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MODELLING THE NON-STATIONARY MOVEMENT OF CRUDE OIL THROUGH PIPES

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Abstract: *In systems used for transporting oil, there are cases when the flow is unsteady. These can be noticed when the pumping starts, or when a valve is suddenly open or closed. The paper shows a numerical model for simulation of unsteady flow of liquids though pipelines, able to take into consideration both elasticity of the system – oil-pipeline – and dissipation phenomena, that is, friction occurring in the system.*

Key words: *pipes, pressure, flow, steady, unsteady*

In pipe systems for the transport of liquid petroleum products there are situations in which the flowing is non-stationary. This situation can occur when starting pumping or at the moment of abruptly acting on an open or close valve.

The article presents an original model of the crude oil non-stationary flow through pipes; the model is capable to take into account both the elasticity of the system crude oil-pipe as well as the dissipated phenomena, more precisely the frictions that appear in the system. We take into account the modelling of the non-stationary crude oil flow phase in order to determine the intensity of the phenomenon and its duration. At the same time we analyse the impact of the elastic waves from the system on different obstacles, phenomenon known as ramming.

1. The Mathematic Modelling of the Non-stationary Movement of Liquids Through Pipes

Starting from the equations of mass conservation and of the impulse in the case of isothermic flow, for a system without losses, we obtain the following equations:

$$\frac{\partial u}{\partial \tau}+\frac{1}{\rho_0}\frac{\partial p}{\partial x}=0, \qquad (1)$$

$$\frac{\partial p}{\partial \tau}+\rho_0 c_0^2\frac{\partial u}{\partial x}=0. \qquad (2)$$

In the case of long pipes we must take into account friction, and in this case the equation (1) becomes

$$\frac{\partial u}{\partial \tau}+\frac{1}{\rho_0}\frac{\partial p}{\partial x}+f\frac{u|u|}{2D}=0. \qquad (3)$$

In the previous equations u and p represent the speed, respectively the pressure of the liquid through the pipes, ρ_0 represents the density of the liquid, and f the coefficient of hydraulic resistance. The compressibility of the environment is characterized by the sound speed c_0 which represents the propagation speed of the perturbances in the system. This is defined according to the elasticity of the system:

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$$c_0 = \sqrt{\frac{E}{\rho}} \qquad (4)$$

$$\frac{1}{E_s} = \frac{1}{E_l} + \frac{D}{\delta E_{OL}} \qquad (5)$$

In relation (5) E_s represents the combined elasticity mode of the oil-pipe system, E_l – the liquid elasticity mode, E_{OL} - the pipe elasticity mode, D – the pipe diameter and δ – the thickness of the pipe wall.

In order to solve the system made up of the differential equations with partial derivatives (2) and (3) we will use the methodology described by *S. Godounov* through which the homogeneous system is solved; the system is made up of the differential equations with partial derivatives (1) and (2), then we correct the solution, taking into account the dissipative term from equation (3). Practically we look for a peculiar solution of the system made up of equations (2) and (3) which fulfils the imposed limit conditions.

The homogeneous system made up of equations (1) and (2) that represents the laws of mass conservation, respectively the impulse conservation, defined in a bidimensional field D(*x,ι*) accept continuous solutions. In order to also analyse the case of discontinuous solutions (continuous only on small fragments), which appear in the case of non-stationary movements, we integrate the homogeneous system of differential equations on field D(*x,ι*) and we apply *Green's formula* to transform the surface equations in curvilinear integrals on contour Γ of the respective field, and the result is

$$\iint_D \left(\frac{\partial u}{\partial \tau} + \frac{1}{\rho_0} \frac{\partial p}{\partial x} \right) = \oint_\Gamma (\rho_0 u dx - p d\tau) = 0 \qquad (6$$

$$\iint_D \left(\frac{\partial p}{\partial \tau} + \rho_0 c_0^2 \frac{\partial u}{\partial x} \right) = \oint_\Gamma \left(\frac{p}{c_0^2} dx - \rho_0 u d\tau \right) = 0 \qquad (7)$$

The system made up of equations (3) and (4) replaces the integral of the vector divergence $[u, p/\rho_0]^t$, $[p, \rho_0 c_0^2 u]^t$ on field D**,** with the vector flux $[\rho_0 u, -p]^t$, $[p/c_0^2, -\rho_0 u]^t$. May $u(x,\tau)$ and $p(x,\tau)$ be solutions of the system made up of equations (1) and (2). If these functions are discontinuous and the respective system is nonsensical, the integral conditions represented by relations (6) and (7) remain valid in this case. We designate the generalized solution of the system made up of equations (1) and (2) any pair of functions $u(x,\tau)$, $p(x,\tau)$ that can be derived on fragments for any close contour Γ, in the semi plan $\tau \geq 0$ relations (3) and (4) are checked.

2. The Numeric Approach of the Model

In order to define the scheme with finite differences we will consider a division of field *D(x,* □), in which we analyse the phenomenon. Thus the field is made up of constant intervals $\Delta x = x_j - x_{j-1}$. The functions that represent the measures that need to be determined (*u, p*) are considered network functions generically designated as *f*. We consider that the values of the functions are constant inside an interval $x_j - x_{j-1}$, the respective value is designated as $f_{j-1/2}^n$ and we consider it to be in the middle of the interval. The current time step is designated as index „*n*", and the new values calculated for the following time step will be designated as index "*n*+1". In the iterative integration process we will use auxiliary values considered at the middle of the temporary interval, these will be designated with capital letters and will have the temporary index "*n*+1/2", for instance $F_j^{n+1/2}$.

$$\oint_{\Gamma} (\rho_0 u dx - p d\tau) = 0 \qquad (8)$$

For the construction of the scheme with finite differences referring to equation (8), we will notice that this is variable on any close contour on field D; thus we can particularize contour Γ as a network node, as it can be seen in figure1.

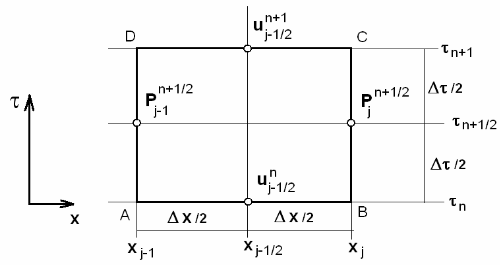

Fig. 1

In the equation there are two functions, and the speed u is integrated along OX axis and pressure p which can be integrated according to time. The contour Γ on which the scheme with finite differences is built is the close contour ABCD. Going on it counter clockwise we will define on each size the corresponding network functions:

- On side AB the value of speed at the temporary moment n

$$u_{j-1/2}^{n} = \frac{1}{\Delta x} \int_{x_{j-1}}^{x_j} u(x, \tau_n) dx . \qquad (9)$$

- On side BC defines the auxiliary measurement P representing the pressure at the middle of the temporary interval

$$P_j^{n+1/2} = \frac{1}{\Delta\tau} \int_{\tau_n}^{\tau_{n+1}} p(x_j, \tau) d\tau . \qquad (10)$$

- On side CD the speed u is defined at the temporary moment $n+1$

$$u_{j-1/2}^{n+1} = -\int_{x_{j-1}}^{x_j} u(x, \tau_{n+1}) dx . \qquad (11)$$

- On side DA the auxiliary measurement P representing the pressure at the middle of the temporary interval

$$P_{j-1}^{n+1/2} = -\frac{1}{\Delta\tau} \int_{\tau_n}^{\tau_{n+1}} p(x_{j-1}, \tau) d\tau . \qquad (12)$$

Applying relation (8) on the contour defined above, we get a relation in finite differences for the speed calculation

$$u_{j-1/2}^{n+1} = u_{j-1/2}^{n} - \frac{\Delta\tau}{\Delta x} \frac{1}{\rho_0} \left(P_j^{n+1/2} - P_{j-1}^{n+1/2} \right). \qquad (13)$$

For the second integral relation we will do a similar thing, meaning we will define a close contour Γ, EFGH on the network nodes, figure 2, and we will get

$$\oint_{\Gamma} \left(\frac{p}{c_0^2} \mathrm{d}x - \rho_0 u \mathrm{d}\tau \right) = 0 . \qquad (14)$$

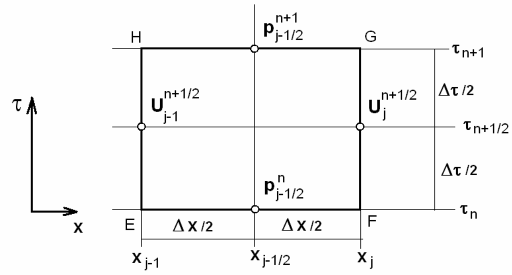

Fig. 2

According to relation (14) on side OX we define the pressure and for speed we use the auxiliary quantities designated and defined at the middle of the temporary interval.

- On side EF the value of the pressure at the temporary moment n is defined

$$p_{j-1/2}^{n} = \frac{1}{\Delta x} \int_{x_{j-1}}^{x_j} p(x, \tau_n) \mathrm{d}x . \qquad (15)$$

- On side FG the auxiliary measurement U is defined representing the speed at the half of the temporary interval in node j

$$U_j^{n+1/2} = \frac{1}{\Delta\tau} \int_{\tau_n}^{\tau_{n+1}} u(x_j, \tau) \mathrm{d}\tau . \qquad (16)$$

- On side GH pressure p at the temporary moment $n+1$ is defined

$$p_{j-1/2}^{n+1} = -\frac{1}{\Delta x}\int_{x_{j-1}}^{x_j} p(x,\tau_{n+1})\mathrm{d}x . \quad (17)$$

- On side HE the auxiliary measurement U is defined; it represents the speed at the half of the temporary interval in node j-1

$$U_{j-1}^{n+1/2} = -\frac{1}{\Delta\tau}\int_{\tau_n}^{\tau_{n+1}} u(x_{j-1},\tau)d\tau . \quad (18)$$

Applying relation (14) on contour EFGH we get the second relation in finite differences

$$p_{j-1/2}^{n+1} = p_{j-1/2}^{n} - \frac{\Delta\tau}{\Delta x}\rho_0 c_0^2\left(U_j^{n+1/2} - U_{j-1}^{n+1/2}\right) (19)$$

The system made up of equations (13) and (14) allows the determination of speed and pressure of thin shock waves, in time and space, with the condition of defining a rule for the calculation of the auxiliary values $P_j^{n+1/2}$, $P_{j-1}^{n+1/2}$, $U_j^{n+1/2}$, $U_{j-1}^{n+1/2}$ at the half of the temporary interval.

In order to define this problem, we consider in the space $(x,\tau);\tau \geq 0$, at the initial moment $\tau = 0$, the following conditions:

$$u_0(x) = u_I, \quad p_0(x) = p_I, \quad x < x^*,$$
$$u_0(x) = u_{II}, \quad p_0(x) = p_{II}, \quad x > x^* . (20)$$

The measures u_I, u_{II}, p_I, p_{II} are constant data that obey at least one of the relations $u_I \neq u_{II}$, $p_I \neq p_{II}$ or both of them simultaneously. The discontinuity defined by relations (20) are propagated in time and space with the sound speed under two discontinuity waves - thin shock waves – represented in Figure 3.

D_I, $x + c_0\tau = x^*$,the left wave, D_{II}, $x - c_0\tau = x^*$, the right wave.

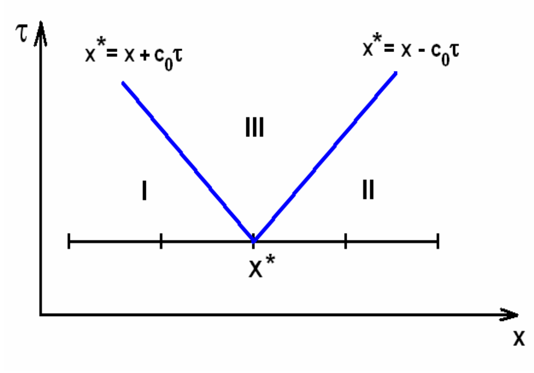

Fig. 3

These discontinuities (D_I, D_{II}) share the space (x, τ) in three zones defined as following:

- Zone I, undisturbed by the shock waves, represented by the points from the left discontinuity with the following properties:

$$u = u_I, p = p_I, \quad x < x^* - c_0\tau .$$

- Zone II, undisturbed by the shock waves, represented by the points from the right discontinuity with the properties:

$$u = u_{II}, p = p_{II}, x < x^* + c_0\tau .$$

- Zone III, disturbed by the shock waves, represented by the points inside the two waves, for which the values of speed and pressure will be designated as U and P, different from u_I, u_{II}, p_I, p_{II}, whose mathematical expressions will be further deduced.
- Functions $u(x,\tau)$ and $p(x,\tau)$ which represent the solution of the system made up of equations (1), (2) are continuous inside fields I, II and III but they are discontinuous on the axes $x^* = x + c_0\tau$ and $x^* = x - c_0\tau$, which separate the above mentioned fields. The two discontinuity steps appear and are formed due to the initial discontinuity $x = x^*$.

From the above mentioned reasons, the issue that we presented above is called the issue of decomposing a discontinuity. Formally we cannot consider functions $u(x,\tau)$ and $p(x,\tau)$ as solutions of the system made up of equations (1), (2) as these are discontinuous. From this cause the functions above represent the generalized solution of the system (1), (2) of the issue of the decomposing of a discontinuity.

If we apply the integral equation (5) on the close contour Γ defined in the vicinity of a discontinuity, as in Figure 4, we get the relation

$$[u]\mathrm{D}-\frac{p}{\rho_0}=0. \qquad (21)$$

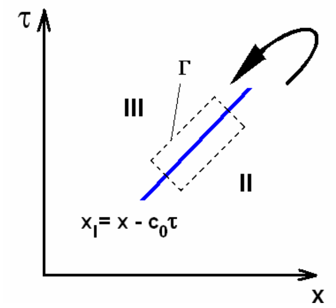

Fig. 4

The symbol $[u]$ represents the sprint of speed on the discontinuity cure, speed defined by

$$D=\frac{dx}{d\tau}\ (D_I=-c_0,\ D_{II}=c_0).$$

Using the notations from the figures, from relation (21) the following system of equations result:

$$\left(U_j^{n+1/2}-u_{j+1/2}^n\right)c_0-\frac{1}{\rho_0}p_{j+1/2}^n=0; \quad (22.a)$$

$$\left(U_j^{n+1/2}-u_{j-1/2}^n\right)(-c_0)-\frac{1}{\rho_0}p_{j-1/2}^n=0 \quad (22.b)$$

Solving the above system we get the auxiliary measurement that represents the speed at the half of the temporal interval, value valid for field III from figure 4.

$$U_j^{n+1/2}=\frac{1}{2}\left(u_{j+1/2}^n+u_{j-1/2}^n\right)-\frac{1}{2\rho_0 c_0}\left(p_{j+1/2}^n-p_{j-1/2}^n\right) \qquad (23)$$

Similarly, from the integral equation (11) applied on the closed contour Γ from the vicinity of the discontinuity curve, we get the relation

$$\left[\frac{p}{c_0^2}\right]D-\rho_0 u=0. \qquad (24)$$

The measure $\left[p/c_0^2\right]$ represents the sprint of pressure on the discontinuity cure. Taking into account the way in which the discontinuity speed D is defined, we get the equation system:

$$\frac{1}{c_0^2}\left(P_j^{n+1/2}-p_{j+1/2}^n\right)c_0-\rho_0 u_{j+1/2}^n=0;$$
$$\frac{1}{c_0^2}\left(P_j^{n+1/2}-p_{j-1/2}^n\right)(-c_0)-\rho_0 u_{j-1/2}^n=0. \qquad (25)$$

By solving the system above we get the expression of auxiliary quantity that represents the value of pressure defined at the half of the temporary interval, value which has sense only in the field III.

$$P_j^{n+1/2}=\frac{1}{2}\left(p_{i+1/2}^n+p_{i-1/2}^n\right)-\frac{\rho_0 c_0}{2}\left(u_{j+1/2}^n-u_{j-1/2}^n\right) \qquad (26)$$

Combining the relations (10), (16), (20) and (23) we get the following equation system in finite differences, which allows us to numerically determine the solution of the equation system (1), (2) on the basis of discontinuity decomposing:

$$u_{j-1/2}^{n+1} = u_{j-1/2}^{n} - \frac{\Delta\tau}{\Delta x}\rho_0\left[\left(\frac{p_{j+1/2}^{n} + p_{j-1/2}^{n}}{2} - \rho_0 c_0 \frac{u_{j+1/2}^{n} - u_{j-1/2}^{n}}{2}\right) - \left(\frac{p_{i-1/2}^{n} + p_{j-3/2}^{n}}{2} - \rho_0 c_0 \frac{u_{j-1/2}^{n} - u_{j-3/2}^{n}}{2}\right)\right] \quad (27)$$

as well as

$$p_{j-1/2}^{n+1} = p_{j-1/2}^{n} - \frac{\Delta\tau}{\Delta x}\rho_0 c_0^2\left[\left(\frac{u_{j+1/2}^{n} + u_{j-1/2}^{n}}{2} - \frac{p_{j+1/2}^{n} - p_{j-1/2}^{n}}{2}\right) - \left(\frac{u_{i-1/2}^{n} + u_{j-3/2}^{n}}{2} - \frac{p_{j-1/2}^{n} - p_{j-3/2}^{n}}{2\rho_0 c_0}\right)\right] \quad (28)$$

In order to analyse the conditions in which the approximation scheme (27), (28) is established we use *Fourier* method and the notation

$$\gamma = c_0 \frac{\Delta\tau}{\Delta x},$$

that is called *Courant's number*. From the stability condition the second degree equation in λ results with the solutions:

$$\lambda_{1,2} = 1 - \gamma(1 - \cos\varphi) \pm i\gamma\sin\varphi. \quad (29)$$

We look for the condition for *Courant's number* that satisfies $|\lambda| \le 1$, meaning

$$|\lambda_{1,2}| = \sqrt{[1 - \gamma(1 - \cos\varphi)]^2 + \nu^2 \sin^2\varphi} = \sqrt{1 - 4\gamma(1-\gamma)\sin^2\frac{\varphi}{2}},$$

so

$$0 \le 1 - 4\lambda(1-\gamma)\sin^2\frac{\varphi}{2} \le 1,$$

$$0 \le 4\gamma(1-\gamma) \le 1, \quad 0 \le \gamma \le 1.$$

Finally the condition for which the scheme is stable results, meaning

$$\gamma = c_0 \frac{\Delta\tau}{\Delta x} \le 1. \quad (30)$$

3. The Validation of the Calculation Method

We made an analysis in order to validate the model; the analysis was made on a 40 kilometer-long pipe that transports crude oil. We considered initially the pipe full of oil and then we analyzed the non-stationary flow from the moment we started pumping to the moment in which the flow became stationary.

We compared the slope of the pressure fall from the non-stationary movement with the slope corresponding to the stationary movement, the result being that they are identical. This thing leads us to the hypothesis that the stationary movement of the crude oil in the pipe is gradually developed behind the discontinuity wave. Practically, beginning and developing the movement of the crude oil through the pipe is done progressively from the pumping head to the supply head, behind the discontinuity wave. The phenomenon is a phenomenon that develops rapidly in time.

In figure 5 the synthesis of the results is presented for the non-stationary period, with friction, of the crude oil.

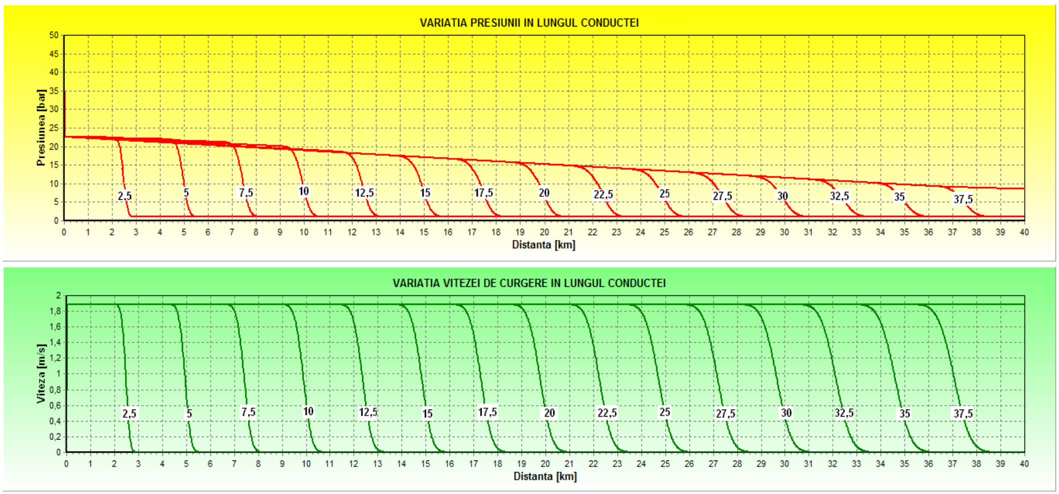

Fig. 5

At the top of the graphic the pressure variation appears, and at the bottom the seconds is inscribed on each curve. Once the pumping process starts along the pipe a pressure wave is created; it propagates along the column due to the environment elasticity.

4. Conclusions

From the analysis of the results above we can notice the influence of the dissipative term, actually the influence of friction on the non-stationary movement. The intensity of a discontinuity wave characterized through the pressure value is diminished as the wave advances in the pipe to the supply head. In spite of these the crude oil speed from speed variation in the non-stationary movement period. The time expressed in behind the wave remains constant, according to the mass conservation law.

We compared the slope of the pressure drop from the non-stationary movement with the corresponding slope of the stationary movement and the result was that they were identical. This led us to the hypothesis that the stationary movement of the crude oil through the pipe is gradually developed behind the discontinuity wave. Practically, beginning and developing the crude oil movement is done progressively from the pumping head to the supply head, behind the discontinuity wave. The phenomenon is a phenomenon that develops rapidly in time.

The numerical model presented in this article describes phenomena from the period of the non-stationary movement, the results obtained with it are according to the experimental measurements.

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