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ANALYTICAL DESCRIPTION OF CYCLIC CURVES FOR PARAMETRICAL MODELLING IN THE CAD SYSTEMS

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Abstract: *Nowadays, the cyclic curves have a wide application in technical branches mainly in mechanics. They are very important for component design and testing simulation for different conditions. Cyclic curves profiles is possible to find in a lot of machine equipments with rotary parts. It is begin in different gears (clock machines), rotary parts of engine, piston pump, accelerometers etc. We can find in the Wankel´s engine but also in the water or wind turbines and other equipments. Cycloid profiles are able to keep a high loads because of they are used in the building industry (bridges, tunnels). The cyclic curves are created trajectory of circle point which moves without slip on a given curve. The paper deals with research of circle moving based on only geometrical basic independently of the time when was realised. For the moving is not important moving speed. The result is a mathematical description of cyclic curves (cycloids, epicycloids, hypocycloids and involutes) which is applied to design module of modern CAD systems which use parametrical modelling by equations.*
Keywords: *cyclic curves, parametric modelling, equations of cyclic curves, CAD systems*

1. INTRODUCTION

The curve is one of the most important the mathematical terms. It is one of the mathematical term which is the most frequently used in the everyday communication. Even though the term is very important and its characteristic is put more precisely by influence of develop various scientific branches. The physical science determine curve like trajectory of moving point. From this characteristic there is possible to say that curve is linear projection of abscissa.
Every CAD (computer aided design) system uses for creating of geometrical models the basic elements like point, abscissa, circle, curve, surface, solid etc. Each software or application can use different series of these basic elements. They have define mathematical model. By the model the CAD system create geometrical form of objects based on input geometrical properties (for example direction point of curve or base, height and placement of prism) by users. There are other possibilities to assign properties like colour, line width, line type or properties for visualization like material, light behaviour ant etc. The CAD systems are able to realize immediate modelling of some geometrical object but also even though the knowledge of geometrical and analytical properties simplify and particularize a work for modelling of complicated assemblies of objects. For object which cannot be directly modelled in the CAD system there is needed to use the geometrical constructions.

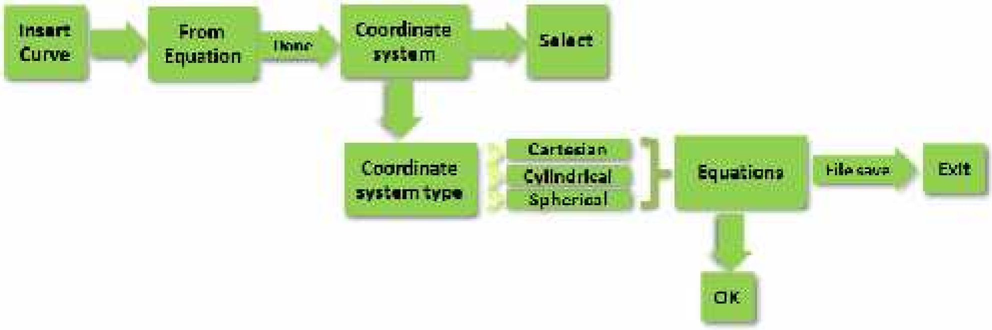


Figure 1: Process of creating parametric curves in ProEngineer

Individual CAD systems uses for modelling different way of curves generating. Most frequent way there is creating spline curve, which goes through direct points. This way is less accurate because it depends of size of mesh points. The bigger mesh point is the less accurate is curve creating. Parametric modelling belongs to the most accurate curve modelling. It defines curves by parameters of coordination systems (Cartesian, cylindrical, spherical). Curves which are generated by the way they model accurate shapes of components. ProEngineer is a CAD system which uses parametric curve modelling. Process of creating parametric curves is illustrated on Fig. 1

2. ANALYTICAL DESCRIPTION OF CYCLIC CURVES

For the parametric curve modelling is the most important the determining of curve equation. This equations is possible to create by analyzing of point motion along cyclic curves. There are classified cycloidal, epicycloidal, hypocycloidal and involutes movements between cyclic movements. In the next part of paper there will be analysed: Cycloidal, epicycloidal and hypocycloidal curves.

2.1. Cycloid

Cycloidal moving is created by a point on the rim of a circular wheel as the wheel rolls along a straight line. Trajectories of cycloidal movement are called the cycloids. The cycloid is a plane curve. In the case if distance of point from centre of the circular rolling wheel is bigger as its radius then the point creates a curtate cycloid. In the case if distance of point from centre of the circular rolling wheel is shorter as its radius then the point creates a prolate cycloid.

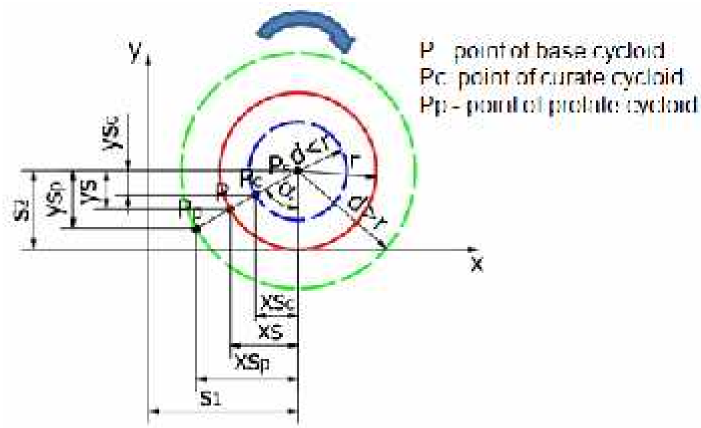


Figure 2: Generation of cycloid curve

From Figure 2 it is observable that centres point of circle P_S moves the length s for angular rotation α. The length s is the same as circumference of circle with angle α. The length s is determined:

$$s = \frac{2\pi r}{360}\alpha \quad \textbf{(1)}$$

where r - is radius generating circle

α - angular rotation of generating circle.

Coordinates of moving point P_0 $[x_s, y_s]$ to centres of circle P_s it may determine by simply goniometric functions:

$$x_s = r.\sin\alpha \quad \textbf{(2)}$$

$$y_s = r.\cos\alpha \quad \textbf{(3)}$$

The final coordinates of point P_0 based on Figure 2 it may describe:

$$x = s - x_s \quad \textbf{(4)}$$

$$y = r - y_s \quad \textbf{(5)}$$

After inputting of equations (1), (2), (3) to equations (4) and (5) it find resulted parametric equation of basic cycloid:

$$x = \frac{2\pi r}{360}\alpha - r\sin\alpha \quad \textbf{(6)}$$

$$y = r - r.\cos\alpha \quad \textbf{(7)}$$

For curate and prolate cycloid it have to change the calculation of coordinates of point P_0 to centres of circle. In the case the coordination will be:

$$x_s = d.\sin\alpha \quad \textbf{(8)}$$

$$y_s = d.\cos\alpha \quad \textbf{(9)}$$

Resulted equations of cycloids are:

$$x = \frac{2\pi r}{360}\alpha - d\sin\alpha \quad \textbf{(10)}$$

$$y = r - d.\cos\alpha \quad \textbf{(11)}$$

For the parametrical description of curve in a text editor it is needed to define variable of angle α which determines interval from zero to maximal angular rotation of moving circle id est.: $ang=t*\alpha_{max}$. There is example of description in the text editor for basic cycloid with radius r =10 and angular rotation angle α_{max} = 720°, for curate cycloid $d = 10$ and prolate cycloid d = 30. Resulted cycloidal curves are illustrated on Figure 3.

Table 1: Parametric description of cycloid equations in the text editor Pro/Engineer

Basic cycloid (red curve)	Curate cycloid (blue curve)	Prolate cycloid (green curve)
r=20	r=20	r=20
ang=t*720	d=10	d=30
s=(2*PI*r/360)*ang	ang=t*720	ang=t*720
xs=r*sin(ang)	s=(2*PI*r/360)*ang	s=(2*PI*r/360)*ang
ys=r*cos(ang)	xs=d*sin(ang)	xs=d*sin(ang)
x=s-xs	x=s-xs	x=s-xs
y=r-ys	y=r-d*cos(ang)	y=r-d*cos(ang)

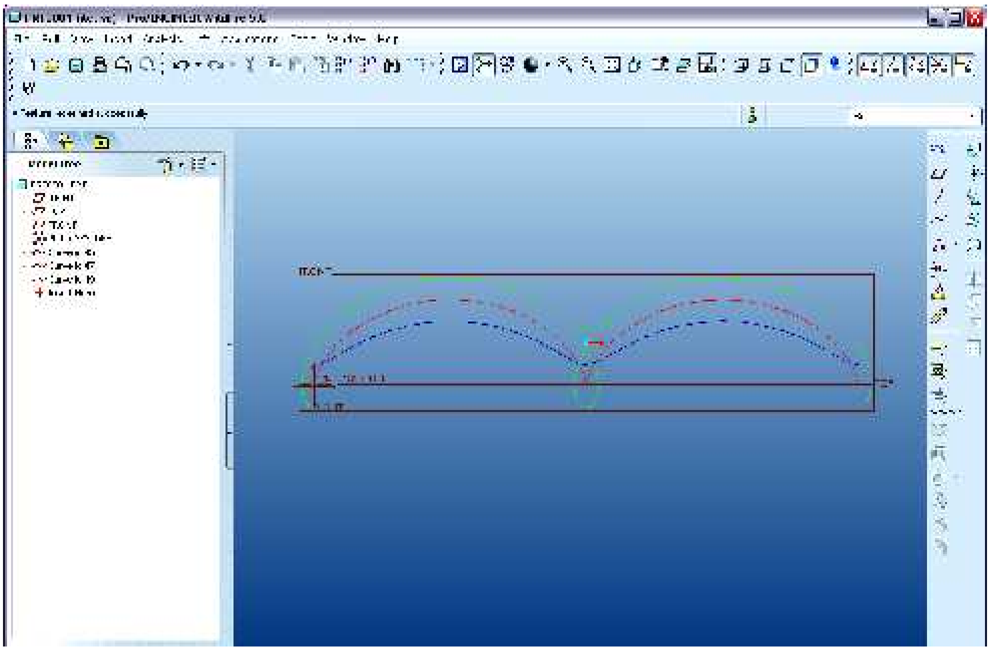


Figure 3: Resulted display of cycloid curves in ProEngineer

2.2. Epicycloid and Hypocycloid

In geometry, a hypocycloid is a special plane curve generated by the trace of a fixed point on a small circle that rolls inside a larger circle and epicycloid is a special curve generated by the trace of fixed point on a small circle that rolls outside a larger circle. It is comparable to the cycloid but instead of the circle rolling along a line, it rolls along a circle.
There will be researched moving center point P_s of small circle with radius r which rolls round a center of coordinates O_1 and moving point which are on circumference of small circle (P, P_c, P_p) and generate cyclic curve (Figure 4). In the first step there is needed to determine a ration between radiuses of small r and larger a circles:

$$\frac{r}{a} = \frac{\alpha}{\beta} \Rightarrow \alpha, \beta \tag{12}$$

where α,β are angles of angular rotation.
In the case we will investigate dependency angle β on angle α. It meant, if angle α rotates about 1° then angle β will rotate $\frac{a}{r}.1°$.
In the next step we need to investigate coordinate (s_1, s_2) of center point of small circle P_s. From Figure 4 is evident, the different between epicycloid and hypocycloid.
For epicycloid, there will be coordinate s_1, s_2:

$$\begin{aligned} s_1 &= (a+r)\cos\beta \\ s_2 &= (a+r)\sin\beta \end{aligned} \tag{13}$$

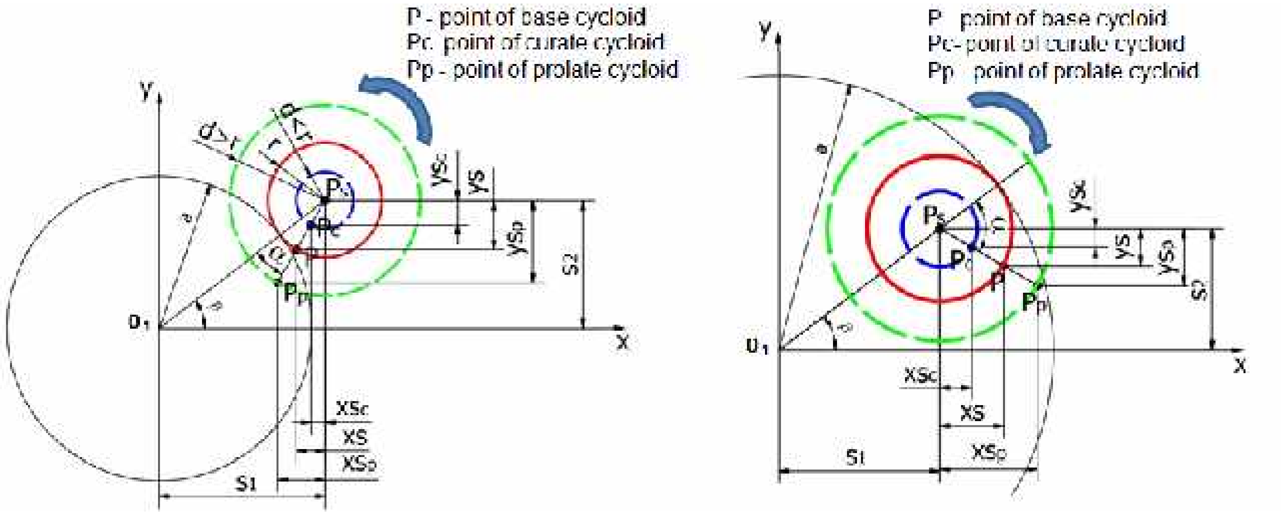
And for hypocycloid there will be coordinate s_1, s_2:

$$s_1 = (a-r)\cos\beta$$
$$s_2 = (a-r)\sin\beta \tag{14}$$

Coordinates of points creating of cyclic curve (P, P_c, P_p) are depended on angle α. Then we can determine length (xs, ys) of the moving in the coordinates:

$$xs = d.\cos\alpha$$
$$ys = d.\sin\alpha \tag{15}$$

Length of point P with index p is determine for prolate curve where $d > r$, for point P with index c is determine for curate curve where $d < r$ and point P without index is determine for basic curve where $r = d$.



(a) epicycloid **(b) hypocycloid**

Figure 4: Generation of epicycloid and hypocycloid curves

Final coordinates x, y of cyclic curve we can determine on based of Figure 4.
Where for epicycloid is valid:

$$x = s_1 - xs$$
$$y = s_2 - ys \tag{16}$$

and for hypocycloid is valid:

$$x = s_1 + xs$$
$$y = s_2 - ys \tag{17}$$

For the description of equation of cyclic curves we have to use system variable "t" It means that the variable "ang" will vary from zero to 360°. The variable "ang" represents angle β. Input parameters are: r, a, d. In the Table 2 are describe equations in text editor for three basic type of hypocycloid and they are illustrated on Figure 5. In the Table 3 are describe equation for three basic type of epicycloid and they are illustrated on Figure 6. These cases are for universal using for modeling cyclic curves in system ProEngineer and it possible to vary for other required cases.

Table 2: Parametric description of hypocycloid equations in the text editor Pro/Engineer

Basic hypocycloid (red curve)	**Curate hypocycloid (blue curve)**	**Prolate hypocycloid (green curve)**
r=20	r=40	r=20
a=200	d=20	d=40
ang=t*360*a/r	a=200	a=200
s1=(a-r)*cos(ang*r/a)	ang=t*360*a/r	ang=t*360*a/r
s2=(a-r)*sin(ang*r/a)	s1=(a-r)*cos(ang*r/a)	s1=(a-r)*cos(ang*r/a)
xs=r*cos(ang)	s2=(a-r)*sin(ang*r/a)	s2=(a-r)*sin(ang*r/a)
ys=r*sin(ang)	xs=d*cos(ang)	xs=d*cos(ang)
x=s1+xs	ys=d*sin(ang)	ys=d*sin(ang)
y=s2-ys	x=s1+xs	x=s1+xs
	y=s2-ys	y=s2-ys

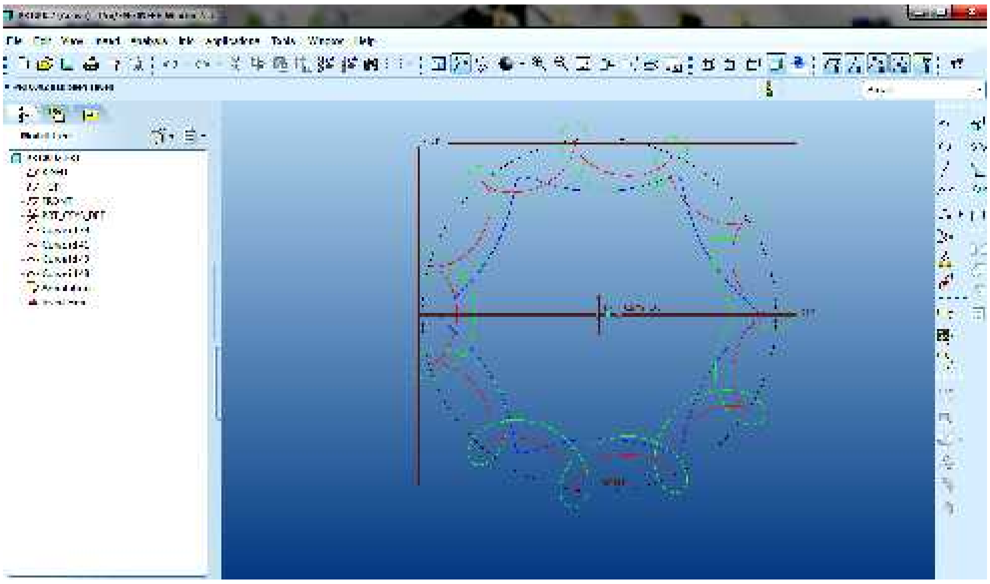


Figure 5: Resulted display of hypocycloid curves in ProEngineer

Table 3: Parametric description of epicycloid equations in the text editor Pro/Engineer

Basic epicycloid (red curve)	Curate epicycloid (blue curve)	Prolate epicycloid (green curve)
r=20	r=40	r=20
a=160	d=20	d=40
ang=t*360*a/r	a=160	a=160
s1=(a+r)*cos(ang*r/a)	ang=t*360*a/r	ang=t*360*a/r
s2=(a+r)*sin(ang*r/a)	s1=(a+r)*cos(ang*r/a)	s1=(a+r)*cos(ang*r/a)
xs=r*cos(ang)	s2=(a+r)*sin(ang*r/a)	s2=(a+r)*sin(ang*r/a)
ys=r*sin(ang)	xs=d*cos(ang)	xs=d*cos(ang)
x=s1-xs	ys=d*sin(ang)	ys=d*sin(ang)
y=s2-ys	x=s1-xs	x=s1-xs
	y=s2-ys	y=s2-ys

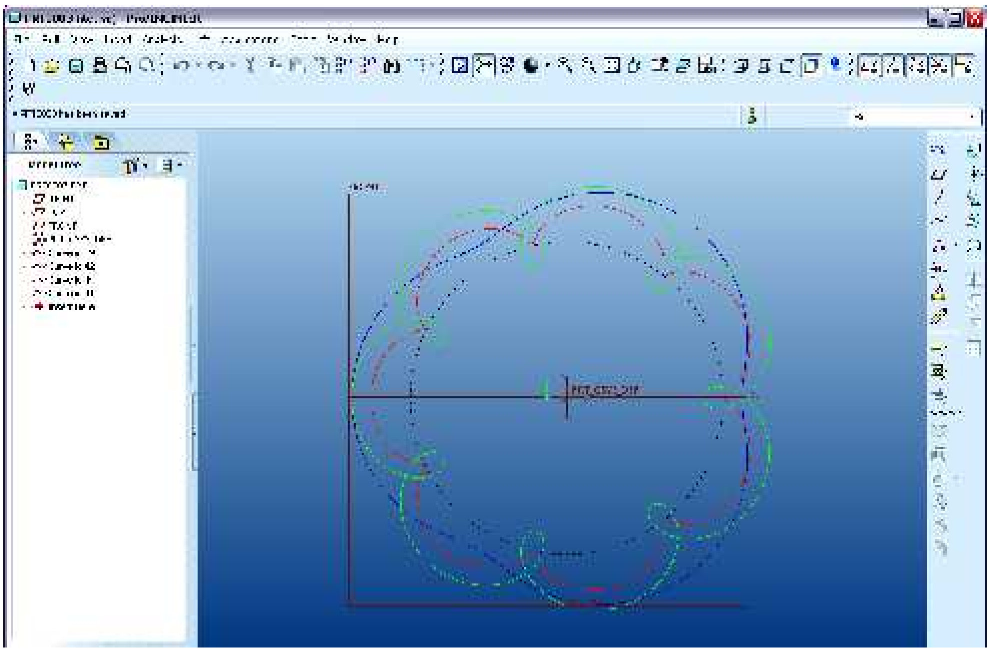


Figure 6: Resulted display of epicycloid curves in ProEngineer

3. CONCLUSION

The modeling of cyclic curves in the CAD systems is possible to use in different areas of a constructional design part or mechanism of mechanism. Each shape can be represented by curve in CAD system. The finding of correct form of equation which represents the curve requires knowledge of mathematical terms and their implementation to the investigation of moving. The paper contains general formulas for equation of cyclic curves. These equations is possible to modify according to requirements of constructional design. There is possible to modify number of cycles by variable *"ang"* or to modify direction of movement by trigonometric function in the parameters *"xs, ys, s1, s2"* or to change basic proportions by parameters *"r, d, a"*.

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