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GEOMETRICAL ASPECTS OF POLYGONAL FRESNEL MIRRORS WITH DOUBLE TRACKING DESIGNED FOR A SOLAR STIRLING ENGINE

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Abstract: *A geometric design solution for a Polygonal (three-sides) Fresnel Mirror suitable for a solar Stirling engine is proposed. The assembly is designed to have a double motion following the daily Sun motion, on horizontal and vertical directions. Also, it takes into account the Sun motion by the Azimuth and the local hour. As an example of practical use, the useful area of the mirrors and the number of Fresnel modules imposed by the SUNMACHINE solar Stirling engine are estimated.*

Key words: *Fresnel mirror, solar Stirling engine, solar energy, geometric design.*

1. Introduction

Due to fluctuating oil prices, depletion of fossil fuel resources, global warming and local pollution, geopolitical tensions and growth in energy demand, alternative energies, renewable energies and effective use of fossil fuels have become much more important than at any time in history. Current and future markets in fossil fuels are subject to volatile price changes in oil and natural gas. National and international energy/environmental crises and conflicts are combining to motivate a dramatic paradigm shift from fossil fuels to reliable, clean and efficient fuels. Using renewable energy sources seems a promising option. However, there are still some serious concerns about some renewable energy sources and their implementation, e.g. (i) capital costs, and (ii) their intermittent nature in power production.

Renewable energy resources – such as wind and solar energies – cannot produce power steadily, since their power production rates change with seasons, months, days, hours etc. The cost issues depend mainly on how research and developments can be successfully carried out in these areas. Extensive public and private researches, and development efforts to achieve technological breakthroughs, are required to bring these technologies to commercial maturity.

Of the developed renewable technologies, Concentrating Solar Power (CSP) is possibly the most adaptable. It can be built in a range of sizes, from a few MW up to several hundred MW [7]. It can be configured with varying levels of storage to suit local weather conditions and to meet the requirements of the local grid operator.

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Different simulation models for solar concentrators for CSP were developed to obtain the irradiance distribution on the absorber [8].

The Linear Fresnel designs use lower cost mirrors than troughs [3], and avoid the need for the expensive heliostats inherent in a power tower design.

The 'Fresnel mirror' type of CSP system is broadly similar to parabolic trough systems but instead of using trough-shaped mirrors that track the sun, it uses long flat mirrors at different angles that have the effect of focusing sunlight on one or more pipes containing heat-collecting fluid which are mounted above the mirrors [10], or on concentrated radiation receiver [5].

The avoidance of large mirror row spacing and receiver heights is an important cost issue in determining the cost of ground preparation, array substructure cost, tower structure cost, steam line thermal losses, and steam line cost. The improved ability to use the Fresnel approach delivers the traditional benefits of such a system, namely small reflector size, low structural cost, fixed receiver position without moving joints, and non-cylindrical receiver geometry. A necessary requirement in this activity was the development of specific ray trace and thermal models to simulate the new concepts [5].

An optimum convex shaped nonimaging Fresnel lens following the edge ray principle is presented in [4]. The lens is evaluated by tracing rays and calculating a projective optical concentration ratio.

Then, the design of a linear Fresnel lens (LFL) according to Fermat's principle is slightly modified with respect to used technology for mass production from glass [6]. Also, a combination of linear Fresnel lenses with PV cells may reduce cost of autonomous solar installations [6].

Relative to the previous work, the present paper proposes an original geometrical solution for a polygonal Fresnel mirror designed to drive a solar Stirling engine.

2. Geometric Design Solution for a Polygonal Fresnel Mirror

Generally, a solar Stirling engine requires a higher temperature at its hot end than a steam generation system. Hence, the concentration factor should have significantly high values that would need large flat Fresnel mirror.

Our geometric design solution leading to a high concentration factor consists of replacing the flat Fresnel mirrors by polygonal modules containing a three side mirror. A solar concentration system using vertical arrays of polygonal modules is illustrated in Figure 1. One can see the two external side mirrors that are inclined compared to the central side mirror so that all the three concentrate the solar radiation in the receiver (see also Figure 2). It results that each polygonal module will increase three times the concentration factor. On the other hand, the receiver may have a smaller dimensions and a fixed position that represent clear advantages when compared to the dish Stirling systems.

3. Optimal rotation angle between sides

In order to determine the optimal rotation angle between the three sides of a Fresnel module mirror, let us considering the situation when the sun rays, the concentrated solar radiation receiver (CSRR) and the mirror are in the same plane (at noon, middle positioned row and column of modules). Once these angles between sides are determined, all the modules will be similar. The difference between their position relative to the Sun rays and CSRR will be imposed by the row and column numbers, as we will see.

A little geometry helps us to determine

the unknown angle $\gamma/2$ in Figure 2. This angle represents the rotation of the upper side mirror with respect to the middle side. By reasons of symmetry, the bottom side mirror is rotated inversely with the same angle $\gamma/2$.

Comparing the triangles BC'C to BSD, one can notice that: $C'C \| BD$ (as both of them are bisector lines for the Sun rays), $C'B \| SD$ while $C'C \perp BC$ and $BD \perp BC$ since the bisector line is perpendicular on the mirror (reflection law). Because of this, the triangles BC'C and BSD are similar ($\Delta BC'C \sim \Delta BSD$); the conclusion is that we identify two equal angles:

$$\angle C'BC \equiv \angle SBD = \frac{\gamma}{2} \tag{1}$$

As BD is the bisector line for the angle $\angle SBR$, it results:

$$\angle SBR = 2 \cdot \angle SBD = \gamma \tag{2}$$

On the other hand, one can notice two angles with one common line and the other two being parallel one to each other, where from:

$$\angle SBR = \angle BRA = \gamma \tag{3}$$

In the triangle BRA:

$$tg(\angle BRA) = tg(\gamma) = \frac{[AB]}{[AR]} = \frac{L}{F} \tag{4}$$

Thus, the rotation angle for the left and right sides mirrors is:

$$\frac{\gamma}{2} = \frac{arctg\left(\frac{L}{F}\right)}{2} \tag{5}$$

Each Fresnel module to be used is identical to this one (Figure 2).

4. Optimal rotation angle in horizontal plane

In Figure 3 one can imagine an upper view of the modules mounted on a same row. Between two modules, there is a distance called horizontal step and noted here P_O.

For simplicity, we consider again the situation when the sun rays, the concentrated solar radiation receiver (CSRR) and the mirror row are in the same plane (at noon, middle positioned row of modules and all the columns).

In this case, the middle positioned module is not rotated. In order to reflect the Sun rays into the same CSRR, all the other columns (of this row) should be rotated by different angles denoted $\gamma_{O,i}/2$ with respect to the vertical position, where *i* represents the number of the column with respect to center (middle positioned) module. We will denote with *i* the columns of modules positioned to the left hand side of the center module when looking from the mirror to the CSRR, and with –i the columns of modules on the right hand side of the center module.

By simple geometrical considerations, following the same reasoning as for the case presented in section 3, taking into account all the above considerations, one can obtain for the first left hand side column:

$$\frac{\gamma_{O,1}}{2} = \frac{arctg\left(\frac{P_O}{F}\right)}{2} \tag{6}$$

Calculating for the second left hand side column, one obtains:

$$\frac{\gamma_{O,2}}{2} = \frac{arctg\left(\frac{2P_O}{F}\right)}{2} \quad (7)$$

When the above two relations are generalized for the i^{th} left hand side column, one gets:

$$\frac{\gamma_{O,i}}{2} = \frac{arctg\left(\frac{i \cdot P_O}{F}\right)}{2} \quad (8)$$

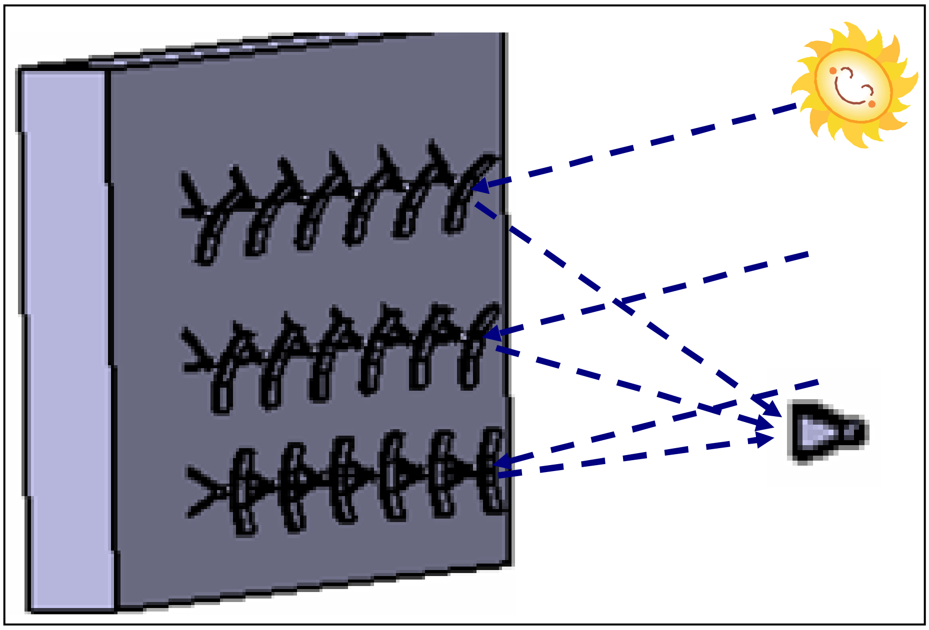

Fig. 1. *Vertical positioned Fresnel mirror modules, oriented on an East-West direction (side view)*

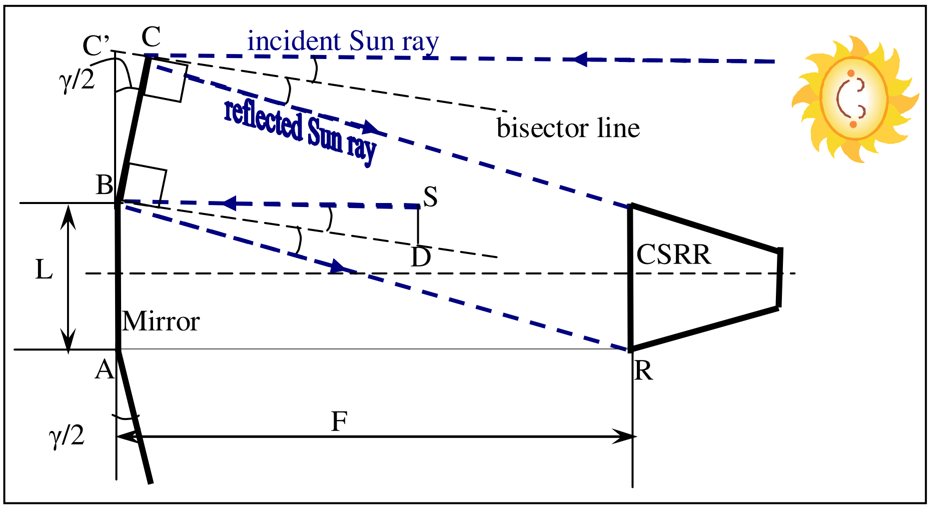

Fig. 2. *Three sides Fresnel mirror – optimal rotation angle between sides*

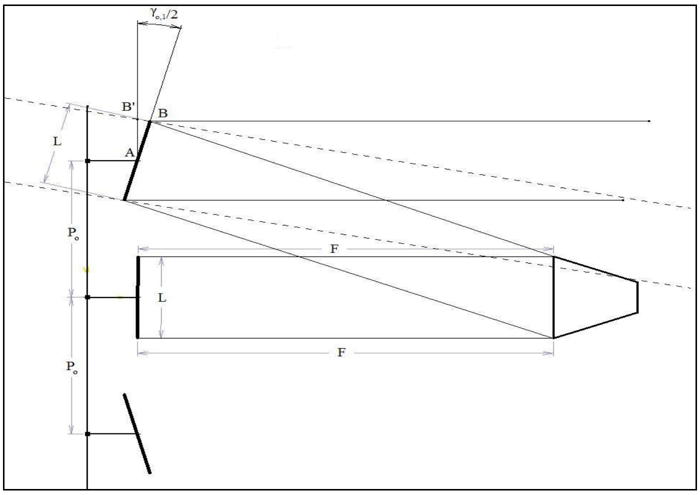

Fig. 3. *Optimal rotation angle in horizontal plane*

So, each column of modules situated on the left hand side of the center column is to be rotated with respect to vertical by $\gamma_{O,i}/2$ angle.

By symmetry, all the right-hand side columns are to be rotated by the same angles, but in opposite direction, as it is represented in Figure 3.

5. Optimal rotation angle in vertical plane

We have seen previously how are rotated the modules of mirrors situated on the left-hand side and right-hand side of the center module. Now we are studying the rotation angle of the rows situated above and below the center row of modules.

In Figure 4 one can imagine side view of the rows mounted on a vertical wall. Between two consecutive columns of modules there is a distance called vertical step and denoted here by P_V.

As for the previous cases, for simplicity, we consider again the situation when the sun rays, the concentrated solar radiation receiver (CSRR) and the mirror center row are in the same plane (at noon, middle positioned row of modules and all the rows from the center column of modules).

We will denote with *i* the columns of modules positioned to above the center row of modules and with *–i* the columns of modules below the center row of modules.

By similar reasoning as for the case presented in section 4, taking into account all the above considerations, one can obtain for the angle of rotation for the first above row:

$$\frac{\gamma_{V,1}}{2} = \frac{arctg\left(\frac{P_V}{F}\right)}{2} \tag{9}$$

For the second above row, it results:

$$\frac{\gamma_{V,2}}{2} = \frac{arctg\left(\frac{2P_V}{F}\right)}{2} \tag{10}$$

By generalizing the above two relations, for the i^{th} row situated above the center row, one gets:

$$\frac{\gamma_{V,i}}{2} = \frac{arctg\left(\frac{i \cdot P_V}{F}\right)}{2} \tag{11}$$

So, each row of modules situated above the center row is to be rotated with respect to vertical by $\gamma_{V,i}/2$ angle. By symmetry, all the rows situated below the center row are to be rotated by the same angles, but in opposite direction, as it is represented in Figure 4.

6. Rotation angle depending on the local time

In Figure 5 one can imagine the case of sunrise and all the rows oriented in such a manner to reflect the incident Sun rays in the same CSRR.

This time, the Sun rays are no more horizontal (as in the previous cases when we have considered noon), but at an angle called "height between horizon" and denoted by *h*. The height between horizon depends on the local time, season, and latitude.

According to this, the center module is inclined by angle *h'* with respect to horizontal (un-inclined) position.

A basic geometry prove: one can notice in Figure 5 that $A'C \perp A'B'$; we build the line $AA'' \| A'O$. This helps us write:

$$\angle A''AB'' + \angle B''AC = 90^\circ \tag{12}$$

Or, taking into account a similar prove as the one detailed in section 3:

$$\frac{h'}{2} + \left(\frac{h'}{2} + h\right) = 90^\circ \tag{13}$$

wherefrom:

$$\frac{h'}{2} = 45^\circ - \frac{h}{2} \tag{14}$$

So, the center column of modules is rotated by this angle (*h'/2*) with respect to horizontal and all the other columns are rotated by this angle plus the previously proved phase difference on horizontal and vertical direction.

The height between horizon is calculated as [7]:

$$\sin(h) = \cos(\varphi)\cdot\cos(H)\cdot\cos(\delta) + \sin(\varphi)\cdot\sin(\delta) \tag{15}$$

where:

φ is the geographic latitude (positive in Nordic hemisphere);

H is the hour angle:

$$H = 15^\circ \cdot (time - 12) \tag{16}$$

δ is the solar declination angle [8, 9]:

$$\delta = 23.45 \cdot \sin\left(360^\circ \frac{284 + n}{365}\right) \tag{17}$$

with *n* the number of the day in an year.

7. Numerical example

Based on technical data of the SUNMACHINE solar Stirling engine [10] and efficiencies of the solar radiation concentrating system [11], the useful area of the mirrors is computed. Thus, an electrical power output of 3 kW with a thermal efficiency of the engine of 25 % and a concentrating system efficiency of 45 % (that takes into account the reflection, interception, and secondary reflection efficiencies) lead to a solar thermal input need of 26.66 kW. By considering the value of 1 kW/m^2 for the maximum solar insolation density, it results a total reflecting area around 27 m^2. It can be provided by 18 polygonal Fresnel modules, each consisting of three mirrors with 1 m x 0.5 m as dimensions.

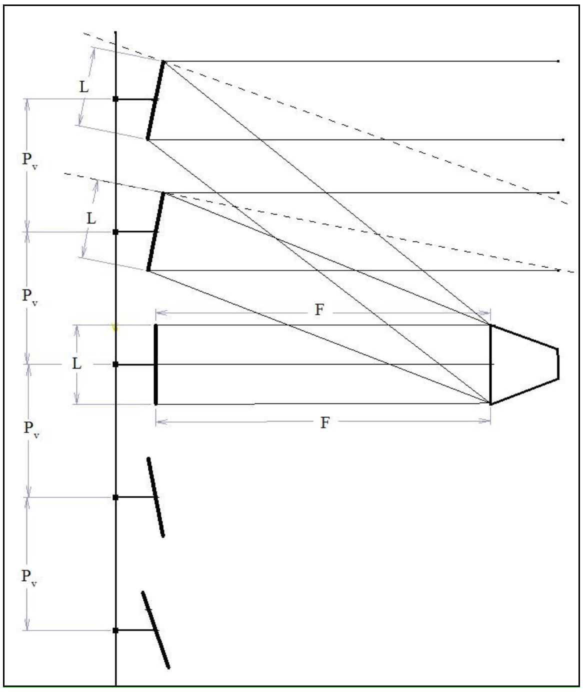

Fig. 4. *Optimal rotation angle in vertical plane*

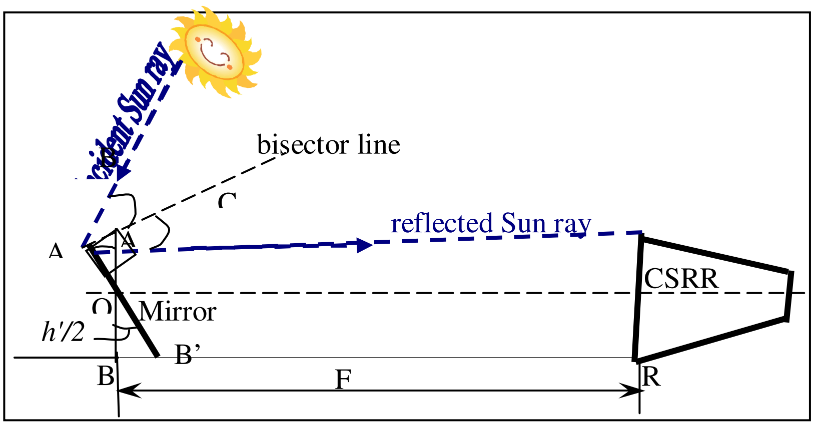

Fig. 5. *Fresnel mirror modules, oriented on an East-West direction (upper view in the morning)*

8. Conclusions

The paper presents an original design of a solar reflector system consisting of polygonal Fresnel mirror. The optimal rotation angles between sides, on horizontal and vertical planes, were determined, as well as the rotation angle depending on the local time. As an example of practical use, the number of polygonal Fresnel modules necessary to drive the SUNMACHINE solar Stirling engine was estimated.

This solar collector system has significant advantages in cost and scalability. It could be small and dispersed, typically on rooftops. It is the most land-efficient solar technology in operation, generating 1.5-to-3 times more power per acre of land than competing solar technologies. This high energy density translates into lower costs, a smaller environmental footprint and greater access to existing power plant and industrial sites.

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