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CALCULATION OF THE UNSTEADY MECHANICAL AND GASOTHERMO-DYNAMIC PROCESSES IN THE FREE ROTATION SUPERCHARGER UNITS OF THE SUPERCHARGED INTERNAL COMBUSTION ENGINE

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Abstract: *The paper deals with the numerical calculation of the unsteady gaso-thermo-dynamical processes in the supercharger units with free rotation of internal combustion engines, including the evaluation of the functioning static mechanical characteristics of the compressor and turbine near the steady functional modes, and solving of the differential equation, of the mechanical system motion therefore the equation of dynamical motion.*

Key words: *mechanical, gasothermodynamic processes, combustion engine.*

1. Introduction

The dynamic bearing, technical index thermo/economical and operating for the supercharged internal combustion engine with supercharger units with free rotation turbocharger are substantially influenced by the unsteady gasothermodynamic and mechanical processes which take place in the turbocharger. Therefore, for the study of working conditions in common, turbocharger plus engine, it is necessary to carry out a model for calculation of unsteady gasothermodynamic and mechanical processes in supercharger units with turbocompressor, therefore it is necessary to establish dynamical behavior of the turbocharger. As a result of this calculation for these processes which take place in the turbochargers, and the knowledge of working conditions in operating the internal combustion engines it is possible to determine the working conditions in common and the watching of engine with the turbocharger.

The paper deals with the numerical calculation of the unsteady gasothermo-dynamic processes in the turbocharger, with the evaluation of the functioning mechanical characteristics of the component elements with Spline functions, as well as with the calculation of the mechanical processes, including the solving of the differential equation of the

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mechanical system motion of the supercharger units using Spline functions.

2. The Calculation of the Unsteady Mechanical and Gasothermodynamic Processes in the Turbocharger

Dynamical behavior of the mechanical mobile equipment internal combustion engine turbocharger with free rotation is given by differential equation:

$$J_{TK} \cdot \frac{d\ \omega_{TK}}{dt} = M_{teT} - |M_{teK}| \tag{1}$$

In case where the quasistationary gas-thermodynamic processes are true the relations :

$$M_{teT} = \frac{\dot{m}_T \cdot l_{tsT} \cdot \eta_T}{\omega_T} \tag{2}$$

$$|M_{teK}| = \frac{\dot{m}_K \cdot |l_{tsK}|}{\omega_K \cdot \eta_K}; \omega_T = \omega_K = \omega_{TK}$$

$$l_{tsT} = \frac{k_T}{k_T - 1} \cdot R_T \cdot T_T \cdot \left(1 - \left(\frac{1}{\pi_T}\right)^{\frac{k_T - 1}{k_T}}\right) \tag{3}$$

$$|l_{tsK}| = \frac{k_K}{k_K - 1} \cdot R_K \cdot T_0 \cdot \left(\pi_K^{\frac{k_K - 1}{k_K}} - 1\right)$$

$$\eta_T = f_{\pi_T}\left(\overline{\dot{m}_T}, \overline{\omega_T}, \overline{h_T}\right)$$

$$\pi_T = \frac{p_T}{p_0'} = \left(\overline{\dot{m}_T}, \overline{\omega_T}, \overline{h_T}\right)$$

$$\eta_K = f_{\pi_K}\left(\overline{\dot{m}_K}, \overline{\omega_K}, \overline{h_K}\right) \tag{4}$$

$$\pi_K = \frac{p_K}{p_0} = \left(\overline{\dot{m}_K}, \overline{\omega_K}, \overline{h_K}\right);$$

$$\overline{\dot{m}_T} = \frac{\dot{m}_T \cdot \sqrt{T_T}}{p_T}$$

$$\overline{\omega_T} = \frac{\omega_{TK}}{\sqrt{T_T}}; \overline{h_T} = \frac{h_T}{h_{Tnom}}; \tag{5}$$

$$\overline{\dot{m}_K} = \frac{\dot{m}_K \cdot \sqrt{T_0}}{p_0}$$

$$\overline{\omega_K} = \frac{\omega_{TK}}{\sqrt{T_0}}; \overline{h_K} = \frac{h_K}{h_{Knom}}; \tag{6}$$

where :

J_{TK} (kgm^2)-mechanical moment of inertia of the rotating mechanical turbocharger components, reduced to its revolution axis:

ω_{TK} (s^{-1})-turbocompressor and gas turbine angular speed;

ω_K (s^{-1})-compressor angular speed;

ω_T (s^{-1})-turbine angular speed;

T_0 (K)-fluid temperature at the entrance of the compressor;

T_k (K)-fluid temperature at the exit of the compressor;

T_T (K)-gas temperature at the entrance of gas turbine;

$\dot{m}_k$ (kg/s)-compressor mass flow rate;

$\dot{m}_T$ (kg/s)-gas turbine mass flow rate;

M_{teT} (Nm)-brake torque of gas turbine;

M_{teK} (Nm)-brake torque of the compressor;

l_{tsT} (J/kg)-specific isentropic shaft work of the turbine;

l_{tsK} (J/kg)-absolute value of the specific isentropic compressor shaft work;

$\eta_{T,K}$ -effective efficiency of gas turbine, compressor respectively;

k_K - compressor fluid adiabatic index;

k_T - gas fluid adiabatic index;

h_{KT} - current control organ position of compressor, turbine respectively;

$R_{K,T}$ (J/kg*K)-specific gas constant of the compressor fluid, exhaust gas respectively;

p_T (N/m^2)-gas pressure at the entrance of gas turbine;

$p_0^{'}$ (N/m^2)-gas pressure at the exit of gas turbine;

p_K (N/m^2)-fluid pressure at the compressor exit;

p_0 (N/m^2)-fluid pressure at the compressor entrance;

$h_{Knom,\ Tnom}$ -current control organ position of the nominal compressor, turbine respectively regim;

From the experimental performance of the gas turbine and the compressor results the following function :

$$M_{teT}=f_{MteT}\left(\omega_{TK},m_T,h_T,p_T,T_T,k_T\right) \quad (7)$$

$$M_{teK}=f_{MteK}\left(\omega_{TK},m_K,h_K,p_K,T_K,k_K\right) \quad (8)$$

The relations (7) and (8) may be written in the general form :

$$M_T = F(x,y,z,u,v,w) \quad (9)$$

To integrate differential equation (1) we can use cubic or polynomial spline functions of several variables [2] defined on the domain :

$$\Omega := \left\{ \begin{array}{l} (x,y,z,u,v,w) \in R^6 \middle| x_0 \le x \le x_N, y_0 \le y \le y_k, z_0 \le z \le z_L, u_0 \le u \le u_M, \\ , v_0 \le v \le v_l, w_0 \le w \le w_J \end{array} \right\}$$

Equation (1) may be written in forms :

$$J_{TK}\frac{dx}{dt} = f_{MteT}\left(x(t),y_1,z_1,u_1,v_1,w_1\right) - f_{MteK}\left(x(t),y_2,z_2,u_2,v_2,w_2\right) \quad (1')$$

$$\frac{dx}{dt} = f\left(x(t),y_1,z_1,u_1,v_1,w_1,y_2,z_2,u_2,v_2,w_2\right) \quad (1'')$$

estimating mechanical unsteady processes in steady gas thermodynamic process conditions of the internal combustion engine turbocharger .

Equation (1'') solution with the spline function support is reduced to approximate the solutions with initial conditions .

Approximate integrating equation (1) involves determining a polynomial spline function with m >=2 degree (m = 3 for cubic spline function) and which belongs from the class $s \in C^{m-1}[a,b]$ with interval $[a,b]$ for variable t (time), ($s \in C^{m-1}[a,b]$ means that function s is of (m-1) class, which represents the fact that s has continuous derivatives to (m-1) degree, inclusive on $[a,b]$ interval);

$$t_k = t_0 + k\cdot h,\ k = 0,1,..N$$

$$\Delta t : a = t_0 < t_1 < t_2 ..... < t_N = b; \quad (10)$$

$$x(t_0) = x_0;\ x^{'}(t_0) = x_0^{'};\ x_0 = s(x_0);\ x_0^{'} = s^{'}(x_0);$$

On the interval $[t_k,\ t_{k+1}]$ spline function s is defined in this way :

$$S(t) = \sum_{j=0}^{m-1} \frac{s^{(j)}(t_k)}{j!} * (t-t_k)^j + \frac{a_k}{m!} * (t-t_k)^m,\ t \in [t_k, t_{k+1}] \quad (11)$$

where $s^{(j)}(t_k)$ is the limit on the left for the order j derivative of the spline function on the interval $[t_k,\ t_{k+1}]$. Values a_k are determined for k=1,2,...N-1 from condition:

$$s'(t_{k+1})=f(s(t_{k+1}), y_1=ct, z_1=ct, u_1=ct, v_1=ct, w_1=ct, y_2=ct, z_2=ct, u_2=ct, v_2=ct, w_2=ct) \quad (12)$$

resulting :

$$a_k = \frac{(m-1)!}{h^{m-1}}\left(f(s(t_{k+1}), y_1 = ct, \ldots) - \sum_{j=0}^{m-1} \frac{s^{(j)}(t_k)}{(j-1)!} h^{j-1} \right) \quad (13)$$

In order to spline polynomial function be unique must be satisfied the conditions :

$$h < \frac{m-1}{L}; 3 \le m \le 5 \quad (14)$$

where L is a Lipschitz constant for the function f.

From the experimental data are built partial spline function order I or II for f_{MteT} and f_{MteK} on each direction.

For order I spline function results [2] :

$$s_{\Delta x}\big|_{I_i} = p_i(x)\big|_{I_i};\ I_i = (x_i, x_{i+1}),\ I_{i+1} = \left(x_{i+1}, x_{i+2}\right)$$

$$\Delta x: a = x_1 < x_2 < \ldots < x_{N-1} < x_N = b \quad (15)$$

$$s_{\Delta x}\big|_{I_{i+1}} = p_{i+1}(x)\big|_{I_{i+1}}$$

$$p_i(x) = f_{Mt_i} + m_i(x - x_i) + \left(3\frac{f_{Mt_{i+1}} - f_{Mt_i}}{h_i^2} - \frac{f_{Mt_{i+1}} + 2f_{Mt_i}}{h_i}\right)(x - x_i)^2 +$$
$$+\left(-2\frac{f_{Mt_{i+1}} - f_{Mt_i}}{h_i^3} + \frac{f_{Mt_{i+1}} + f_{Mt_i}}{h_i^2}\right)(x - x_i)^3, h_i = x_{i+1} - x_i,\ i = 1,2,\ldots,N-1 \quad (16)$$

$m_j = Dp_i(x_j)$ first order derivatives in the x_j for $p_i(x)$;

Values m_j results from the conditions $D^2 p_i(x_{i+1}) = D^2 p_{i+1}(x_{i+1})$:

$$h_{i+1}m_i + 2(h_i + h_{i+1})m_{i+1} + h_i m_{i+2} = 3\left(h_i + \frac{f_{Mt_{i+2}} - f_{Mt_{i+1}}}{h_{i+1}} + h_i\frac{f_{Mt_{i+1}} - f_{Mt_i}}{h_i}\right);$$

$$i = 1,2,\ldots N-2;$$

$$2m_1 = 2F'_{Mt_1},\ \text{or},\ 2m_1 + m_2 = 3\frac{F_{Mt_2} - F_{Mt_1}}{h_1} - \frac{h_1}{2}F''_{Mt_1} \quad (17)$$

$$2m_N = 2F'_{Mt_N},\ \text{or},\ m_{N-1} + 2m_N = 3\frac{F_{Mt_N} - F_{Mt_{N-1}}}{h_{N-1}} + \frac{h_{N-1}}{2}F''_{Mt_N}$$

where: F'_{Mt_1}, F'_{Mt_N}, F''_{Mt_1}, F''_{Mt_N} are the approximate values for first and second order derivatives of the given spline function, on the ends of the interval;

If the approximate solution of equation (1) is a spline function with m >=5 then it is adequate to use spline function with deficiency because of convergence, therefore we can write conform [2]:

$$s(t_0 + t) = x_0 + \frac{x_0' t}{1!} + \sum_{i=2}^{m} c_i \frac{t^i}{i!},\ t \in [t_0, t_0 + (m-1)h] \quad (18)$$

known x_0, on, $[t_0, t_1]$; the values c_i, i=2,3,..,m result from conditions:

$$s'(t_0+jh)=f\left(t_0+jh,\ s(t_0+jh),\ y_1=ct\right),$$
$$j=1,2,m-1 \tag{19}$$

Knowing c_2,......, c_m there are determined $s(t_1)$ *and* $s'(t_1)$.

For the interval [t_1, t_2] we can write :

$$s(t_1+t)=s(t_1)+\frac{s'(t_1)}{1!}(t-t_1)+\sum_{i=2}^{m}c_i^{(1)}\cdot\frac{(t-t_1)^i}{i!},\ t\in[t_1,t_2] \tag{20}$$

the values c_i, i = 2, 3, .., m result from conditions:

$$s'(t_1+jh)=f\left(t_1+jh,\ s(t_1+jh),\ y_1=ct\ldots\right),$$
$$j=1,2,\ldots,m-1 \tag{21}$$

The unique solution for c_i, I = 2, 3,..,m determined for each interval, gives spline function consistence and, because of its construction, $s\in C^1[t_0,t_N]$.

In the case of the unsteady gas thermodynamic processes study, near to stationary processes and for the unsteady mechanical processes, equation (1) becomes:

$$J_{TK}\frac{d(\Delta\omega_{TK})}{dt}-\left(\left(\frac{\partial M_{teT}}{\partial\omega_{TK}}\right)_0-\left(\frac{\partial M_{teK}}{\partial\omega_{TK}}\right)_0\right)\Delta\omega_{TK}=\left(\frac{\partial M_{teT}}{\partial\dot{m}_T}\right)_0*\Delta\dot{m}_T-\left(\frac{\partial M_{teK}}{\partial\dot{m}_K}\right)_0*\Delta\dot{m}_K+$$
$$+\left(\frac{\partial M_{teT}}{\partial h_t}\right)_0*\Delta h_t-\left(\frac{\partial|M_{teK}|}{\partial h_k}\right)_0*\Delta h_k+\left(\frac{\partial M_{teT}}{\partial p_T}\right)_0*\Delta p_T-\left(\frac{\partial|M_{teK}|}{\partial p_k}\right)_0*\Delta p_k+ \tag{22}$$
$$+\left(\frac{\partial M_{teT}}{\partial T_T}\right)_0*\Delta T_T-\left(\frac{\partial|M_{teK}|}{\partial T_0}\right)_0*\Delta T_0+\left(\frac{\partial M_{teT}}{\partial k_T}\right)_0*\Delta k_T-\left(\frac{\partial|M_{teK}|}{\partial k_k}\right)_0*\Delta k_k$$

where:

$\Delta\omega_{TK}$, $\Delta\dot{m}_k$, $\Delta\dot{m}_T$, Δh_t, Δh_k,

Δp_T, Δp_k, ΔT_T, ΔT_0, Δk_T, Δk_k

-- the deviations of the values from the stationary values, noted by " 0 ".

Using spline function which approximate M_{teT}, M_{teK} using (16, 17) on each direction we calculate the derivatives which are in equation (22), for each direction and then solve differential equation (22) using spline function with the solution of the form (11, or 18, 20) and obtain the approximate solution **3.**

3. Conclusions

The algorithm shown coupled with the algorithm for calculus of the motor process [1] permits the gas-thermodynamic and mechanical process study for the turbocharged internal combustion engine.

Using the programme for interpolation of functions with two or three variables with the help of Spline functions, from [2], it is possible to estimate with a good approximation the characteristics of gas turbines and turbocompressors for turbochargers on the basis of experimental characteristics known.

Using the equations recommended in the paper it is possible to calculate the terms from differential equation of rotative mechanical system motion, solve the equation, establishing in this case with a good approximation the working conditions of turbocharger. If we know these different working conditions for the turbocharger and the different working conditions of internal combustion engines with big probability, it is possible to establish the common working conditions of internal combustion engine and turbocharger.

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