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# INFLUENCES OF THE MINIATURISED INERTIAL SENSORS ERRORS ON THE NAVIGATION SOLUTION IN A BIDIMENSIONAL SDINS IN VERTICAL PLANE

**T.L. Grigorie[1], M. Lungu[1], I.R. Edu[1], R. Obreja[1]**
[1]University of Craiova, Avionics Division, Craiova, ROMANIA, e-mail: lgrigore@elth.ucv.ro, mlungu@elth.ucv.ro, edu_ioana_raluca@yahoo.com, radu@sistemeuroteh.ro

***Abstract:*** *The paper deals with the study of the influence of the inertial sensors errors on the position, speed and attitude of a bidimensional strap-down inertial navigation system (SDINS) in a vertical plane. To test this influence we put up MATLAB/SIMULINK models for the acceleration and rotation sensors based on the sensors data sheets and on the IEEE equivalent models for the inertial sensors. The models can be used in the numerical simulation of the strap-down inertial navigation system, close by real conditions from the point of view of the distortions suffered by the useful acceleration and rotation signals at the passing through any type of accelerometers or gyros desired to be implemented in the navigator. These models have the advantage to work independent from each of the sensors' errors and thus to study their influence on the inertial navigator.*
***Keywords:*** *Strap-Down, Inertial Navigation, Miniaturised Inertial Sensors, Errors, Sensors models*

## 1. INTRODUCTION

Taking into account the safety and security requirements of increasingly severity, currently imposed on air transport, aerospace navigation systems provide continuously research topics in the scientific community in the field and beyond. Also, military domain and special civilian applications assign miniaturization and redundancy requirements that make this area one of the top in current technology. Extension of the inertial navigators and of the GPS on land and maritime navigation applications, but also in robotics applications, led to an increased interest in these systems beyond the borders of aerospace industry. Worldwide, increasingly more companies involved in specific research activities, as well as in activities to achieve miniature inertial sensors or even miniature inertial navigation systems.

The here presented work is a part of a research project concerning the development of *high-precision strap-down inertial navigators, based on the connection and adaptive integration of the nano and micro inertial sensors in low cost networks, with a high degree of redundance*, financed by National Council for Scientific Research in Higher Education (CNCSIS) in Romania.

The paper deals with the study of the influence of the inertial sensors errors on the position, speed and attitude of a bidimensional Strap-Down Inertial Navigation System (SDINS) in a vertical plane. To test this influence we put up MATLAB/SIMULINK models for the acceleration and rotation sensors based on the sensors data sheets and on the IEEE equivalent models for the inertial sensors. The models can be used in the numerical simulation of the strap-down inertial navigation system, close by real conditions from the point of view of the distortions suffered by the useful acceleration and rotation signals at the passing through any type of accelerometers or gyros desired to be implemented in the navigator. These models have the advantage to work independent from each of the sensors' errors and thus to study their influence on the inertial navigator.

On the other way, an analytic error model of the SDINS is developed and software implemented. The development of the model aims to create a mechanism whereby a SDINS developer can accomplish the estimation, still in the navigator design stage, of the impact of errors, affecting the desired to be used miniaturized inertial sensors, on the precision of position, speed and attitude calculation. In a first phase, the navigator mechanization equations are presented. Based on these equations, and considering the embedded inertial sensors errors as small perturbations, an analytical error model resulted for the studied navigator. The obtained model is constituted by five coupled differential equations and contains five variables: a variable representing the attitude angle error ($\delta\theta$), two variables characterizing the vehicle speed error in navigation reference frame ($\delta v_{xl}$, $\delta v_{zl}$), and two variables characterizing the errors of vehicle positioning in navigation reference frame ($\delta x_l$, $\delta z_l$). The model inputs are the errors of the three inertial sensors used in the strap-down navigator: $\delta f_{xv}$ and $\delta f_{zv}$ for the two accelerometers, and $\delta\omega_{yv}$ for the gyro.

The paper is organized as follows: Section 2 presents the navigator basic equations, Section 3 focuses on the accelerometers and gyros error models, while the Section 4 talks about the influences of the sensors errors in the solution of navigation; finally, some conclusions are drawn in Section 5.

## 2. NAVIGATOR MECHANIZATION EQUATIONS

The position and the speed of a vehicle can be obtained through the direct integration of the general equation of the inertial navigation, relative to the navigation frame ([1]-[5]). In the case of the present navigation problem one chooses as a navigation frame the horizontal local frame $Ox_ly_lz_l$ (SOL). It can be approximated with an inertial frame considering the length and the specific of the mission. As a consequence, in the solving of the navigation problem there will be implied two reference frames: the horizontal local frame (SOL) and the vehicle frame (SV) (Fig. 1). The system being one of a bidimensional type in vertical plane, will be considered just the $x$ and $z$ axis for the determination of the position and of the speed. This implies the using of a three inertial sensors in the navigator sensing system: two accelerometers installed on the $x_v$ and $z_v$ axis of the SV frame, and a gyro installed on the $y_v$ axis of the same frame. We consider the following notations: $\vec{r}$ - position vector of the vehicle in SOL frame, $\vec{v}$ - the speed of the vehicle relative to the SOL frame, $\vec{v}_{xv}$, $\vec{v}_{zv}$ - the components of the vehicle speed on the SV frame axis, $\vec{v}_{xl}$, $\vec{v}_{zl}$ - the components of the vehicle speed on the SOL frame axis, $\vec{\omega}$ - the angular speed of the vehicle relative to the SOL frame, $\omega_{yv}$ - the component of the $\vec{\omega}$ speed on the $y_v$ axis of the SV frame (gyro reading), $f_{xv}, f_{zv}$ - the components of the specific force $\vec{f}$ in the SV frame (accelerometric readings).

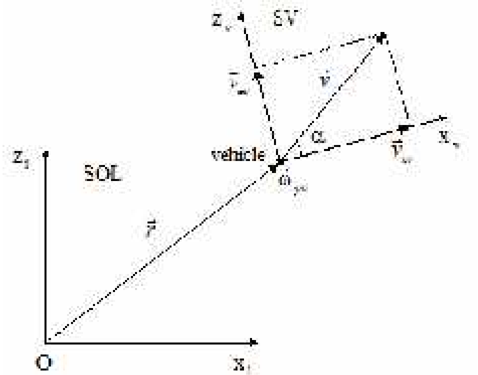


**Figure 1:** The relative position of the SOL and SV reference frames

The output $\vec{f}$ of the accelerometers is influenced by the gravitational field; this is a resultant between the kinematic acceleration of the carrier vehicle $\vec{a}$ and the gravity acceleration $\vec{g}$ :

$$\vec{f} = \vec{a} - \vec{g}. \tag{1}$$

$\vec{f}$ is well known in the literature as specific force vetor [2], [3]. The kinematic acceleration can be expressed with the relation:

$$\vec{a} = (\mathrm{d}\vec{v} / \mathrm{d}t) + \vec{\omega} \times \vec{v}, \tag{2}$$

Therefore, the equation (1) results under the next form:

$$\vec{f} = (\mathrm{d}\vec{v} / \mathrm{d}t) + \vec{\omega} \times \vec{v} - \vec{g}. \tag{3}$$

Projecting the equation (3) on the SV frame axis and retaining the equations on the $x$ and $z$ axis one obtains:

$$f_{xv} = (\mathrm{d}v_{xv} / \mathrm{d}t) + \omega_{yv} v_{zv} - g_{xv}, \;\; f_{zv} = (\mathrm{d}v_{zv} / \mathrm{d}t) - \omega_{yv} v_{xv} - g_{zv}. \tag{4}$$

$g_{xv}$, $g_{zv}$ - are the components of the gravity acceleration in SV frame and are obtained with the next transformation:

$$[g_{xv} \;\; g_{yv} \;\; g_{zv}]^T = R_l^v [g_{xl} \;\; g_{yl} \;\; g_{zl}]^T, \tag{5}$$

where $R_l^v$ is the rotation matrix between the SOL and SV frames, and $g_{xl}$, $g_{yl}$, $g_{zl}$ are the components of the gravity acceleration in SOL frame having the expressions:

$$g_{xl} = 0, \;\; g_{yl} = 0, \;\; g_{zl} = -g_{\mathrm{local}}. \tag{6}$$

$g_{\mathrm{local}}$ is the value of the local gravity acceleration and this may be introduced in the navigation processor.

The determination of the $R_l^v$ attitude matrix supposes the preliminary calculation of the pitch angle $\theta$, which is obtained through the numerical integration of the equation:

$$\dot{\theta} = \omega_{yv}, \tag{7}$$

and using the gyro reading $\omega_{yv}$. On the other way, because the considered navigator is a bidimensional one, the roll and the yaw vehicle angles are chosen to have a particular zero value ($\varphi$=0 and $\psi$=0) in the $R_l^v$ matrix calculation.

Once the attitude problem solved (pitch angle $\theta$ and attitude matrix $R_l^v$ calculated), the vehicle speed components on the vehicle reference frame $x_v$ and $z_v$ axes are deduced as a result of the eq. (4) numerical integration. As a consequence, the components of the vehicle speed components on the SOL reference frame axes results with next coordinate transformation equation:

$$[v_{xl} \;\; v_{yl} \;\; v_{zl}]^T = R_v^l [v_{xv} \;\; v_{yv} \;\; v_{zv}]^T = (R_l^v)^T [v_{xv} \;\; v_{yv} \;\; v_{zv}]^T, \tag{8}$$

and considering $v_{yv}$=0. Finally, the vehicle position relative to the navigation reference frame is deduced by numerical integration of the $\vec{v}$ components in SOL reference frame as follows:

$$x_l = x_{l0} + \int_{t_{n-1}}^{t_n} v_{xl}\, dt, \quad z_l = z_{l0} + \int_{t_{n-1}}^{t_n} v_{zl}\, dt, \tag{9}$$

$x_{l0}$ and $z_{l0}$ are the initial coordinates of the vehicle in the navigation reference frame.

## 3. ACCELEROMETERS AND GYROS ERROR MODELS

The present studies of the strap-down inertial navigation systems related to the involved sensors suppose analyze of the systems in order to estimate the sensors errors after that compensate them in order limit the influence of the inertial sensors error in the solution of navigation. In this way, based on the previous exposed models, many calibration techniques, off-line and on-line, were developed for the inertial sensors, coupled or un-coupled, with the mechanization model of the strap-down inertial navigation systems. Because, the most numerical simulations of strap-down inertial navigation systems performed in the design phase, presented in the literature, suppose the application of clean acceleration and rotation signals to the system input, without errors and noises, the study of the designed navigation systems errors is made without taking into account the inertial sensors errors. To put up a complex study for a navigation system, near by the real conditions, which will include the real errors of the used sensors, equivalent models can be analytically established and software implemented both for accelerometric and gyrometric sensors. What is important in these models is that they will consider the parameters from the real sensors data sheets.

After a study related to the parameters given by producers in a series of accelerometers data sheets, an equation describing a simplified model was chosen for accelerometers [6]

$$a = (a_i + Na_i + B + k_c a_c + \nu)(1 + \Delta K / K). \tag{10}$$

The proposed model covers the main errors of the acceleration sensors: bias, scale factor error, sensitivity axis misalignment, cross axis sensitivity and noise. In equation (10) we have: $a$ - sensors output acceleration (disturbed signal) expressed in m/s$^2$, $a_i$ - applied acceleration (m/s2), $N$ -sensitivity axis misalignment (radians), $B$ - bias (expressed in percents of span), $a_c$ - cross-axis acceleration (m/s$^2$), $k_c$ - cross-axis sensitivity (expressed in percents of $a_c$), $\nu$ - sensor noise (given by its density $\nu_d$ expressed in $\mu\mathrm{g}/\sqrt{\mathrm{Hz}}$ , $K$ - scale factor (expressed in mV/$g$), and $\Delta K$ - scale factor error (percents of $K$).

Implementing software the analytical model (10), the Matlab/Simulink model in Fig. 2.a was obtained. Based on the acceleration sensors parameters variation limits given in the data sheets, the obtained model is generally valid for all acceleration sensors, and, as a consequence, can be used to numerical simulate the influences of theirs errors on the navigation solution (position, speed and attitude channels).

The "Accelerometer model" block has as inputs the acceleration $a_i$ applied along of the sensitivity axis and the cross-axis acceleration $a_c$ (acceleration applied in a perpendicular plane), and as output the disturbed acceleration $a$.

As in the accelerometers case, after a detailed study of some specific sites and data sheets for a number of gyros, miniaturized or not, study related to the parameters given by producers for gyros, the model described by the next equation was chosen for gyro sensors [7]

$$\omega = (\omega_i + S \cdot a_r + B + \nu)(1 + \Delta K / K). \tag{11}$$

The proposed model covers the main errors of the angular speed sensors: bias, scale factor error, sensitivity to the acceleration applied on an arbitrary direction, and noise. In equation (11) we have: $\omega$ - sensors output angular speed (disturbed signal) expressed in $^o$/s, $\omega_i$ - applied angular speed ($^o$/s), $S$ - sensitivity to the acceleration $a_r$ applied on an arbitrary direction (($^o$/s)/$g$), $B$ - bias (expressed in percents of span), $\nu$ - sensor noise (given by its density $\nu_d$ expressed in $(^\circ/\mathrm{s})/\sqrt{\mathrm{Hz}}$ ), $K$ - scale factor (expressed in mV/($^o$/s)), $\Delta K$ - scale factor error (percents of $K$).

Implementing software the analytical model (11), the Matlab/Simulink model in Fig. 2.b was obtained. Based on the gyro sensors parameters variation limits given in the data sheets, the obtained model is generally valid for all gyro sensors, and, as a consequence, can be used to numerical simulate the influences of theirs errors on the navigation solution (position, speed and attitude channels).

The "Gyro model" block has as inputs the angular speed $\omega_i$ applied along of the sensitivity axis and the acceleration $a_r$ applied on an arbitrary direction, and as output the disturbed angular speed $\omega$. It may be noted as a new element that the model allows having a coupling between gyros triad and accelerometers triad through the acceleration $a_r$ applied on an arbitrary direction. The acceleration $a_r$ is the resultant acceleration acting on the inertial navigator, and can be calculated starting from the three IMU accelerometers outputs. In numerical simulations carried out so far to the inertial navigation systems these couplings were practically nonexistent. Usually, the signals from the IMU six inertial sensors for the classical configuration (three gyros and three accelerometers) are completely decoupled.

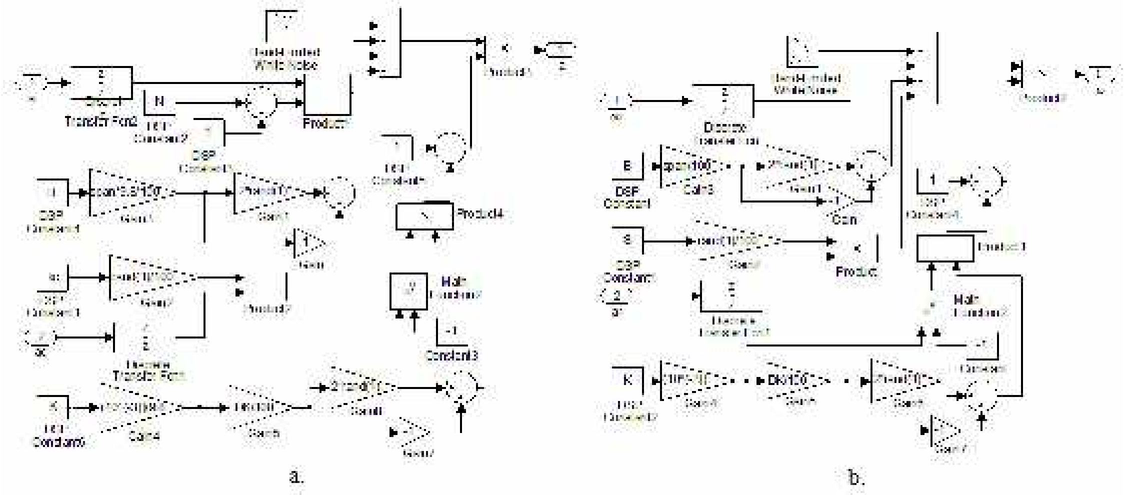

**Figure 2:** Matlab/Simulink inertial sensors models

The inertial sensors models can be successfully used in the numerical simulation of any strap-down inertial navigation system, to create similar conditions with the real ones from the point of view of the distortions that affects the useful acceleration or angular speed signals, when passing through any acceleration or angular speed detection device desired to be implemented in the sensing block of the navigator.

## 4. IMPACT OF THE SENSORS ERRORS ON THE SOLUTION OF NAVIGATION

Starting from the previously presented navigator mechanization equations, the Matlab/Simulink model in Fig. 3 is obtained for our navigator. The block „SV in SOL" models the coordinates change from the SV reference frame in the SOL reference frame, while the block „SOL in SV" models the inverse coordinates change. Grouping the schema in Fig. 3.a the block in Fig. 3.b "Bidimensional vertical navigator" is obtained. Navigator inputs are the gyro reading $\omega_{yv}$ and the accelerometric readings $f_{xv}$, $f_{zv}$; the outputs are the vehicle attitude information, expressed by the pitch angle $\theta$, the vehicle coordinates $x_l$ and $z_l$ in the SOL reference frame, expressing its position relative to the navigation reference frame, and the vehicle speed information relative to the SOL reference frame, given by the components $v_{xl}$ and $v_{zl}$.

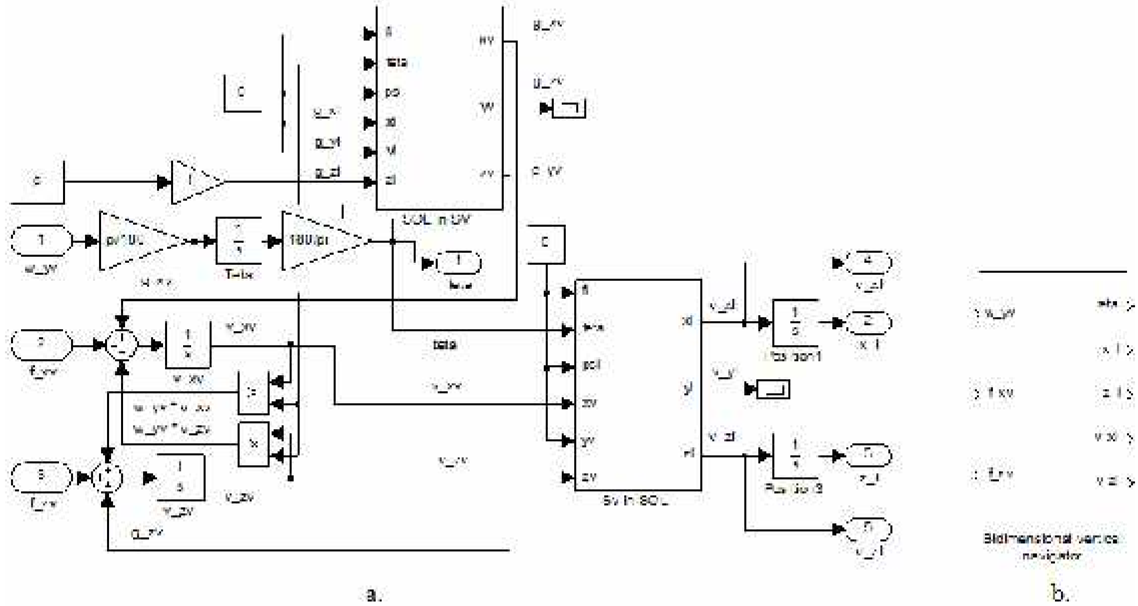


**Figure 3:** Navigator Matlab/Simulink model

For the sensors errors influence study on the navigation algorithm one uses the models in Fig. 2, and the navigator model in Fig. 3. With these models it results the Simulink model in Fig. 4. The "IDEAL" and "REAL" blocks are by the form in Fig. 3.b, its inputs being acceleration and rotation signals non-disturbed by the inertial sensors errors, respectively disturbed by the inertial sensors errors. The blocks "Acc" and "Gyro", from the input of the simulation model, are error models for accelerometers and for gyro, which considers the biases, the scale factor errors, and the noises of the sensors. Their outputs are applied to the "REAL" block. The values of the input constants are considered to be ideal signals, non-disturbed by the inertial sensors errors, these being applied to the "IDEAL" block. The evaluation of the errors induced in the navigator by the inertial sensors is put up by means of the calculation of the differences between the quantities obtained to the outputs of the "IDEAL" and "REAL" blocks for position, speed and attitude. For validation we have chosen two MEMS accelerometers and a MEMS gyro, having the parameters in Table 1 and $\delta f_{xv}$, $\delta f_{zv}$ and $\delta\omega_{yv}$ errors characteristics in Fig. 5.

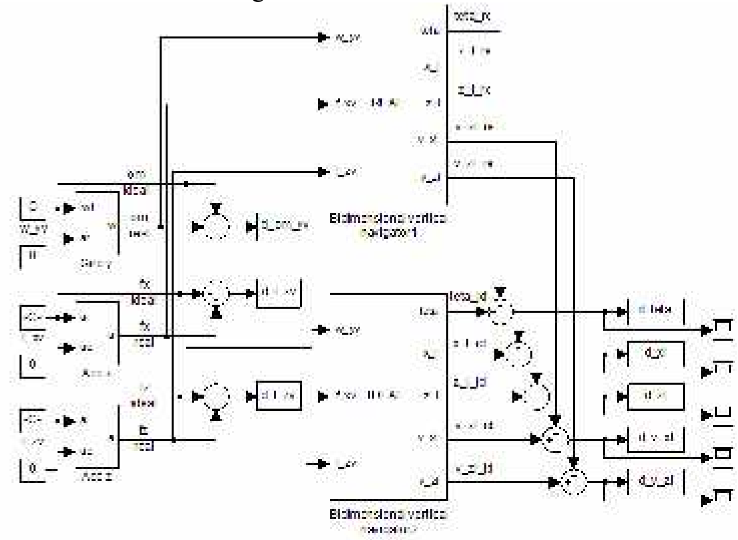

**Figure 4:** Simulation model

**Table 1:** Parameters of the inertial sensors used in numerical simulations

| Sensor | Bias | Scale factor error | Noise density |
|---|---|---|---|
| x Accelerometer | -0.54 $m/s^2$ | -1% | 230 μg/ $Hz^{1/2}$ |
| y Gyro | 0.175 °/s | -0.6 % | $2.5 \cdot 10^{-3}$ (°/s)/ $Hz^{1/2}$ |
| z Accelerometer | 0.45 $m/s^2$ | 1.4% | 255 μg/$Hz^{1/2}$ |

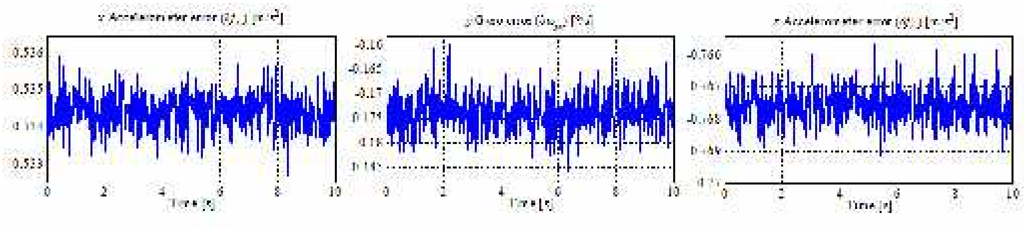

**Figure 5:** Inertial sensors errors

In the first phase numerical simulations are put up for different times (10 s, 1 min) resulting in the absolute maximal values of the attitude, position and speed errors in Table 2 and the graphic characteristics of the errors in Fig. 6.

**Table 2:** Absolute maximal values of the navigator errors for different simulation times

| Simulation time | Attitude angle error [°] | Positioning errors [m] | | Speed errors [m/s] | |
|---|---|---|---|---|---|
| | $\delta\theta$ | $\delta x_l$ | $\delta z_l$ | $\delta v_{xl}$ | $\delta v_{zl}$ |
| 10 s | 1.7398 | 21.3598 | 38.6072 | 3.7361 | 7.7404 |
| 60 s | 10.4370 | $1.9528 \cdot 10^2$ | $1.3873 \cdot 10^3$ | 25.7699 | 45.4607 |

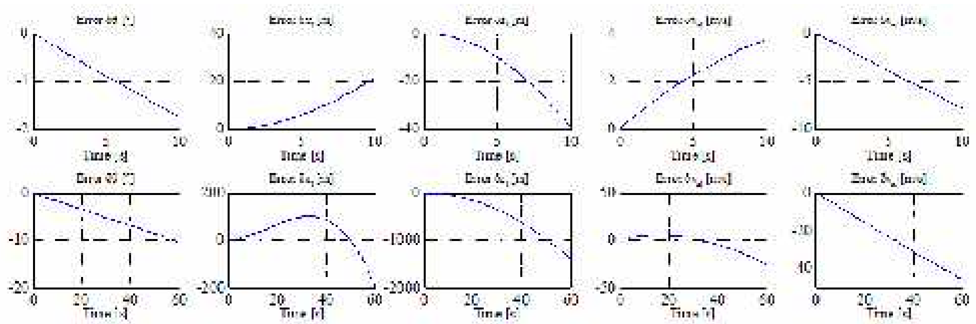

**Figure 6:** Navigator errors for 10 seconds, respectively 1 min, simulation time

The errors values after 10 s simulation time are: the attitude angle error $\delta\theta$ =-1.74°, the errors of vehicle speed in navigation reference frame $\delta v_{xl}$=3.73 m/s, and $\delta v_{zl}$=-7.74 m/s), and the errors of vehicle position in navigation reference frame $\delta x_l$=21.36 m and $\delta z_l$=-38.6 m. One can observe the high level of the SDINS errors' after only 10 s at null inputs, level due to the low quality of the miniaturised inertial sensors used in simulations (see the values in Table 1).
Further were made simulations with 1 min simulation time for different cases in that occur or not the values of certain categories of errors which influence the inertial sensors, with the goal to find out these weights in the final errors of the inertial navigator. Keeping the previous initial conditions and the sensors noise, one considers the following variants during the simulation: bias and scale factor error null for accelerometers and for gyro, the accelerometers bias non-null and the others errors null, the accelerometers scale factor error non-null and the others errors null, the accelerometers with all errors non-null and the gyro with all errors null, the gyro bias non-null and the others errors null, the gyro scale factor error non-null and the others errors null, the gyro with all errors non-null and the accelerometers with all errors null. It results the absolute maximum values for the attitude, position and speed in Table 3.

**Table 3:** Absolute maximal values of the navigator errors for different inertial sensors errors

| Sensors errors | Attitude angle error [°] | Positioning errors [m] | | Speed errors [m/s] | |
|---|---|---|---|---|---|
| | $\delta\theta$ | $\delta x_l$ | $\delta z_l$ | $\delta v_{xl}$ | $\delta v_{zl}$ |
| Noise only | $2.3955\cdot10^{-3}$ | 0.2785 | $7.2268\cdot10^{-3}$ | $8.8888\cdot10^{-3}$ | $4.9188\cdot10^{-4}$ |
| $B_{Acc} \neq 0$ | $2.3955\cdot10^{-3}$ | $9.7131\cdot10^{2}$ | $8.0969\cdot10^{2}$ | 32.3786 | 26.9896 |
| $\Delta K_{Acc} \neq 0$ | $2.3955\cdot10^{-3}$ | 0.2833 | $2.4722\cdot10^{2}$ | $8.7949\cdot10^{-3}$ | 8.2408 |
| $B_{Acc} \neq 0$ & $\Delta K_{Acc} \neq 0$ | $2.3955\cdot10^{-3}$ | $9.6159\cdot10^{2}$ | $1.0682\cdot10^{3}$ | 32.0546 | 35.6083 |
| $B_{Gyro} \neq 0$ | 10.5 | 49.3740 | $1.0769\cdot10^{3}$ | 3.2894 | 53.7854 |
| $\Delta K_{Gyro} \neq 0$ | $2.3812\cdot10^{-3}$ | $7.2226\cdot10^{-3}$ | 0.2768 | $4.9188\cdot10^{-4}$ | $8.8367\cdot10^{-3}$ |
| $B_{Gyro} \neq 0$ & $\Delta K_{Gyro} \neq 0$ | 10.4370 | 48.8028 | $1.0707\cdot10^{3}$ | 3.2507 | 53.4725 |
| All errors non-null | 10.4370 | $1.9528\cdot10^{2}$ | $1.3873\cdot10^{3}$ | 25.7699 | 45.4607 |

## 5. CONCLUSIONS

From the obtained values for the errors of the solution of navigation (attitude, position and speed) in Table 2 can be concluded that after 1 minute the navigator will be in the degraded mode due to the strong influences of the sensors especially in the vertical channel where acts the most important component of the gravity acceleration; the positioning error on the vertical channel (*z* axis) is with approximately one order of magnitude bigger than in the horizontal channel (*x* axis). All errors are divergent, the attitude error and the speed error on vertical channel having an approximate linear carriage, while the other ones comport an approximate parabolic carriage.
According with the numerical results in Table 3, the accelerometers scale factor errors and biases have null weights in the attitude angle error. On the other way, because of the presence of the gravity acceleration, the gyro bias and scale factor error influences on the positioning and speed errors are significant (especially on the vertical channel); the biggest gyro influence in the attitude channel error is due to the bias. For accelerometers, the decisive weights in the position and speed final errors are the biases weights, followed by the scale factor error weights.
Can be observed, also, that the sensors errors combination is useful sometimes; in some cases this combination produces the decrease of the absolute maximal values of the navigator errors in the general case (with all errors) as against the particulars cases.
The present study is very important from the point of view of the fact that it offers a pre-evaluation method for the errors induced by the inertial sensor in such strap-down inertial navigation system just on the base of the sensors data sheets; the inertial sensors choice is a very important step in the inertial navigators design.

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