UNIVERSITATEA Transilvania BRAŞOV

2nd International Conference
"Advanced Composite Materials Engineering"
COMAT 2008
9 – 11 October 2008, Brasov, Romania

# STIFFNESS EVALUATION OF SOME ADVANCED COMPOSITE LAMINATES UNDER OFF-AXIS LOADING SYSTEM

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***Abstract:*** *The paper presents the stiffness evaluation of some advanced composite laminates based on epoxy resin reinforced with HM-, HS carbon- and Kevlar49 fibers, with plies sequence [0/90/0/90], [0/45/-45/90] and [45/-45/45/-45], laminates subjected under off-axis loading system.*
***Keywords:*** *stiffness, HM carbon, HS carbon, kevlar49, laminates*

## 1. INTRODUCTION

It is well known that composite laminates with aligned reinforcement are very stiff along the fibers direction, but also very weak in the transverse direction. The solution to obtain equal stiffness of laminates subjected in all directions within a plane is by stacking and bonding together plies with different fibers orientations [1-3].
A composite laminate (fig. 1) formed by a number of unidirectional reinforced laminas subjected regarding to the loading scheme presented in fig. 2 is considered. The elasticity law for a unidirectional lamina $K$ is:

$$\begin{bmatrix} \sigma_{xx\,K} \\ \sigma_{yy\,K} \\ \tau_{xy\,K} \end{bmatrix} = \begin{bmatrix} r_{11\,K} & r_{12\,K} & r_{13\,K} \\ r_{12\,K} & r_{22\,K} & r_{23\,K} \\ r_{13\,K} & r_{23\,K} & r_{33\,K} \end{bmatrix} \cdot \begin{bmatrix} \varepsilon_{xx\,K} \\ \varepsilon_{yy\,K} \\ \gamma_{xy\,K} \end{bmatrix}, \tag{1}$$

where $r_{ijK}$ represent the transformed stiffness, $\sigma_{xxK}$ , $\sigma_{yyK}$ are the mean stresses of $K$ lamina on x- respective y-axis and $\tau_{xyK}$ represent the mean shear stress of K lamina against the x-y coordinate system.
The balance equations of the laminate structure are:

$$n_{xx} = \underline{\sigma}_{xx} \cdot t = \sum_{K=1}^{N} \left( \sigma_{xxK} \cdot t_K \right) = \sum_{K=1}^{N} n_{xxK}, \tag{2}$$

$$n_{yy} = \underline{\sigma}_{yy} \cdot t = \sum_{K=1}^{N} \left( \sigma_{yyK} \cdot t_K \right) = \sum_{K=1}^{N} n_{yyK}, \tag{3}$$

$$n_{xy} = \underline{\tau}_{xy} \cdot t = \sum_{K=1}^{N} \left( \tau_{xyK} \cdot t_K \right) = \sum_{K=1}^{N} n_{xyK}, \tag{4}$$

where $n_{xx}$ , $n_{yy}$ are the normal forces on the unit length of the laminate on x- respective y-axis and $n_{xy}$ represents the shear force, in plane, on the unit length of the laminate against the x-y coordinate system. $\underline{\sigma}_{xx}$ , $\underline{\sigma}_{yy}$ are the normal stresses on x- respective y-axis of the laminate, $\underline{\tau}_{xy}$ represent the shear stress of the laminate against the x-y coordinate system. $t_K$ , $t$ represent the thickness of the $K$ lamina respective the laminate thickness, $n_{xxK}$ , $n_{yyK}$ are forces on the unit length of $K$ lamina on x- respective y-axis directions and $n_{xyK}$ is the shear force in plane, on the unit length of $K$ lamina against the x-y coordinate system. Beside the balance equations it must be determined the geometric conditions also to compute the stresses. For composite laminates these conditions imply that all laminas are bonded together and withstand, in a specific point, the same strains $\varepsilon_{xx}$, $\varepsilon_{yy}$, $\gamma_{xy}$ as well as for the entire laminate:

$$\varepsilon_{xxK} = \varepsilon_{xx}\,,$$
$$\varepsilon_{yyK} = \varepsilon_{yy}\,, \tag{5}$$
$$\gamma_{xyK} = \gamma_{xy}\,,$$

for all $K$ layers.

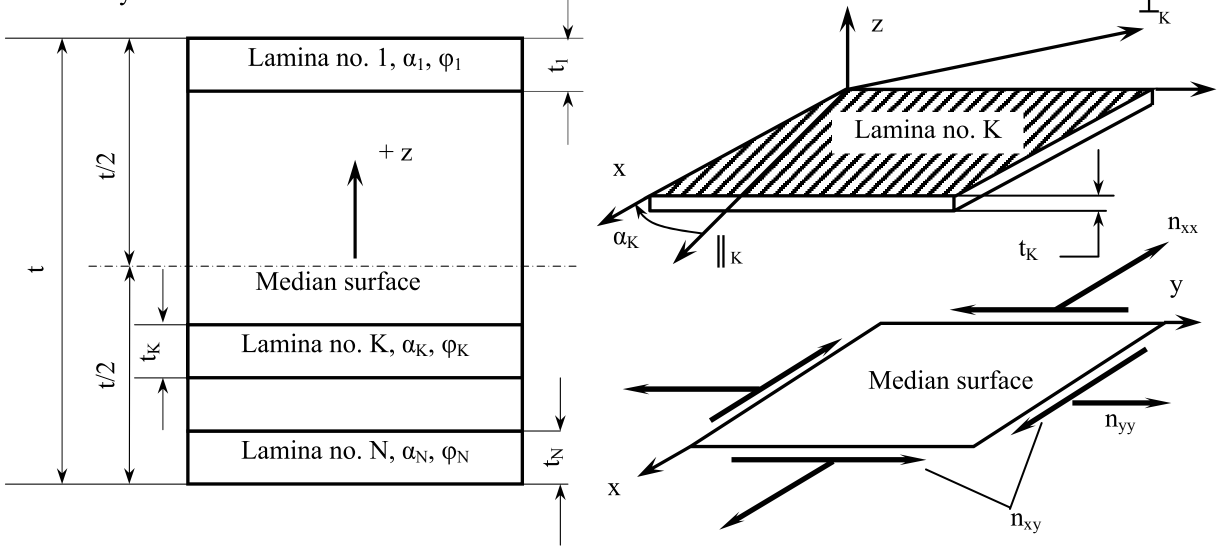


**Figure 1:** Constructive scheme of a composite laminate

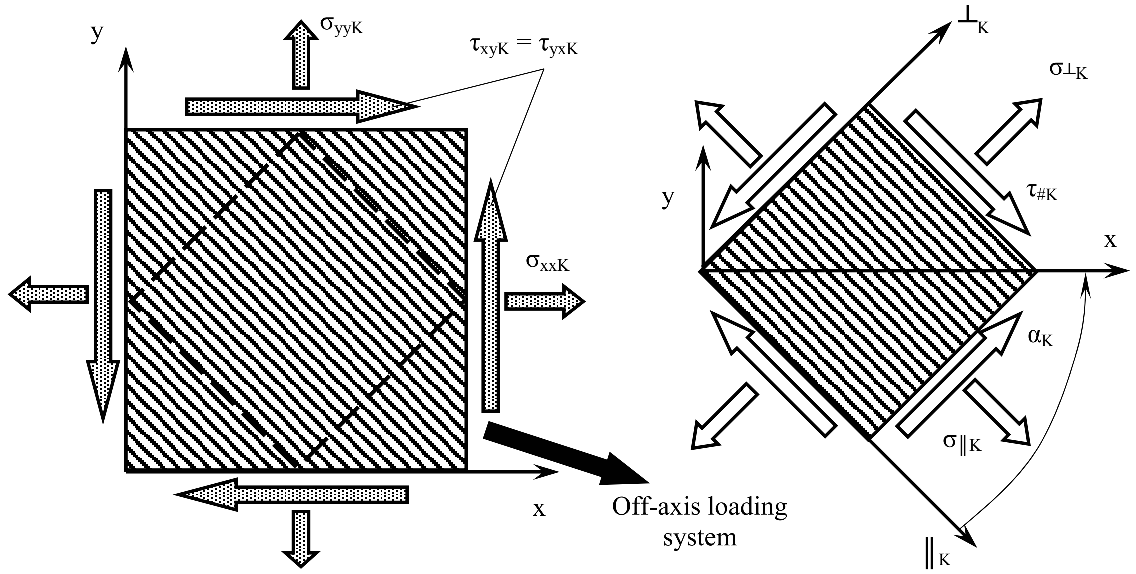


**Figure 2:** Loading scheme of a composite laminate

According to equations (1)-(5), the elasticity law for entire laminate can be computed [4]:

$$\begin{bmatrix} \underline{\sigma}_{xx} \\ \underline{\sigma}_{yy} \\ \underline{\tau}_{xy} \end{bmatrix} = \begin{bmatrix} \sum\limits_{K=1}^{N}\left(r_{11K}\cdot\frac{t_K}{t}\right) & \sum\limits_{K=1}^{N}\left(r_{12K}\cdot\frac{t_K}{t}\right) & \sum\limits_{K=1}^{N}\left(r_{13K}\cdot\frac{t_K}{t}\right) \\ \sum\limits_{K=1}^{N}\left(r_{12K}\cdot\frac{t_K}{t}\right) & \sum\limits_{K=1}^{N}\left(r_{22K}\cdot\frac{t_K}{t}\right) & \sum\limits_{K=1}^{N}\left(r_{23K}\cdot\frac{t_K}{t}\right) \\ \sum\limits_{K=1}^{N}\left(r_{13K}\cdot\frac{t_K}{t}\right) & \sum\limits_{K=1}^{N}\left(r_{23K}\cdot\frac{t_K}{t}\right) & \sum\limits_{K=1}^{N}\left(r_{33K}\cdot\frac{t_K}{t}\right) \end{bmatrix} \cdot \begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{bmatrix}, \tag{6}$$

where the laminate stiffness $\underline{r}_{ij}$ are:

$$\underline{r}_{ij} = \sum_{K=1}^{N}\left(r_{ijK}\cdot\frac{t_K}{t}\right). \tag{7}$$

So, the laminate elasticity law becomes:

$$\begin{bmatrix} \underline{\sigma}_{xx} \\ \underline{\sigma}_{yy} \\ \underline{\tau}_{xy} \end{bmatrix} = \begin{bmatrix} \underline{r}_{11} & \underline{r}_{12} & \underline{r}_{13} \\ \underline{r}_{12} & \underline{r}_{22} & \underline{r}_{23} \\ \underline{r}_{13} & \underline{r}_{23} & \underline{r}_{33} \end{bmatrix} \cdot \begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{bmatrix}, \qquad \textbf{(8)}$$

Computing the laminate strains as a function of stresses, the expressions (8) are:

$$\begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{bmatrix} = \begin{bmatrix} \underline{c}_{11} & \underline{c}_{12} & \underline{c}_{13} \\ \underline{c}_{12} & \underline{c}_{22} & \underline{c}_{23} \\ \underline{c}_{13} & \underline{c}_{23} & \underline{c}_{33} \end{bmatrix} \cdot \begin{bmatrix} \underline{\sigma}_{xx} \\ \underline{\sigma}_{yy} \\ \underline{\tau}_{xy} \end{bmatrix}, \qquad \textbf{(9)}$$

where $\underline{c}_{ij}$ represents the laminate compliance tensor. This tensor can be computed as a function of elastic constants. Thus [5-6]:

$$E_x = \frac{1}{\underline{c}_{11}}; \quad G_{xy} = \frac{1}{\underline{c}_{33}}; \quad \upsilon_{xy} = -E_x \cdot \underline{c}_{12}. \qquad \textbf{(10)}$$

It is obvious that the laminate will exhibit different elastic constants if the loading system is applied at a randomly angle, $\Phi$, to the x-y coordinate system.

## 2. EXAMPLES OF ADVANCED COMPOSITE LAMINATES

The architectures of some advanced composite laminates based on epoxy resin reinforced with HM-, HS carbon- and Kevlar49 fibers are presented in fig. 3, laminates taken into account at stiffness evaluation.

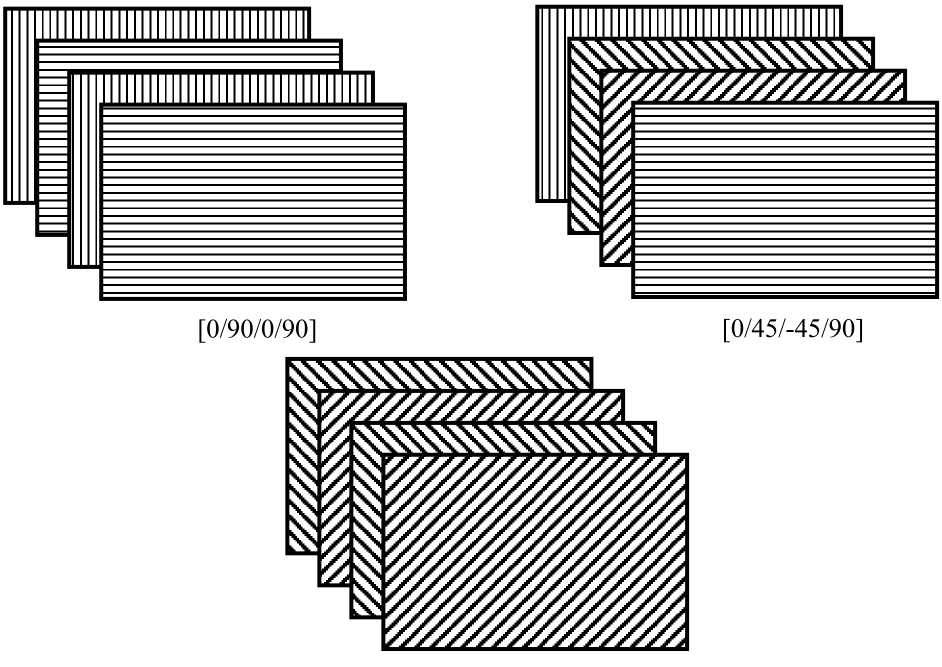


**Figure 3:** Architectures of some advanced composite laminates

Carbon fibers of type HM (high modulus) present a value of Young modulus larger than 300 GPa. High strength (HS) carbon fiber is a general purpose, cost effective carbon fiber, designed for industrial and recreational applications and is usually used for non structural components of aircrafts. Kevlar 49 aramid fiber is characterized by low-density and high-tensile strength and modulus. These properties are the key to its successful use as reinforcement for plastic composites in aircraft, aerospace, marine, automotive, other industrial applications, and in sports equipment. It is available in continuous-filament yarns, chopped fiber, woven and unidirectional fabrics, tissues or veils and tapes for reinforcement applications. Kevlar 49 aramid is used in high-performance composite applications where lightweight, high strength and stiffness, vibration damping and resistance to damage and fatigue are key properties. Reinforced composites can save up to 40% of the weight of glass-fiber composites at equivalent stiffness [7-8].

## 3. RESULTS

The elastic constants $E_x$, $G_{xy}$ and $\upsilon_{xy}$ for fibers volume fractions of $\varphi = 0.5$ are presented in figs. 4 – 12.

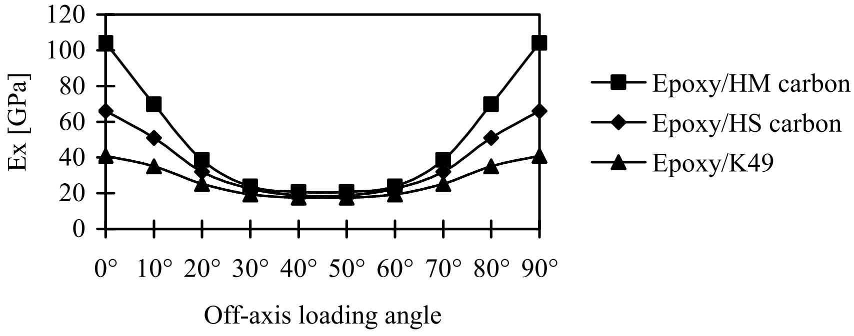


**Figure 4:** $E_x$ Young modulus for a [0/90/0/90] epoxy based composite laminate

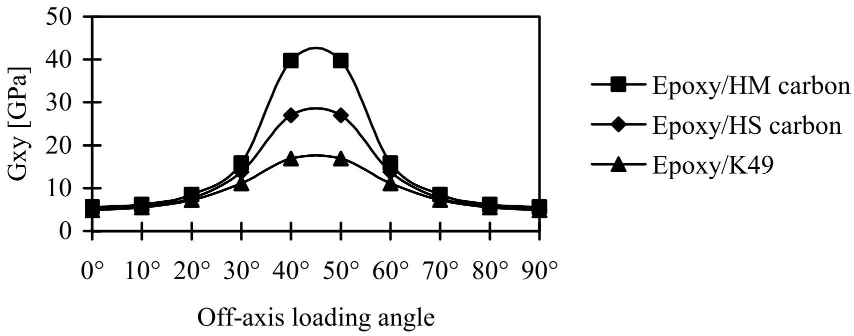


**Figure 5:** $G_{xy}$ shear modulus for a [0/90/0/90] epoxy based composite laminate

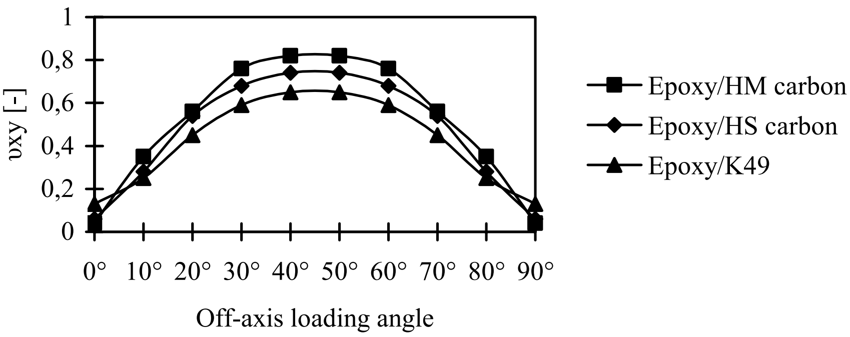


**Figure 6:** $\upsilon_{xy}$ Poisson ratio for a [0/90/0/90] epoxy based composite laminate

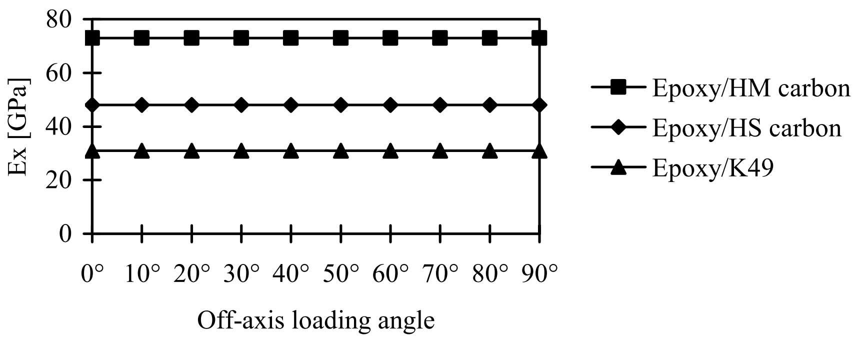


**Figure 7:** $E_x$ Young modulus for a [0/45/-45/90] epoxy based composite laminate

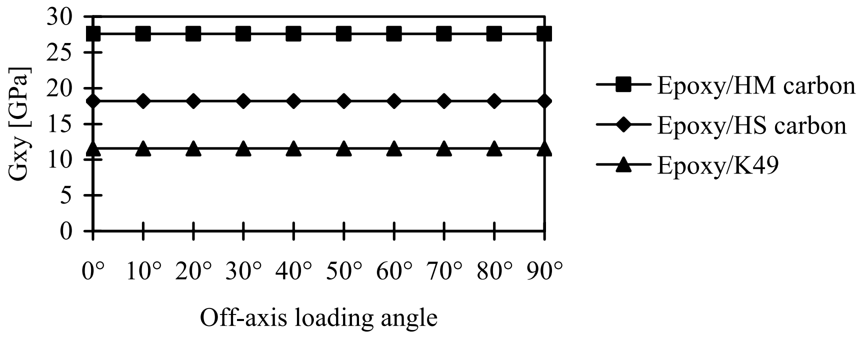


**Figure 8:** $G_{xy}$ shear modulus for a [0/45/-45/90] epoxy based composite laminate

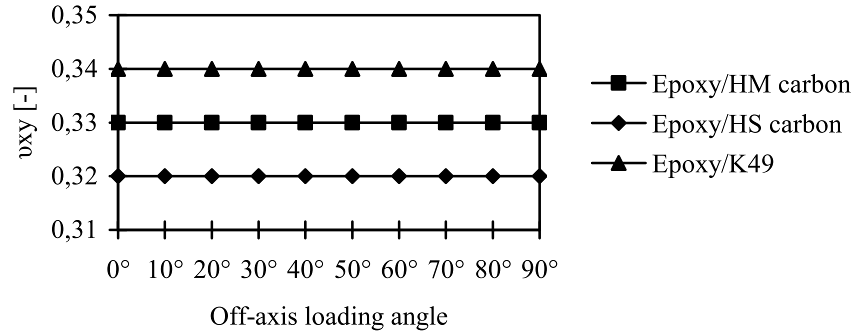


**Figure 9:** $\upsilon_{xy}$ Poisson ratio for a [0/45/-45/90] epoxy based composite laminate

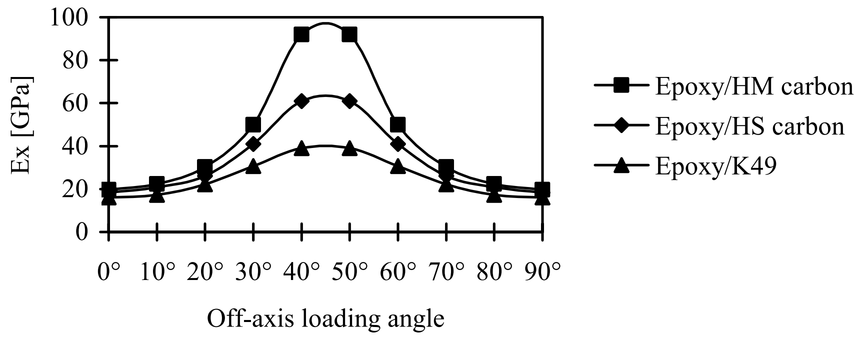


**Figure 10:** $E_x$ Young modulus for a [45/-45/45/-45] epoxy based composite laminate

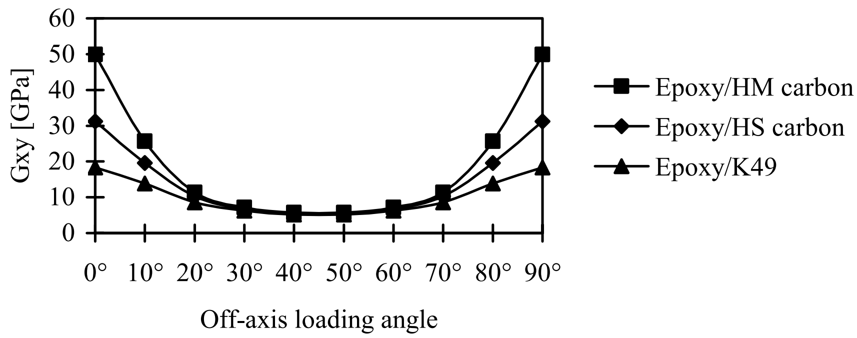


**Figure 11:** $G_{xy}$ shear modulus for a [45/-45/45/-45] epoxy based composite laminate

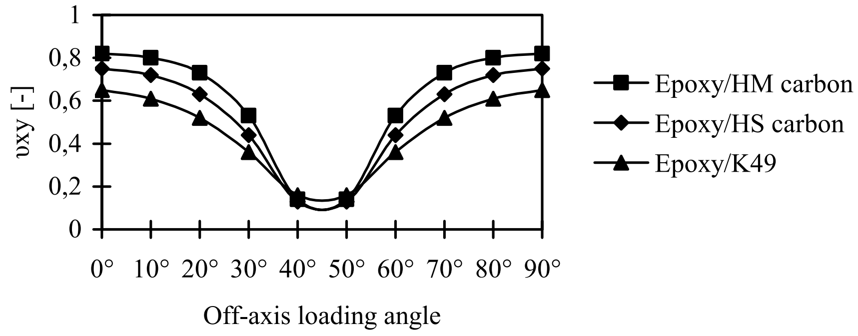


**Figure 12:** $\upsilon_{xy}$ Poisson ratio for a [45/-45/45/-45] epoxy based composite laminate

## 4. CONCLUSION

Tensile-shear interactions lead to distortions and local micro-structural damage and failure, so in order to obtain equal stiffness in all off-axis loading system, a composite laminate have to present balanced angle plies, e.g. [0/45/-45/90]. Under off-axis loading, normal stresses produce shear strains (and of course normal strains) and shear stresses produce normal strains (as well as shear strains). This tensile-shear interaction is also present in laminates (fig. 13), but does not occur if the loading system is applied along the main axes of a single lamina or if a laminate is balanced.

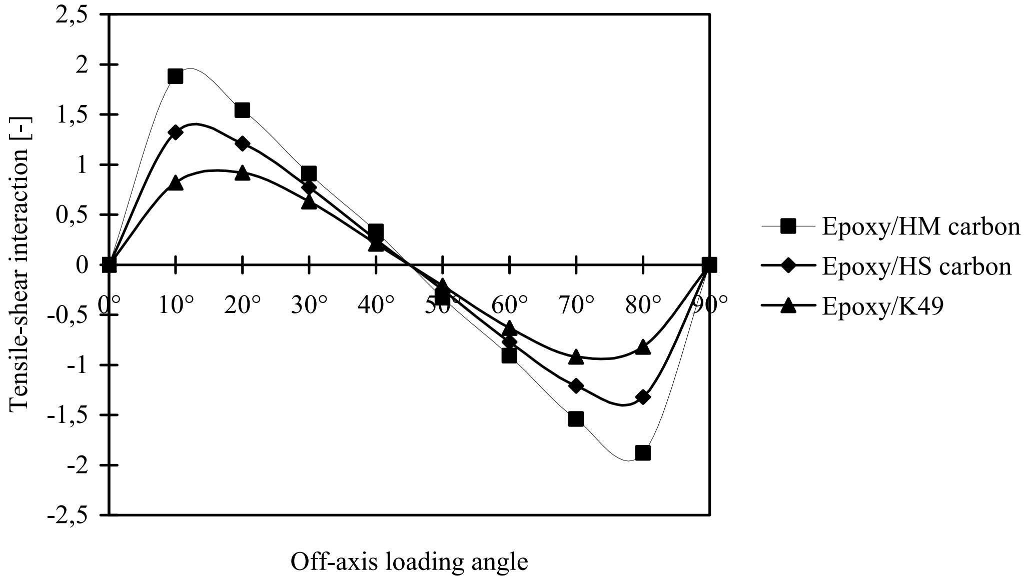


**Figure 13:** Distribution of tensile-shear interaction in a [0/90/0/90] composite laminate

## REFERENCES


[1] Hull D. and Clyne T.W.: An Introduction to Composite Materials, CUP, 1996.
[2] Clyne T.W. and Withers P.J.: An Introduction to Metal Matrix Composites Materials, CUP, 1993.
[3] Piggott M.R.: Load Bearing Fibre Composites, Pergamon Press, 1980.
[4] Teodorescu, D.H.: Basics and Mechanics of PMCs Reinforced with Fibers, Transilvania University Publ. House, 2007 (in Romanian).
[5] Chou T.W.: Microstructural Design of Fibre Composites, CUP, 1992.
[6] Harris B.: Engineering Composite Materials, Institute of Metals, 1986.
[7] Kelly A.: Concise Encyclopedia of Composite Materials, Pergamon Press, 1994.
[8] Ashby M.F.: Materials Selection in Mechanical Design, Pergamon Press, 2nd ed., 1999.