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IPLEMENTATION OF PARAMETRIC CURVES TO THE DESIGN OF TRUE INVOLUTE GEAR PROFILE

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Abstract: *Gears are very wide used parts of machines and machine equipments by which there is realised transmission and transformation of mechanical energy and movement. Because of there is a very important to focus on increase of accurate of their design in CAD systems. The most difficult part of gear wheels is an involute and fillet profile which requires a good knowledge of mathematical and kinematic equations. Within of the modelling there is the most accurate modelling by parametric curves. Paper deals with design of mathematical model of real involute profile of gear wheels by parametric curves in CAD system ProEngineer.*
Keyword: *true involutes, gear profile, parametric modelling, ProEngineer, CAD systems*

1. INTRODUCTION

Gear wheel are very used parts of machines and machine equipments. They are basic elements by which there is realised transmission and transformation of mechanical energy and movement in machines. Tooth faces have complex curve surfaces because of their modelling in the CAD systems is a very difficult and requires a good knowledge of mathematical and kinematic equations. Despite the fact that gears was developed some century ago, they are continuing in progress of design now. They are designed by CAD systems by 2D or 3D design which is used for other kinematic and dynamic simulations. For finding the most accuracy output results there is needed to define each input elements and to create an accuracy virtual model. In the present CAD systems there are a lot of component libraries which contain a lot of gear components. Question of accuracy is ambiguous.
For modelling of gear wheels there are not possible to use only elementary curve and surface with simply analytical description (line, circle, sphere, conic...) and are not possible to use neither approximation by abscissa or plane surfaces, which are inaccurate. The different between approximation involute curve by design accelerator in AutoDesk Inventor and accurate involute created by parametric curve in ProEngineer is on Figure 1. The inaccurate is showed by red colour.

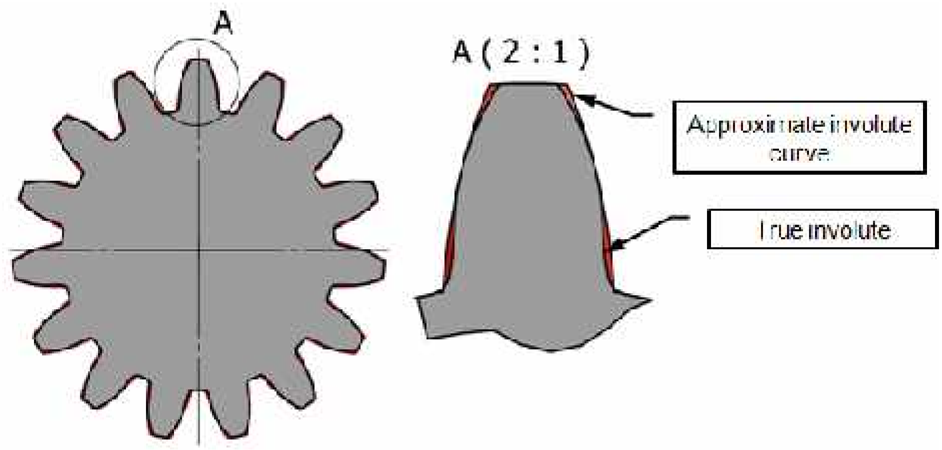

Figure 1: Comparison involute gear profile created AutoDesk Inventor and ProEngineer

In the computer graphics there is very often used parametrical curve and surface, which are able to create general shapes by setting of set direction points which required shape describe. Parametrical curve have the using for font definition, for determining trajectory of moving objects for animation, for templating etc.

2. ANALYTICAL DESCRIPTION OF AN INVOLUTE SPUR GEAR

The pitch circle is the reference circle of tooth element proportions (Figure 2). The circular pitch is the distance between gear neighbouring teeth measured along the pitch circle. The circular pitch is an arc of the pitch circle. The distance between two neighbouring teeth of the rack-cutter (Figure 4) is a segment of a straight line is equal to circular pitch. Diametral pitch is represented as

$$p_c = \pi.m \quad \textbf{(1)}$$

and is defined as the number of teeth of the gear per inch of its diameter. The module *m* is represented as

$$m = \frac{d}{z} \Rightarrow d = m.z \quad \textbf{(2)}$$

where *d* is diameter of pitch circle

z is number of teeth.

Relations for cad equations are base on the following Figure 2, where input parameters are m, z, α (pressure angle α=20°).

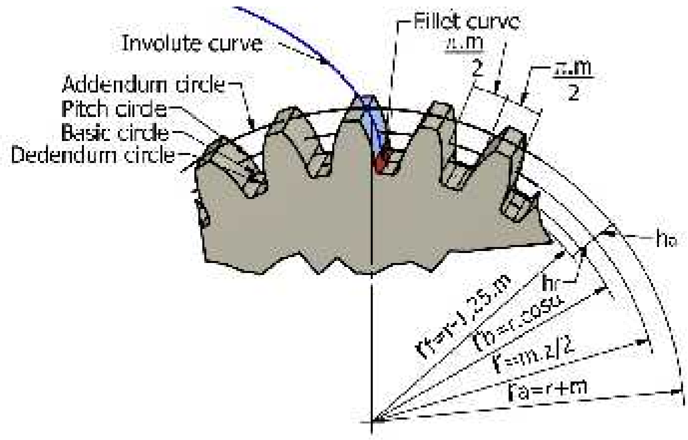

Figure 2: Description of involutes spur gear

For the design spur gear wheel in ProEngineer we need parametric equations of input construction circle. The parametric equations of input circle curves in text editor are in Table 1: There is a system variable "t" that varies from 0 to 1 over the length of the curve.

Table 1: Parametric equations of input circles in the text editor for ProEngineer

Pitch circle	Basic circle	Addendum circle	Dedendum circle
x = r * cos (t * 360) y = r * sin (t * 360) z = 0	x = rb * cos (t * 360) y = rb * sin (t * 360) z = 0	x = ra * cos (t * 360) y = ra * sin (t * 360) z = 0	x = rf * cos (t * 360) y = rf * sin (t * 360) z = 0

If we are generated four input construction circle curves we need to create two basic profile curves: Involutes curve and fillet curve.

2.1.Generation fillet curve

The fillet of the gear is a curve that interconnects the working part of the tooth profile of the gear with the dedendum circle. The Figure 3 (a) shows the tooth of a rack-cutter for generation of fillet curve of gear. Let us now consider the case where the gear fillet is generated by point M - the edge of the rack-cutter (Figure 3 (b)). The equation of the gear fillet may be derived as the trajectory of edge M that is traced out in coordinate system S_2. This trajectory may be represented by the matrix equation

$$r_2 = M_{21}.r_1^{(M)} = \begin{bmatrix} \cos\varphi & \sin\varphi & 0 & r\left(\frac{2\pi\varphi}{360}\cos\varphi - \sin\varphi\right) \\ \sin\varphi & -\cos\varphi & 0 & r\left(\frac{2\pi\varphi}{360}\sin\varphi + \cos\varphi\right) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.r_1^{(M)}. \quad \textbf{(3)}$$

Here, $r_1^{(M)}$ is the column matrix that represents in S_1 the coordinates of edge *M*.

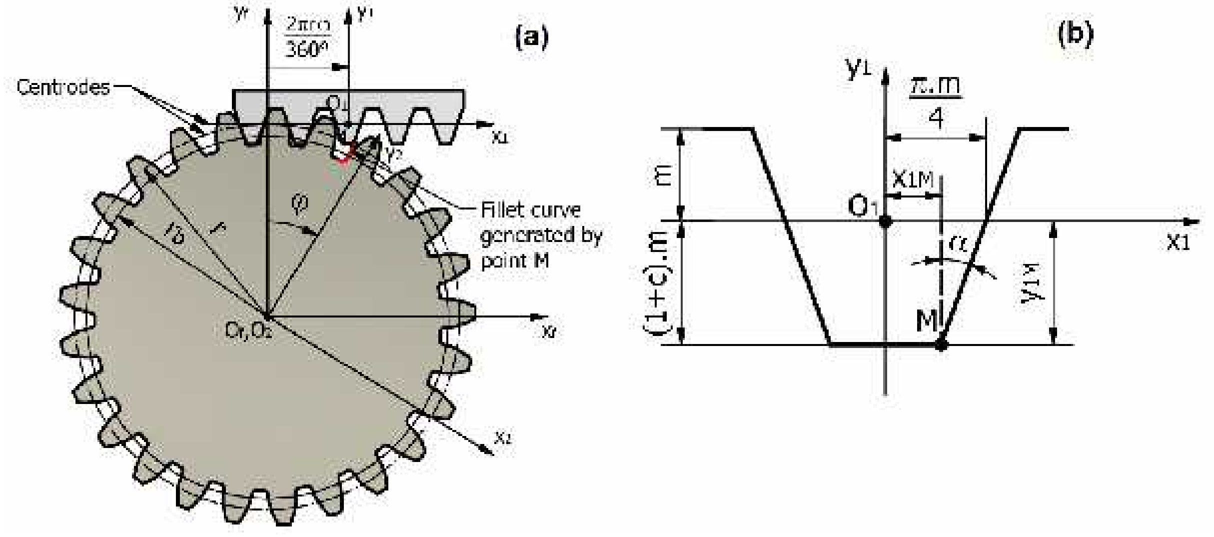

Figure 3: Generation of gear fillet by edge point M of rack-cutter

The final parametric equation defined in Cartesian coordinate system are:

$$x = x_{1M}\, cos(\varphi) + y_{1M}\, sin(\varphi) + r\left(\frac{2\pi\varphi}{360} cos(\varphi) - sin(\varphi)\right)$$
$$y = x_{1M}\, sin(\varphi) - y_{1M}\, cos(\varphi) + r\left(\frac{2\pi\varphi}{360} cos(\varphi) + sin(\varphi)\right) \quad \textbf{(4)}$$
$$z = 0$$

where x_{1M}, y_{1M} are:

$$x_{1M} = \frac{\pi.m}{4} - m(1+c)tg(\alpha)$$
$$y_{1M} = m(1+c) \quad \textbf{(5)}$$

Resulted parametric equations of fillet curve described in text editor are in Table 2:

Table 2: Parametric equations of fillet curve in text editor ProEngineer

Fillet curve
x1=(PI*m/4)-c*m*tan(alfa) y1=m+c*m ang=t*(90) s=(2*PI*r*ang/360) x=(x1*cos(ang))+(y1*sin(ang))+(s*cos(ang))-(r*sin(ang)) y=(x1*sin(ang))-(y1*cos(ang))+(s*sin(ang))+(r*cos(ang)) z=0

End of fillet curve is determine by place where fillet curve goes through basic curve. We may create point there. For modelling involute we need to determine new coordinate system. This system will be go through normal planes. One plane go through create point and axis of gear wheel, second plane is normal to first. Coordinate system will be created by these planes and will be oriented by x- axis toward to created point and z - axis will be oriented toward to axis of wheel. After these steps we may create involute which will be start on end of fillet curve.

2.2.Generation involute curve

Consider the particular case when the evolute *E* is a circle. The involute *I* for such a case is the tooth profile for a spur gear. The evolute, the circle of radius *rb* (Figure 2), is called the *base circle*. Two branches of an involute curve are shown in Figure 4. They are generated by point *P* of the straight line that rolls over the base circle clockwise and counterclockwise, respectively. Each branch represents its respective side of the tooth.

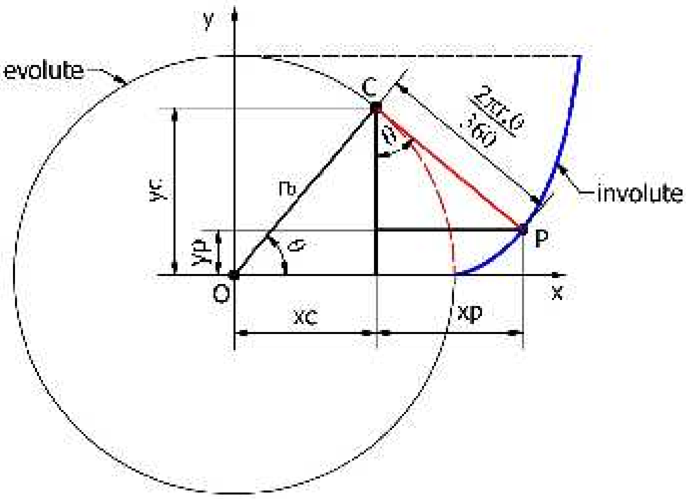

Figure 4: Generation of involute curve

Development of the mathematical equations for the involute curve uses simple trigonometry. The equations can be obtained referring to Figure 4. The involute starts from the base circle and its parametric equation are in Table 3.

Table 3: Parametric equations of involute curve in text editor ProEngineer

Involute curve
rb= /* real value ang=t*90 s=(PI*r*t)/2 xc=rb*cos(ang) yc=rb*sin(ang) x=xc+(s*sin(ang)) y=yc-(s*cos(ang)) z=0

In the case we need to create new coordination system because involute curve have to be connected with fillet curve. We have to create point which goes through fillet curve and basic circle. In the next step we need create two planes. The first goes through the created point and centre axis of gear wheel, second plane is normal to first and goes through centre axis. Coordinate will be oriented: x-axis is first plane, y-axis is second plane and z-axis is centre axis.

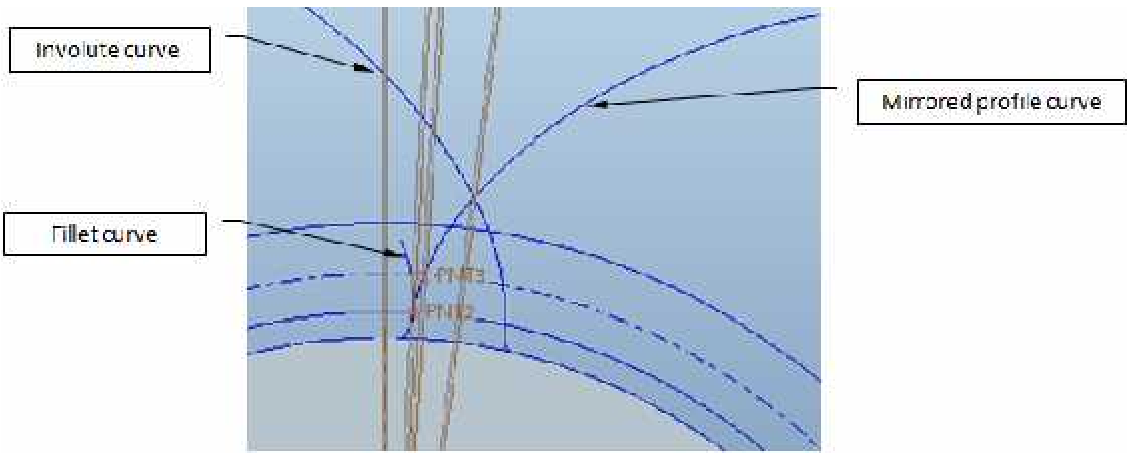

Figure 5: Gear profile with involute and fillet curve

For creating gear teeth we need to mirror filler and involute curve. At first there we need to create new point which goes through pitch circle and involute curve and create new plane which goes through this point and centre axis of wheel. Last created plane which will be used for mirroring will be rotated from previous plane about angle:

$$angle = \frac{360}{4z} \quad \textbf{(6)}$$

where z is number of teeth.

Last step will be a creating of new sketch where we use displaying of used geometry (fillet curve, involute curve, mirrored curve, dedendum and addendum circle). For teeth creating we have to remove unnecessary geometry by cut tool. We finish sketch and use extrude for creating gear teeth. More teeth we create by pattern tool.

3. CONCLUSION

3D modelling of gear wheel requires a good knowledge of mathematical and kinematical equations which describes the movement of tool which generates real gear profile. The involute part of gear tooth profile is generated by straight line part of basic rack. The fillet curve is generated by horn of rack. Every the movement is possible to describe by parametrical equations which describe parametrical curve. These curves are the basic for displaying of final shape of gear tooth model. The modelling by parametrical curves requires a good knowledge of work with sketch tools (sketch, mirror, display of uses geometry, cut etc.) and basic work with modelling in ProEngineer.

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