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OPTIMAL CONDITIONS IN TRANSIENT OPERATING REGIME FOR REFRIGERATION SYSTEMS ACCORDING TO EQUILIBRIUM THERMODYNAMICS MODELLING

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Abstract: *The optimization of the transient regime is an important issue to be solved in nowadays engineering. The proposed paper deals with this topic applied to refrigeration machines treated in equilibrium thermodynamics. In transient regime, one of the most widespread problems is to determine the existence of a variation law for the refrigerant temperature that would convey to minimum energy consumption or maximum coefficient of performance.*

Key words: *transient regime, refrigeration machines, equilibrium thermodynamics, optimization.*

1. Introduction

The refrigeration systems have widespread applications and mostly in refrigeration and air conditioning fields. Therefore numerous valid contributions are dedicated to their study and optimization. An extensive study on the theory of irreversible heat transfer refrigeration systems has been done [1]. Other works give formulae for the *COP* and cooling rate of an endoreversible refrigeration machine [2] and also when maximum specific cooling load is required [3]. An analytical model for predicting general performance characteristics of an irreversible Carnot cycle machine has been achieved recently [4-6]. It may be applied to direct or reverse cycle machines. The optimization procedure focused on several objectives, namely maximum useful effect, minimum consumption or minimum total dissipation. Several operational and dimensional constraints were introduced in the model. Internal and external irreversibilities of the cycle were also taken into account by the internal and total entropy generation. The approach concerning the entropic analysis of machines and processes [7-10] becomes an important tool for the design of real operating machines.

The authors of the present paper brought their contributions in the field [4-6],

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adding some results to the previous cited works. The developed model provides an analytical method for optimizing the performance of an irreversible Carnot refrigerating machine for which dimensional or performance constraints were imposed. This method has applicability to the design of the refrigerating machines in terms of selection, design and optimization of the main parameters. The results of the model were compared with data from a refrigeration machine under operating conditions. The degree of correlation between the analytical and operational data indicated that the analysis accurately predicts the performance in terms of COP, power consumption and total entropy generation.

The next step in developing this subject was to consider transient operating regime. Since each analysis starts with the simplest cases, the paper deals with the case of equilibrium thermodynamics (thermostatic thermodynamics) refrigeration machines.

In transient regime, one of the most widespread problems is to determine the existence of a variation law for the refrigerant temperature that would be accompanied by imposed energy consumption. Another problem is to determine the minimum attainable refrigerant temperature to be reached in a given time interval with a given consumption of energy. Or, by imposing a given value for the system energetic consumption, to determine the refrigerant temperature variation law that will be attended.

2. Mathematical Analytical Model

A model for the study of transient regime in reversed cycle thermal machines with two heat reservoirs is presented. The mathematical model basically consists of the First and Second Laws of Thermodynamics applied to the cycle and the heat transfer equations at the source and sink. The entropy generation term considers the internal irreversibility of the cycle. Also, the sink source in fact is considered with finite thermal capacity, while for the heat source it is considered a constant temperature one. Equilibrium thermodynamics (no mass or heat gradients between working fluid and sources) is applied.

The schematic representation of the studied case is shown in Figure 1.

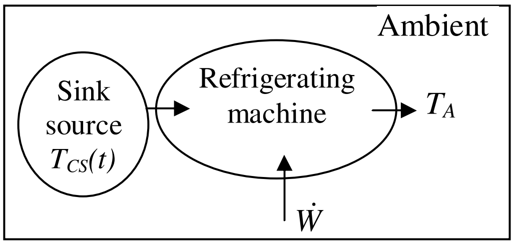

Fig. 1. *"Thermostatic" machine*

The First Law of Thermodynamics for transient operating conditions is:

$$\dot{E}_{in} - \left|\dot{E}_{out}\right| = \frac{dE_{sys}}{dt} \tag{1}$$

When applied to the refrigerating machine, equation (1) becomes:

$$\dot{W} = \dot{Q}_C + \dot{Q}_H = \dot{Q}_C - \left|\dot{Q}_H\right| \tag{2}$$

where the indices "*C*" and "*H*", respectively are for cold side and hot side; $\dot{W}$ is the consumed power (negative), while $\dot{Q}$ represents the thermal load (positive if received by the working fluid or negative if rejected to the source sink), corresponding to English sign convention.

The energy balance equation applied to the refrigerating chamber (cold source) gives:

$$\dot{Q}_C = -\left(mc_p\right)_{CS} \frac{dT_{CS}}{dt} \qquad (3)$$

which is positive since the cold source temperature T_{CS} decreases in time t. The product $\left(mc_p\right)_{CS}$ represents the thermal capacity of the cold source fluid and will be denoted by C_{CS}, in J/K.

The Second Law of Thermodynamics applied to the cycle gives:

$$\frac{\dot{Q}_H}{T_H} + \frac{\dot{Q}_C}{T_C} + \dot{S}_i = 0 \qquad (4)$$

where $\dot{S}_i$ is the internal entropy generation.

Considering thermal equilibrium (no heat gradient between working fluid temperature and heat reservoirs: $T_{CS} = T_C$ and $T_{HS} = T_H = T_A = const.$) and replacing $\dot{Q}_H$ from equation (2) in equation (4), one could get:

$$\frac{dT_{CS}}{dt}\left(\frac{1}{T_A} - \frac{1}{T_{CS}}\right) = -\frac{\dot{W}}{C_{CS} T_A} - \frac{\dot{S}_i}{C_{CS}} \qquad (5)$$

By integrating this equation and taking into account the analytical expression for the internal entropy generation and the imposed considered constraints, one could obtain the variation law for the cold source temperature, T_{CS} as a function of time t.

Let us denote the variation law for the cold source temperature T_{CS} by x as it is the unknown. The variation law in time will be $x(t)$.

The mathematical procedure for searching the minimum of a function (unknown function $x(t)$) is based on the Euler method for which the objective function (OF) and the constraints (C) are considered. The modified Euler equation allows us to find an integral function having a maximum or minimum value among all functions having this property:

$$\frac{\partial H}{\partial x} - \frac{d}{dt}\left(\frac{\partial H}{\partial x'}\right) = 0 \qquad (6)$$

which will give the function $x(t)$, where the Hamiltonian H is:

$$H(x, x', t) = OF + \lambda \cdot C \qquad (7)$$

This equation is to be solved for different cases, function on the imposed constraints and considered objective function.

3. Different study cases

Some study cases could be considered. For example:

Study A: for imposed power consumption $\dot{W}$ we look for the cold source temperature variation law in time.

Study B: for imposed power consumption $\dot{W}$ we look for the cold source temperature minimum value reached after a time period t_f.

Study C: for imposed cold source temperature T_{CSf} reached after a certain final time t_f, we look for that variation law of the cold source temperature T_{CS} which assures minimum energy consumption W_{min}, if there exists such a law.

Study D: the same problem as for study B, but considering heat losses through the cold source insulation.

Study E: the same problem as for study C, but considering heat losses through the cold source insulation.

For each study case, different sub-cases could be considered as function of the entropy generation variation law. In the absence of an empirical one, in technical literature, three laws have been proposed [11]:

a) constant: $\dot{S}_i = const$ (8a)

b) linear variation law with temperature:

$$\dot{S}_i = const \cdot \left(T_A - T_{CS}\right) \qquad (8b)$$

c) logarithmic variation law with

temperature: $$\dot{S}_i = const \cdot \ln \frac{T_A}{T_{CS}} \quad (8c)$$

If other variation laws are available, they could be used.

4. Study A

For imposed power consumption $\dot{W}$ we look for the cold source temperature variation law in time.

Let us consider firstly the sub-case (8a).

By separating the variables in equation (5), one obtains:

$$\left(\frac{1}{T_A}-\frac{1}{T_{CS}}\right)dT_{CS} = \underbrace{\left(-\frac{\dot{W}}{C_{CS}T_A}-\frac{\dot{S}_i}{C_{CS}}\right)}_{const}dt \quad (9)$$

By integrating this equation and computing the constant integral (taking into account that at moment $t = 0$ the cold source temperature is equal to the ambient temperature $T_{CS}(0) = T_A$), which resulted to be $1-\ln T_A$, one gets:

$$\frac{T_{CS}}{T_A} - \ln\frac{T_{CS}}{T_A} = t\left(-\frac{\dot{W}}{C_{CS}T_A}-\frac{\dot{S}_i}{C_{CS}}\right)+1 \quad (10)$$

which gives the variation law of T_{CS}.

The refrigerating load is:

$$\dot{Q}_0 = -C_{CS}\frac{dT_{CS}}{dt} = \frac{1}{\left(\frac{1}{T_A}-\frac{1}{T_{CS}}\right)}\left(\frac{\dot{W}}{T_A}+\dot{S}_i\right) \quad (11)$$

Some graphical results are illustrated in Figures 2-5.

Similarly one can solve the other sub-cases (8b) and (8c). For example, for the case (8b), one gets a nonlinear equation in x that is to be integrated:

$$\left(\frac{1}{T_A}-\frac{1}{x}\right)x' - \frac{K_S}{C_{CS}}x = -\frac{\dot{W}}{C_{CS}T_A} - \frac{K_S T_A}{C_{CS}} \quad (12)$$

where K_S is the constant that appears in the expression of $\dot{S}_i$ - equation (8b).

By integrating this equation (numerical results), one can get the variation law for the cold source temperature (here x).

5. Study B

For imposed power consumption $\dot{W}$ we look for the cold source temperature minimum value reached after a time period t_f.

In order to solve this problem, we write the expressions of the Hamiltonian as considering the power consumption as constraint and the objective function the minimum attainable cold source temperature at a time period t_f:

$$H(x, x', t) = x' + \lambda \cdot \dot{W} \quad (13)$$

Replacing $\dot{W}$ from equation (5), the Hamiltonian becomes:

$$H = x' + \lambda\left[\left(\frac{T_A}{x}-1\right)C_{CS}x' - T_A\dot{S}_i\right] \quad (14)$$

Let us considering a general expression for the internal entropy generation $\dot{S}_i$ as a function of x (as it is the case 8b or 8c). The modified Euler equation (6) is composed of the following derivatives:

$$\frac{\partial H}{\partial x} = -\lambda\frac{T_A}{x^2}C_{CS}x' - \lambda T_A\frac{\partial \dot{S}_i}{\partial x} \quad (15)$$

$$\frac{\partial H}{\partial x'} = 1 + \lambda\left(\frac{T_A}{x}-1\right)C_{CS} \quad (16)$$

$$\frac{d}{dt}\left(\frac{\partial H}{\partial x'}\right)=-\lambda\frac{T_A}{x^2}C_{CS}x' \qquad (17)$$

Replacing these derivatives in equation (6), we get:

$$-\lambda T_A\frac{\partial \dot{S}_i}{\partial x}=0 \qquad (18)$$

which obviously represents a constant internal entropy generation solution, being a contradiction with the considered hypothesis (a function of x). This means that for the above stated problem (“thermostatic” machine with imposed power consumption) there is no minimum attainable cold source temperature in a given time period t_f. Some interesting results are obtained for other constraints of the problem (e.g. considering losses between sources, finite heat transfer between sources and the working fluid, etc), but they are not the aim for the present paper.

6. Study C

For imposed cold source temperature T_{CSf} reached after a certain time duration t_f, we look for that variation law of the cold source temperature T_{CS} which assures minimum energy consumption W_{min}.

Thus the objective function is:

$$\dot{W}=-C_{CS}x'\left(1-\frac{T_A}{x}\right)-T_A\dot{S}_i \qquad (19)$$

And the Hamiltonian becomes:

$$H=-C_{CS}x'\left(1-\frac{T_A}{x}\right)-T_A\dot{S}_i+\lambda x' \qquad (20)$$

For the case of constant $\dot{S}_i$ – equation (8a), the derivatives composing the modified Euler equation (6) are:

$$\frac{\partial H}{\partial x}=-C_{CS}T_A\frac{x'}{x^2} \qquad (21)$$

$$\frac{\partial H}{\partial x'}=-C_{CS}\left(1-\frac{T_A}{x}\right)+\lambda \qquad (22)$$

$$\frac{d}{dt}\left(\frac{\partial H}{\partial x'}\right)=-C_{CS}T_A\frac{x'}{x^2} \qquad (23)$$

Replacing them in equation (6) will get:

$$-C_{CS}T_A\frac{x'}{x^2}+C_{CS}T_A\frac{x'}{x^2}=0 \qquad (24)$$

which is a mathematical identity, so the problem has one physical solution satisfied whatever is x.

Let us consider now the case (8b). The objective function is:

$$\dot{W}=-C_{CS}x'\left(1-\frac{T_A}{x}\right)-T_AK_S\left(T_A-x\right) \qquad (25)$$

where K_S is the constant that appears in the expression of $\dot{S}_i$ - equation (8b).

After computing the three derivatives from the Hamiltonian and replacing them in equation (6), we obtain:

$$T_AK_S=0 \qquad (26)$$

which is obviously a contradiction with our hypothesis (K_S = 0 means no internal entropy generation).

For the case (8c) we obtain:

$$\dot{W}=-C_{CS}x'\left(1-\frac{T_A}{x}\right)-T_AK_S\left(\ln T_A-\ln x\right) \qquad (27)$$

and equation (6) becomes:

$$T_AK_S\frac{1}{x}=0 \qquad (28)$$

which doesn’t have any physical solution.

But generally speaking, considering internal entropy generation as a function of x and x', one could obtain:

$$\frac{d}{dt}\left(\frac{\partial \dot{S}_i}{\partial x'}\right)-\frac{\partial \dot{S}_i}{\partial x}=0 \quad (29)$$

which doesn't provide a variation law for the cold source temperature. So, the considered problem doesn't have a physical solution.

7. Study D

For imposed power consumption $\dot{W}$ we look for the cold source temperature minimum value reached after a time duration t_f, considering heat losses through the cold source insulation.

In this case, equation (3) becomes:

$$\dot{Q}_C=-C_{CS}\frac{dT_{CS}}{dt}+K_L\left(T_A-T_{CS}\right) \quad (30)$$

where K_L is the heat loss conductance.

The power consumption becomes:

$$\dot{W}=\left(\frac{T_A}{x}-1\right)\left[C_{CS}x'-K_L\left(T_A-x\right)\right]-T_A\dot{S}_i \quad (31)$$

The associated Hamiltonian is:

$$H=x'+\lambda\cdot\dot{W} \quad (32)$$

After computing the three derivatives and replacing them in equation (6), we get a solution for x:

$$x=\frac{T_A}{\sqrt{\frac{T_A}{K_L}\left[\frac{\partial \dot{S}_i}{\partial x}-\frac{d}{dt}\left(\frac{\partial \dot{S}_i}{\partial x'}\right)\right]}} \quad (33)$$

If one takes into account the expression of internal entropy generation (which could be a function on x and x'), this solution gives us the possible cold source temperature variation law, obtained with minimum energy consumption.

8. Study E

For imposed cold source temperature T_{CSf} reached after a certain time t_f, we look for that variation law of the cold source temperature T_{CS} which assures minimum energy consumption W_{min}, but considering heat losses through the cold source insulation.

In this case, instead of equation (3) we use equation (30).

Supposing that the internal entropy generation depends on x, we get the Hamiltonian from equation (32) with $\dot{W}$ given by (31).

Solving the equation (6), we get a solution for x:

$$x=\frac{T_A}{\sqrt{1+\frac{T_A}{K_L}\frac{\partial \dot{S}_i}{\partial x}}} \quad (34)$$

By introducing the expression of internal entropy generation, the problem is solved.

9. Results and Discussions

For study A some graphical results are illustrated in Figures 2-5. Equation (10) imposes the variation law for cold source temperature T_{CS} as a function of time t under the imposed conditions.

Figure 2 represents a sensitivity study with respect to the internal entropy generation constant value. One can notice that higher values of $\dot{S}_i$ constraints the system to reach a certain value of the cold source temperature after a longer time period.

In figure 3, the refrigerating load is represented, as a function of time and also internal entropy generation constant values, respectively in Figure 4, the coefficient of performance.

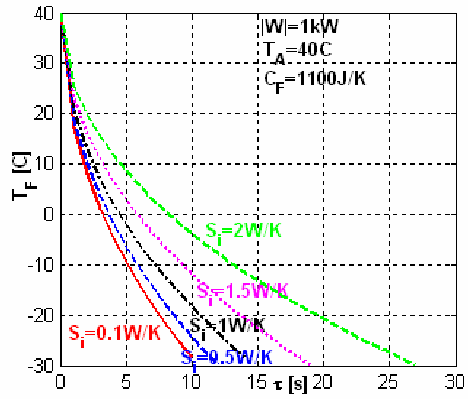

Fig. 2. *Cold source temperature as a function of time duration and* $\dot{S}_i$.

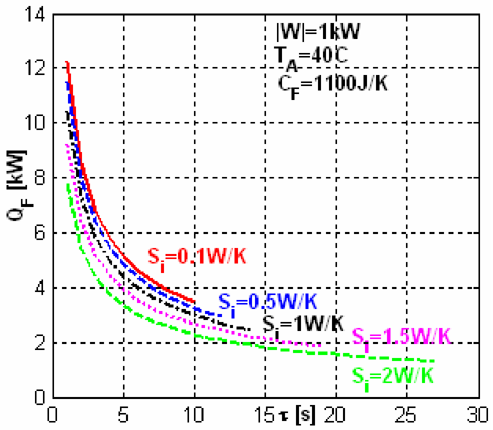

Fig. 3. *Refrigerating load as a function of time duration and* $\dot{S}_i$.

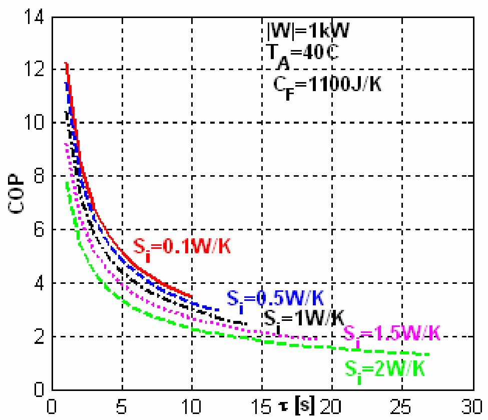

Fig. 4. *Coefficient of performance as a function of time duration and* $\dot{S}_i$.

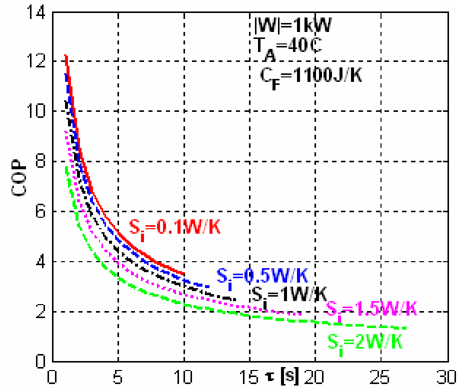

Fig. 5. *Refrigerating load as a function of time duration and thermal capacity* C_{CS}.

The main conclusion of this analytical development is related to the possible solution for T_{CS} in different cases. The considered hypotheses are very important. In our future studies, we are going to consider experimental relations for $\dot{S}_i$.

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