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FEM BUCKLING BEHAVIOR STUDIES ON COMPOSITE PLATES WITH INITIAL IMPERFECTIONS

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Abstract: *In the paper, the buckling behavior analysis of a clamped, composite laminated quadratic plate under a uniaxial and shear in-plane loading is presented. The imperfection is considered as the initial deformation due to the manufacturing operations: cosine shape in both of the longitudinal and transverse direction. Usually, the initial deformation mode appears in the form of the fundamental mode of the buckling or vibration. The boundary conditions are considered as the usual condition of the structural composite ship panels. On the plate sides the loading is considered as an uniform pressure. Variation of the maximum transverse displacement versus the in-plane load (displacement controlled after nonlinear buckling analysis) for three cases. Maximum initial deformation magnitude is considered as a rate from the thickness, t, that is* $wo = \alpha \cdot t$. *The parametric calculus was done for various values of loading ratio.*
Keywords: *imperfect plates, composites, laminated, buckling*

1. INTRODUCTION

The outstanding performance of laminated composite plates, in terms of high strength properties and low specific weight have found an increasing use in many engineering areas, especially in marine and shipbuilding, where the corrosion is a dominant key factor. The lightweight structural parts of the ship hull are stiffened thin-walled plates or shells. So, an important key factor in the design analysis of these types of structures is overall buckling behavior.

A lot of studies were developed in last decade for postbuckling analysis of laminated imperfect plates. In [1] postbuckling behavior of rectangular orthotropic laminated composite plates with initial imperfection under inplane shear load was investigated in a closed-form analytical manner.

In [2] a higher-order finite strip method based on the higher-order shear deformation plate theory is developed for postbuckling analysis of laminated composite plates with initial geometric imperfection subjected to progressive end shortening.

A postbuckling analysis is presented in [3] for a uniaxial in-plane loaded, simply supported, composite laminated rectangular plate resting on a “softening” non-linear elastic foundation. The analysis uses a perturbation technique to determine the buckling loads and postbuckling equilibrium paths, taking initial geometrical imperfection into account.

In [4] the authors present the validation of finite element models against a series of plate tests that were performed within this framework and parametric studies that were carried out to identify the effects of geometric imperfections on the ultimate compressive strength of composite plates with three alternative lay-up configurations.

A methodology to evaluate the influence of the imperfections on the buckling and postbuckling behavior of the composite plates under shear and compression loading, used in ship hull structure, is presented in [5]. The parametric calculus was done for various positions of delaminations and seven values of loading ratios.

Postbuckling analysis is essential to predict the capacity of composite plates carrying considerable additional load before the ultimate load is reached, and manufacturing-induced geometric imperfections often reduce the load-carrying capacity of composite structures.

In order to fully exploit the lightweight potential of such thin-walled structures, it is of practical importance to consider load ranges beyond bifurcational buckling and to develop analysis methods that allow for a postbuckling analysis and design to be used in day-to-day engineering practice.

The behaviour of ship deck plating normally depends on a variety of influential factors, such as geometric/material properties, loading characteristics, initial imperfections, boundary conditions and deterioration arising from interlaminar fatigue cracking. The analysis is presented for a uniaxial and bi-axial in-plane loading, clamped, composite laminated quadratic plate. The imperfection is considered as the initial deformation due to the manufacturing operations: cosine shape in both of the longitudinal and transverse direction. Usually, the initial deformation mode appears in the form like the fundamental mode of the buckling or vibration. The boundary conditions are considered as the usual condition of the structural composite ship panels. On the plate sides the loading is considered as an uniform pressure. Variation of the maximum transversal displacement versus the inplane load (displacement controlled after nonlinear buckling analysis) for three cases, obtained in numerical analysis are performed.
For maximum initial deformation magnitude three values are considered: 1.06mm, 3.2mm and 9.6mm.
A methodology to evaluate buckling and postbuckling behavior of the ship deck composite plates with imperfections under biaxial compression loading is presented. The parametric calculus was done for various values of loading ratio k=q/p (fig.1). The numerical tests are analyzed with respect to combined boundary biaxial compression buckling on the imperfect composite plates.
The results are be presented, in terms of deflection, evolution of buckling load versus transversal displacement in the middle point of the panel, for various loading ratios k and transversal imperfection magnitude.

2. BUCKLING THEORY OF ORTHOTROPIC PLATES

Recently a considerable effort has been dedicated towards the development of fast and reliable design procedures for buckling, postbuckling and collapse analyses of fibre composite stiffened panels. It is well-known that thin-walled structures made of carbon fibre reinforced plastics are able to tolerate repeated buckling without any change in their buckling behaviour. However, it has yet to be established, how deep into the postbuckling regime loading one can go without severely damaging the structure, and how this can be reliably predicted by fast and accurate simulation procedures.
In this trend, in this paper, a methodology to establish the buckling and postbuckling loads of the laminated plates with transversal imperfection is analysed.

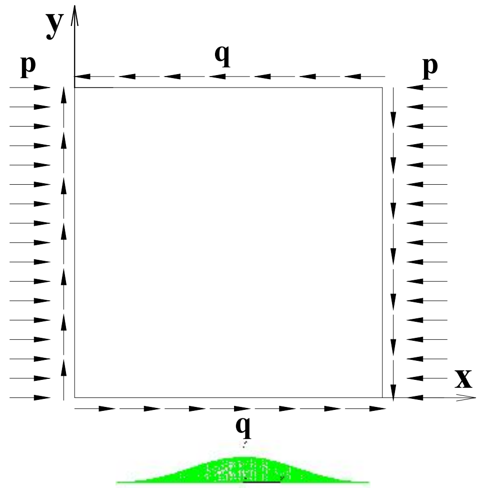


Figure 1: In-plane loading of imperfect plate

The state of equilibrium of a plate deformed by forces acting in the plane of the middle surface is unique and the equilibrium is stable if the forces are sufficiently small. If, while maintaining the distribution of forces constant at the edge of the plate, the forces are increased in magnitude, there may arise a time when the basic form of equilibrium ceases to be unique and stable and other forms become possible, which are characterized by the curvatures of the middle surface.
The equation of the deflected surface of symmetrically laminated plates for combined axial loading is

$$D_{11}\frac{\partial^4 w}{\partial x^4}+2(D_{12}+D_{66})\frac{\partial^4 w}{\partial x^2 \partial y^2}+D_{22}\frac{\partial^4 w}{\partial y^4}-N_x\frac{\partial^2 w}{\partial x^2}-2N_{xy}\frac{\partial^2 w}{\partial x \partial y}=0 \tag{1}$$

where D_{11}, D_{12}, D_{22}, D_{66}, are the orthotropic plate stiffnesses, calculated according to the equation

$$D_{ij}=\frac{1}{3}\sum_{k=1}^{N}\overline{Q}_{ij}^{k}\left(\bar{z}_k^3-\bar{z}_{k-1}^3\right) \tag{2}$$

The thickness and position of every ply can be calculated from the equation

$$t_k = z_k - z_{k-1} \tag{3}$$

and

$$\bar{z}_k = z_{k-1} + \frac{t_k}{2}$$

Linear buckling of beams, membranes and plates has since been studied extensively. A linearized stability analysis is convenient from a mathematical viewpoint but quite restrictive in practical applications. What is needed is a capability for determining the nonlinear load-deflection behaviour of a structure. Considerable effort has also been expended on this problem and two approaches have evolved: class-I methods, which are incremental in nature and do not necessarily satisfy equilibrium; and class-II methods, which are self-correcting and tend to stay on the true equilibrium path ([6]).

Historically, class-I was the first finite element approach to solving geometrically non-linear problems ([7]). In this method the load is applied as a sequence of sufficiently small increments so that the structure can be assumed to respond linearly during each increment.

To solving of geometrically and material nonlinear problems, the load is applied as a sequence of sufficiently small increments so that the structure can be assumed to respond linearly during each increment ([8]).

For each increment of load, increments of displacements and corresponding increments of stress and strain are computed. These incremental quantities are used to compute various corrective stiffness matrices (variously termed geometric, initial stress, and initial strain matrices) which serve to take into account the deformed geometry of the structure. A subsequent increment of load is applied and the process is continued until the desired number of load increments has been applied. The net effect is to solve a sequence of linear problems wherein the stiffness properties are recomputed based on the current geometry prior to each load increment. The solution procedure takes the following mathematical form

$$\left(\mathbf{K} + \mathbf{K}_{\mathrm{I}}\right)_{\mathrm{i-1}} \Delta\mathbf{d}_{\mathrm{i}} = \Delta\mathbf{Q} \tag{4}$$

where

K is the linear stiffness matrix,

K_I is an incremental stiffness matrix based upon displacements at load step i-1,

$\Delta\mathbf{d}_i$ is the increment of displacement due to the i–th load increment,

$\Delta\mathbf{Q}$ is the increment of load applied.

The correct form of the incremental stiffness matrix has been a point of some controversy. The incremental approach is quite popular (this is the procedure applied in all studies in this chapter). This is due to the ease with which the procedure may be applied and the almost guaranteed convergences if small enough load increments are used.

The plate material is damaged according to a specific criterion.

For various materials classes three dimensional failure criteria are developed. These include both isotropic and anisotropic material symmetries, and are applicable for macroscopic homogeneity. In the isotropic materials form, the properly calibrated failure criteria can distinguish ductile from brittle failure for specific stress states. Although most of the results are relevant to quasi-static failure, some are for time dependent failure conditions as well as for fatigue conditions.

The buckling load determination may use the Tsai-Wu failure criterion in the case if the general buckling does not occurred till the first-ply failure occurring. In this case, the buckling load is considered as the in-plane load corresponding to the first-ply failure occurring.

The Tsai-Wu failure criterion provides the mathematical relation for strength under combined stresses. Unlike the conventional isotropic materials where one constant will suffice for failure stress level and location, laminated composite materials require more elaborate methods to establish failure stresses. The strength of the laminated composite can be based on the strength of individual plies within the laminate. In addition, the failure of plies can be successive as the applied load increases. There may be a first ply failure followed by other ply failures until the last ply fails, denoting the ultimate failure of the laminate. Progressive failure description is therefore quite complex for laminated composite structures. A simpler approach for establishing failure consists of determining the structural integrity which depends on the definition of an allowable stress field. This stress field is usually characterized by a set of allowable stresses in the material principal directions.

The failure criterion is used to calculate a failure index (F.I.) from the computed stresses and user-supplied material strengths. A failure index of 1 denotes the onset of failure, and a value less than 1 denotes no failure. The failure indices are computed for all layers in each element of your model. During postprocessing, it is possible to plot failure indices of the mesh for any layer.

The Tsai-Wu failure criterion (also known as the Tsai-Wu tensor polynomial theory) is commonly used for orthotropic materials with unequal tensile and compressive strengths. The failure index according to this theory is computed using the following equation ([9], [10]).

$$F.I. = F_1 \cdot \sigma_1 + F_2 \cdot \sigma_2 + F_{11} \cdot \sigma_1^2 + F_{22} \cdot \sigma_2^2 + F_{66} \cdot \sigma_6^2 + 2F_{12} \cdot \sigma_1 \cdot \sigma_2 \tag{5}$$

where

$$F_1 = \frac{1}{R_1^T} - \frac{1}{R_1^C};\ F_{11} = \frac{1}{R_1^T \cdot R_1^C};\ F_2 = \frac{1}{R_2^T} - \frac{1}{R_2^C};\ \ F_{22} = \frac{1}{R_2^T \cdot R_2^C};\ F_{66} = \frac{1}{R_{12}^2}. \tag{6}$$

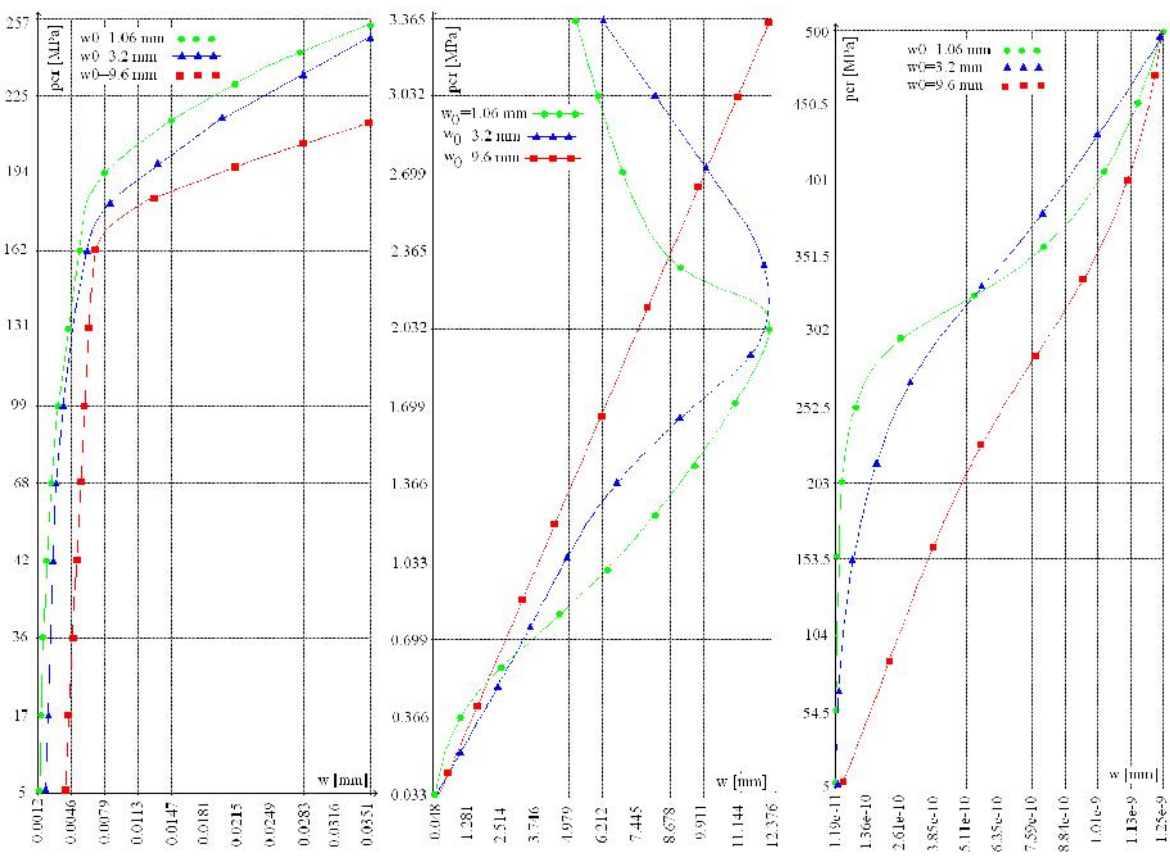
The coefficient F_{12}, which represents the parameter of interaction between σ_1 and σ_2, is to be obtained by a mechanical biaxial test. In the equations (6), the parameters R_i^C, R_i^T are the compressive strength and tensile strength in the material in longitudinal direction (i=1) and transversal direction (i=2). The parameter R_{12} is in-plane shear strength in the material 1-2 plane.
According to the Tsai-Wu failure criterion, the failure of a lamina occurs if

$$F.I. > 1 \tag{7}$$

The failure index in calculated in each ply of each element. In the ply where failure index is greater than 1, the first-ply failure occurs, according to the Tsai-Wu criterion. In the next steps, the tensile and compressive properties of this element are reduced by the failure index. If the buckling did not appeared until the moment of the first-ply failure occurring, the in-plane load corresponding to this moment is considered as the buckling load.

3. ANALYSIS OF AXIAL AND SHEAR BUCKLING OF COMPOSITE PLATES

In the study, the numerical analysis was carried out using COSMOS/M finite element package software. The square plates (320x320mm), are made of E-glass/polyester concerning 16 biaxial layers having the orthotropic directions and thickness (4.96mm) according to the topological code of the plate: $[0_2/45/90_2/45/0_2]_s$.



a) pure compression (q=0) b) q/p=0.4 c) pure shearing (p=0)

Figure 2: Variation of axial loading p (shear q) versus transversal displacement

The material characteristics, determined in experimental tests are:
Ex=38.6 GPa, Ey=8.27 GPa, Ez=8.27 GPa, Gxy=4.14 GPa, Gxz=4.14 GPa, Gyz = 4.6 GPa;
μ_{xy}=0.3, μ_{yz}=0.42, μ_{xz}=0.3; R_x^T=1.062 GPa, R_y^T=0.031 GPa, R_y^C=0.118 GPa, R_{xy}=0.72 GPa.
The direction of the axial loading is considered along the symmetry geometrical axis of the plate.
The loading was considered to be a combination between axial compression (p) and shear (q) acting on the plate sides as it is presented in the figure 1. The combination is determined by the loading ratio k=q/p. For the loading

ratio, 6 values were considered: 0 (pure axial compression); 0.2; 0.4; 0.6; 0.8; 1.0 and ∞ (pure shearing). In the table 1, the values of ultimate strength (buckling load) for the all analyzed loading ratios, for the three cases of initial transversal imperfection magnitude are given.
In the figure 2, the variation of the axial loading p (q for pure shearing) versus the magnitude of the transversal displacement are presented for pure compression, loading ratio equal to 0.4 and pure shearing for the three cases of initial transversal imperfection magnitude are presented.

Table 1: Buckling load [MPa]

k=q/p	w_0=0	w_0=1.06 mm	w_0=3.2 mm	w_0=9.6mm
0	1.94	5.103	6.612	1.028
0.2	0.243	0.288	0.222	0.134
0.4	0.211	0.266	0.111	0.087
0.6	0.243	0.288	0.088	0.071
0.8	0.244	0.277	0.111	0.071
1.0	0.285	0.305	0.124	0.081
∞	1.468	2.330	2.448	2.764

Table 2: Buckling load [MPa] for all cases, according to Tsai-Wu criterion

k= p/q	FAIL	w_0=0	w_0=1.06 mm	w_0=3.2 mm	w_0=9.6mm
0	FAIL 1	2.561	1.89	1.77	6.9
	FAIL 2	-	1.93	1.81	1.38
0.2	FAIL 1	2.499	0.706	0.605	0.403
	FAIL 2	-	1.413	1.312	1.211
0.4	FAIL 1	2.499	0.706	0.504	0.403
	FAIL 2	-	1.311	1.211	0.999
0.6	FAIL 1	1.874	0.504	0.504	0.302
	FAIL 2	-	0.908	0.876	0.802
0.8	FAIL 1	1.562	0.403	0.403	0.302
	FAIL 2	-	0.806	0.706	0.706
1	FAIL 1	1.249	0.302	0.302	0.302
	FAIL 2	-	0.706	0.563	0.563
∞	FAIL 1	2.0	1.5	1.5	1.5

For each case in the parametric calculus, according to the changing in the slope of the curves, the value of the buckling load (determined by graphical method, by drawing a tangent line in the point of suddenly changing the curve slope) is presented in the table 1.
In the table 2, the values of ultimate strength (buckling load) determined by Tsai-Wu criterion for all cases are presented. As it is seen, the first failure for all cases occurs for the tension case (FAIL 1).

4. CONCLUSION

This paper presented a detailed numerical investigation on the post-buckling behaviour of composite panels subjected to shear and axial loads.
From the comparative analysis it can be concluded that the numerical model provides a good approximation to the actual behaviour of imperfect plates. Thus, it can be used as an useful analysis tool in order to develop and establish new design rules for the composite ship structures. Moreover, the phenomenon study requires a stress analysis in order to improve the evaluation of the structural response of the composite plates with transversal imperfection.
As it is seen in the table 2, the buckling load is decreasing as the magnitude of the transversal deformation is increasing. But for the magnitude of 1.06 mm the plate behavior looks to be as a corrugated plate, having a good endurance to the buckling.
The proposed methodology accounts for failure, material non-linearity/degradation, geometric imperfections and geometric non-linearity effects. The numerical results indicated that the post-buckling behaviour of the panels prior to collapse was not significantly affected by the geometric imperfections and their magnitudes.
The numerical results obtained from the tests and allow to reach certain conclusions related to the behaviour of imperfect composite plate under combined axial and shear loads.

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