in order to analyze the influence of suspension design on all the performance functions.
In this paper using approximate decoupling we obtain simple single degree of freedom models from a high order automotive suspension model. Each simple model involved a small number of parameters and enabled easy analysis of the performance of some suspension functions.

2. DYNAMICAL MODEL AND PERFORMANCE INDEXES

2.1 Dinamical model

Since many of the proposed electronic suspension being considered today are independent, i.e. using local sensor information and control law, the quarter car model show in Fig. 1 has been considered in this paper.
We used the following notation: m_{us} is the equivalent unsprung mass consisting of the wheel and its moving parts; m_s is the sprung mass, i.e., the part of the whole body mass and the load mass pertaining to only one wheel; k_t is the elastic constant of the tire, whose damping characteristics have been neglected.
The state component $x_1(t)$ is the deformation of the suspension with respect to (wrt) the static equilibrium configuration, taken as positive when elongating; $x_2(t)$ is the vertical absolute velocity of the sprung mass m_s.
The state component $x_3(t)$ is the deformation of the tire wrt the static equilibrium configuration, taken as positive when elongating.
The state component $x_4(t)$ is the vertical absolute velocity of the unsprung mass m_u; $u(t)$ is the control force produced by the actuator.
The signal $x_0(t)$ represents the disturbance, it coincides with the absolute vertical velocity of the point of contact of the tire with the road.

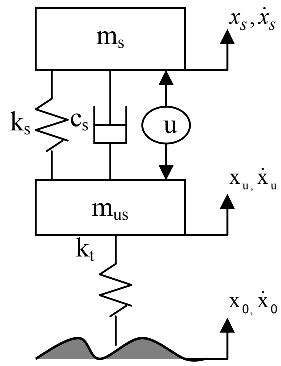


Figure 1: Quarter car model

If assume that the tire does not leave the ground, the liniarized equations of the motion are

$$\begin{bmatrix} m_s & 0 \\ 0 & m_u \end{bmatrix} \begin{Bmatrix} \ddot{x}_s \\ \ddot{x}_u \end{Bmatrix} + \begin{bmatrix} c_s & -c_s \\ -c_s & c_s \end{bmatrix} \begin{Bmatrix} \dot{x}_s \\ \dot{x}_u \end{Bmatrix} + \begin{bmatrix} k_s & -k_s \\ -k_s & k_s + k_t \end{bmatrix} \begin{Bmatrix} x_s \\ x_u \end{Bmatrix} = \begin{Bmatrix} 0 \\ k_t \end{Bmatrix} x_0 + \begin{Bmatrix} 1 \\ -1 \end{Bmatrix} u \tag{1}$$

$$M\,\ddot{\mathbf{x}} + C\,\dot{\mathbf{x}} + K\,\mathbf{x} = D_1\,x_r + D_2\,u \tag{2}$$

The parameter values are taken from reference [2] and are listed in Table 1.

Tabel 1: numerical parameter

$m_s = 240\,Kg$	$m_u = 36\,Kg$
$c_s = 1000\,N \cdot \mathrm{sec}/m$	$c_t = 0$
$k_s = 16000\,N/m$	$k_t = 160000\,N/m$

2.2 Suspension performance and tradeoffs

The purpose of this paragraph is to point out that are some inherent performance limitation for any suspension that acting between two sprung masses. The principal source of discomfort for the driver is the acceleration transmissibility η, or ride comfort index [2], [4]. In order to improve passenger confort the acceleration transfer function

$$H_A(s) = \frac{\ddot{x}_s(s)}{\dot{x}_0(s)} = \frac{k_t s (c_s s + k_s)}{d(s)} \tag{3}$$

from the road disturbance to the car body acceleration should be small in the frequency range from 0 – 70 rad/s. In (3) $d(s)$ is characteristic polynom.

At the same time it is necessary to ensure that the transfer function $H_{RS}(s)$ from the road disturbance to the suspension deflections, rattle space transfer function [nic]

$$H_{RS}(s) = \frac{x_s(s) - x_u(s)}{\dot{x}_0(s)} = -\frac{k_t m_s s}{d(s)} \tag{4}$$

is small enough to ensure that even very rough road profiles do not cause the deflection limits to be reached. As shown in [3] the acceleration transfer function $H_A(s)$ has an invariant point at $\omega_1 = \sqrt{k_t / m_u}$. For the parameter value listed

in Table 1, $\omega_1 = 56{,}6\ rad/s$. Similarly, the suspension deflection transfer function $H_{RS}(s)$ has a zero at the rattle space frequency, $\omega_2 = \sqrt{k_t(m_u + m_s)}$ **.** For the parameter value listed in Table 1, $\omega_2 = 24{,}06\ rad/s$.
The tradeoff between passenger comfort and suspension deflection is captured by the fact that is not possible to simultaneously keep both the above transfer functions small around the tyrehop frequency and in the low-frequency range. In [2] it is shown that a small reduction in $H_A(s)$ at low frequencies and in the vicinity of the tyrehop frequency results in a large increase in $H_{RS}(s)$ at these frequencies and vice versa.

3. REDUCED MODELS

This paper utilized approximate decoupling to obtain simple single degree of freedom models from a high order automotive suspension model. Each simple model involved a small number of parameters and enabled easy analysis of the performance of some suspension functions.
In order to study the effects of specific suspension parameters on the suspension performance, we calculate the natural frequencies and mode shapes of the suspension system and then transform to a new set of coordinates in which the two equations of motion are approximately decoupled. For the particular case where the tire stiffness is much higher than the suspension stiffness, we make the approximations [4] $k_s + k_t \approx k_t - k_s \approx k_t$ which then results in the natural frequencies

$$\omega_1 = \sqrt{k_s/m_s}\ ,\ \omega_2 = \sqrt{k_t/m_u} \quad \textbf{(5)}$$

Let $P = [\mathbf{p}^{(1)}\ \mathbf{p}^{(2)}]$ the mass normalized modal matrix. The following change of coordinates

$$\boldsymbol{\eta} = P^T M\ \mathbf{x} \quad \textbf{(6)}$$

results in decoupled equations of motion in the new modal coordinates

$$\ddot{\boldsymbol{\eta}} + \Lambda\boldsymbol{\eta} + P^T C P\dot{\boldsymbol{\eta}} = P^T D_1 x_0 \quad \textbf{(7)}$$

In the case of passive suspension system, for numerical parameter listed in Table 1 he two new decoupled coordinates can therefore be approximated by

$$\eta_1 = -15.4 x_s \ \text{ for } \ |x_s| \geq |x_u| \quad \textbf{(8)}$$

$$\eta_2 = -5.9 x_u \ \text{ for } \ |x_u| \geq |x_s| \quad \textbf{(9)}$$

The two approximate decoupled equations are:

a) sprung mass mode approximation for $|x_s| >> |x_u|$

$$m_s\ddot{x}_s + c_s\dot{x}_s + k_s x_s = c_s\dot{x}_0 + k_s x_0 \quad \textbf{(11)}$$

b) unsprung mass mode approximation for $|x_u| >> |x_t|$,

$$m_u\ddot{x}_u + c_s\dot{x}_u + k_t x_u = k_t x_0 \quad \textbf{(12)}$$

For judge effectiveness of the suspension system we are looking at the approximate transfer function:

- acceleration transfer function

$$\frac{1}{s}H_A(s) \approx \frac{x_s}{x_0} = \frac{c_s s + k_s}{m_s s^2 + c_s s + k_s} \quad \textbf{(13)}$$

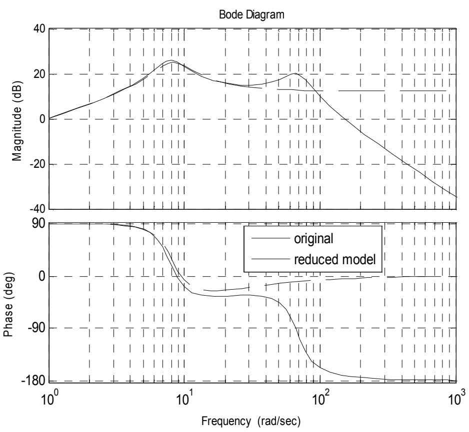
- rattle space transfer function

$$sH_{Rs}(s) \approx \frac{x_s - x_0}{x_0} = -\frac{m_s s^2}{m_s s^2 + c_s s + k_s} \quad \textbf{(14)}$$

The tire deflection transfer function in this case can be approximate by

$$H_{TD}(s) \approx \frac{x_u - x_0}{x_0} \approx \frac{-m_u s^2}{m_u s^2 + c_s s + k_t} \qquad \textbf{(15)}$$

To evaluate the accuracy of the approximate transfer functions of equations (13) and (15), Figures 2 and 3 show a comparison between the original and approximate transfer functions. It is clear that the approximate transfer function (13) matches the original transfer function (16) well for frequency range $\omega < \omega_1$ and the approximate transfer function (15) matches the original transfer function $H_{TD}(s) = (x_u - x_0)/\dot{x}_0$ well for the frequency range $\omega > 0.5\omega_2$.



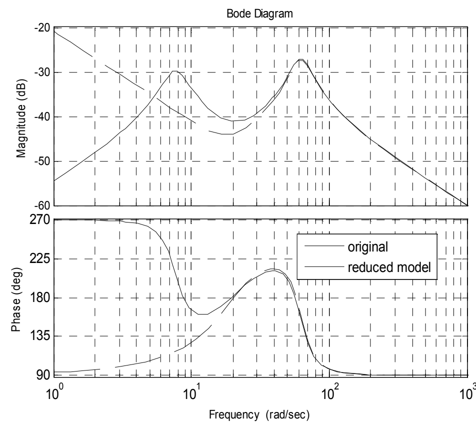


Figure2. Bode for $H_A(s)$ sprung mass approximate mode **Figure 3** Bode for $H_{TD}(s)$ unsprung mass approximate mode

By inspection of the simple second order transfer functions in equations (13) and (14) above, it is clear that changes in the suspension stiffness k_s, and in the suspension damping c_s, will lead to the changes in the transfer function H_A (s) and H_{RS} (s). A softer suspension (lower k_s) leads to an improvement in ride quality by reducing the first resonant frequency and hence causing the roll-off in the transfer function H_A(s) to start at a lower frequency. An increase in suspension damping c_s, reduces or eliminates the resonant peak corresponding to the sprung mass natural frequency. Thus the ride quality transfer function $H_A(s)$ will be significantly improved at the sprung mass frequency. However, due to the impact of c_s, on the numerator in equation (13), the higher damping introduces high frequency harshness in H_A (s) by causing a slower roll-off.

The increase in suspension damping will have no detrimental effects on the suspension deflection transfer function H_{RS}(s). By examining the simple second order transfer function in equation (14), it is clear that an increase in tire stiffness reduces tire deflection by reducing the low frequency asymptote of $H_{TD}(s)$.

4. CONCLUSION

Using the approximately decoupled models, the following conclusions on suspension design were obtained:

1. Decreasing suspension stiffness improves ride quality and road holding. However, it increases rattle space requirements.
2. Increased suspension damping reduces resonant vibrations at the sprung mass frequency. However, it also results in increased high frequency harshness.
3. Increased tire stiffness provides better road holding but leads to harsher ride at frequencies above the unsprung mass frequency.

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