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ANALYSIS OF COMPOSITE RECTANGULAR PLATES BASED ON THE CLASSICAL LAMINATED PLATE THEORY

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ABSTRACT: *In this paper are developed some analytical solutions for static analysis of thin orthotropic plates, based on the Classical Laminated Plate Theory (CLPT).*
The considered reasons for the solutions were to exactly satisfy the boundary conditions and to verify as close as possible the differential equation of the plate. The weighted residue method was considered to optimise the chosen analytical solutions. Interesting evaluations were performed for different types of functions, especially with respect to the orthotropic answer of the plate.
Article contains a comparative analysis of analytical results with the numerical (FEM) and those obtained experimentally.
KEY WORDS: *composite plates, orthotropic, static analysis, simply supported edge, clamped edge.*

1. Assumptions

To establish the constitutive equation of flat plates, is considering Kirchhoff Love hypothesis, "weak" as follows: We accept as working hypotheses to determine the constitutive equation of composite flat plate, assumptions of Kirchhoff-Love, extended as follows [5]:
a. During deformations, the normal to the median of the plate remain straight and normal to the deformed middle surface ($\gamma_{xz} = \gamma_{yz} = 0$ wich implies $\tau_{xz} = \tau_{yz} = 0$). Although efforts unit τ_{xz}, τ_{yz} are very small compared with the normal, we extend the assumption considering their gradient high enough $\frac{\partial \tau_{xz}}{\partial z} \neq 0,\ \frac{\partial \tau_{yz}}{\partial z} \neq 0$.
b. During deformation of the plate its thickness is constant, equivalent with $\varepsilon_z = 0$.
c. We accept the hypothesis of small strains, wich approximates:
$\varepsilon_x = -z \cdot \frac{\partial^2 w}{\partial x^2},\ \varepsilon_y = -z \cdot \frac{\partial^2 w}{\partial y^2},\ \gamma_{xy} = -2 \cdot z \cdot \frac{\partial^2 w}{\partial x \partial y}$, where z is the distance from the point where displacements are been computed and the midplane.

2. ANALYSIS FOR COMPOSITE PLATE EQUATIONS

2.1. Case of clamped edge

For a rectangular plate, on which pressure p is evenly distributed, using the assumptions from the first chapter, we obtain a generalization of Sophie-Germain equation in particular case, when ox-axis is oriented along the fiber direction, [5],[8]:

$$Q_{11} \cdot \frac{\partial^4 w}{\partial x^4} + Q_{22} \cdot \frac{\partial^4 w}{\partial y^4} + 2 \cdot (Q_{12} + 2 \cdot G_{12}) \cdot \frac{\partial^4 w}{\partial x^2 \partial y^2} = \frac{12 \cdot p}{h^3}, \quad (1)$$

where E_1, E_2 are elasticity moduli in the longitudinal and transversal directions, respectively, G_{12} is the shear modulus in the plane of the ply, and ν_{12} is the Poisson coefficient,

$$Q_{11} = \frac{E_1}{1-\nu_{12}\cdot\nu_{21}},\quad Q_{22} = \frac{E_2}{1-\nu_{12}\cdot\nu_{21}},\quad Q_{12} = \frac{\nu_{21}\cdot E_1}{1-\nu_{12}\cdot\nu_{21}} = \frac{\nu_{12}\cdot E_2}{1-\nu_{12}\cdot\nu_{21}}.$$

For example, we consider a flat plate with size $a = 300mm, b = 200mm$, thickness $h = 1.5\ mm$, plate made by incorporating the 8 layers of glass fiber fabric, type EWR $300\,g/mp$, Polylite 440-M888 resin, with technology VARTM (Vacuum Assisted Resin Transfer Molding), at SC STRAERO SA. For each layer of fabric fibers have been removed on one direction (2 of 4 fibers). (Fig. 1(a))

Sails were arranged to have the same warp and weft direction, laminate being treated as orthotropic material. The main directions of elasticity are the warp and weft. To determine the elasticity modules were made standard flat specimens, the size 25×300.

Following tensile testing (SC STRAERO SA), were obtained values $E_1 = 22263MPa$, $\nu_{12} = 0.18$, and $E_2 = 19035MPa.$

In the same time, the orthotropic plate with clamped edge is required to stretching and bending.

Using weights of 1.2 kg and flat specimens size (24×100), were obtained values $E_1 = 22065MPa,\quad E_2 = 17079MPa$.

For this, we used the relations:

$$v = \frac{F\cdot\ell^3}{48\cdot E\cdot I},\quad I = \frac{c\cdot h^3}{12},\ \text{which implies}\ E = \frac{F\cdot\ell^3}{4\cdot v\cdot c\cdot h^3} = \frac{M\cdot g\cdot\ell^3}{4\cdot v\cdot c\cdot h^3}.$$

The orthotropic plate was driven by a pressure $p = 0.00465\ MPa$, uniformly distributed over the entire surface.

To ensure better clamped conditions, contour plate was reinforced (fig. 1(b)).

In table. 1 are presents the comparative results between the experimental measurements and the numerical results (using ANSYS- shell Elastic 4 node 63, mesh 10), taking into account the elasticity modules in stretching.

In table 2 are presents the comparative results between the experimental measurements and the numerical results, taking into account the elasticity modules in bending.

For a better approximation, we consider an average for modules of elasticity, respectively $E_1 = 22164MPa,\quad E_2 = 18057MPa.$ (table.3)

Mathematic, clamped edge for $x = 0$, $x = a$ and $y = 0,\quad y = b$ follows:

$$\begin{cases} w(0,y) = w(a,y) = 0, & \dfrac{\partial w}{\partial x}(0,y) = \dfrac{\partial w}{\partial x}(a,y) \\ w(x,0) = w(x,b) = 0, & \dfrac{\partial w}{\partial y}(x,0) = \dfrac{\partial w}{\partial y}(x,b) = 0 \end{cases}. \tag{2}$$

In [8], Reddy proposed solution for equation (1), as:

$$w(x,y) = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}\cdot\left(\frac{x}{a}\right)^{i+1}\cdot\left(\frac{y}{b}\right)^{j+1}\cdot\left(1-\frac{x}{a}\right)^2\cdot\left(1-\frac{y}{b}\right)^2,$$ and solved it for $m = n = 1$.

In this paper, using Ritz method, we look for a solution as

$$w(x,y) = \sum_{i=1}^{n}\sum_{j=1}^{m} c_{ij}\cdot x^{i+1}\cdot(a-x)^{i+1}\cdot y^{j+1}\cdot(b-y)^{j+1}. \tag{3}$$

The coefficients $\left(c_{ij}\right)_{i,j}$ will be determined using the weighted residue method.

For $p = 0.00465\ MPa$, $E_1 = 22164MPa,\quad E_2 = 18057MPa,$ we get coefficients (Maple):

$c_{11} = 1.29378\cdot 10^{-16},\quad c_{12} = -3.2159\cdot 10^{-21},\quad c_{13} = 2.09204\cdot 10^{-26},\quad c_{21} = -3.3471\cdot 10^{-21},$

$c_{31} = 1.5746\cdot 10^{-26},\quad c_{22} = 1.2466\cdot 10^{-25},\quad c_{33} = -4.9444\cdot 10^{-35},$

for $m = n = 3$.

In this case, the maximum value of the arrow is $w\left(\frac{a}{2},\frac{b}{2}\right) \approx 3.053mm.$

Using finite element method (shell Elastic 4 node 63, mesh 10, ANSYS) we obtain a maximum displacement $w_{\max} = w\left(\frac{a}{2},\frac{b}{2}\right) \approx 3.055\ mm$.

In table 4 are presented results comparison between the analytical (Maple) and numerical solution.

To validate the solutions we performed measurements on the same plate, driven uniformly with different pressures ($p = 0.00305 MPa$, table 5, and $p = 0.004 MPa$, table 6).
In fig. 2 (a,b,c) are experimental graphics solutions for requests made.

Given the need for several sets of measurements of orthotropic plate arrows, depending on the conditions imposed on the shape and position in plan, I chose the option to apply a pressure plate perpendicular, evenly distributed over the entire surface with a pressurized air chamber.
Solution readily allows repeated measurements, and application of pressure change.
Measurement of air pressure in the chamber and so the pressure plate application is made with a liquid manometer. To measure arrows of deformed plate, we use inductive displacement transducer.
In these conditions, the entire system covers the following modules:

- Application module orthotropic plate
- Displacement transducer module in the horizontal plane *xoy*
- Arrow measuring module
- Acquisition and storage module measurements (steper electric motor EM-257, 17PM-K212-P1T for moving the transducer 12 on the *ox* and EM-464, 2Y28AD2 for moving the transducer 12 on the *oy*). .

Air chamber pressure was made from a thick $1\,mm$ rubber sheet $340 \times 240\ mm$ and, on where were bonded two valves, one for charging and one for connection to the manometer.
Deformable wall of the chamber is a plastic foil. (Fig. 1(c))

2.2. Tables

Table 1: *(* $p = 0.00465 MPa$, $E_1 = 22263 MPa$, $E_2 = 19035 MPa$)

ANSYS (mm)	EXPERIMENTALLY (mm)	Difference (%)
0.0208	0.303	-1.0
0.401	0.439	-3.8
1.004	1.056	-5.1
1.619	1.7	-8.1
2.145	2.248	-10.3
2.539	2.651	-11.2
2.788	2.861	-7.3
2.894	2.893	0.1
2.859	2.878	-1.9
2.682	2.807	-12.5
2.36	2.48	-12.0
1.898	2.001	-10.3
1.319	1.383	-6.4
0.691	0.701	-1.0
0.164	0.166	-0.2

Table 2: ($p = 0.00465 MPa$, $E_1 = 22065 MPa$, $E_2 = 17079 MPa$)

ANSYS (mm)	EXPERIMENTALLY (mm)	Difference (%)
0.0222	0.0303	-0.8
0.431	0.439	-0.8
1.066	1.055	3.1
1.764	1.7	6.4
2.348	2.248	10.0
2.788	2.651	13.7
3.07	2.861	20.9
3.19	2.893	29.7
3.15	2.878	27.2
2.949	2.807	14.2
2.587	2.48	10.7
2.071	2.001	7.0
1.431	1.383	4.8

0.746	0.701	4.4
0.176	0.166	1.0

Table 3: *(* $p = 0.00465 MPa$, $E_1 = 22164 MPa$, $E_2 = 18057 MPa$)

ANSYS (mm)	EXPERIMENTALLY (mm)	Difference (%)
0.0215	0.0303	-0.9
0.416	0.439	-2.4
1.044	1.055	-1.1
1.689	1.7	-1.1
2.242	2.248	-0.6
2.658	2.651	0.7
2.871	2.861	1.0
2.923	2.893	3.0
2.871	2.878	-0.7
2.809	2.807	0.2
2.469	2.48	-1.1
1.981	2.001	-2
1.373	1.383	-1
0.717	0.701	1.6
0.17	0.166	0.4

Table 4: *(* $p = 0.00465 MPa$, $E_1 = 22164 MPa$, $E_2 = 18057 MPa$)

ANSYS	0.022	0.415	1.689	2.242	2.658	2.923	2.998	2.809	2.469	1.98	0.717	0.17
Maple	0.022	0.416	1.688	2.241	2.657	2.921	2.996	2.808	2.467	1.98	0.717	0.17

Table 5: *(* $p = 0.00305 MPa$, $E_1 = 22164 MPa$, $E_2 = 18057 MPa$)

ANSYS (mm)	EXPERIMENTALLY (mm)	Difference (%)
0.014	0.015	0.11
0.272	0.31	3.8
1.109	1.105	-0.3
1.471	1.508	3.7
1.743	1.723	-2
1.917	1.916	-0.1
1.966	1.982	1.6
1.843	1.845	0.2
1.619	1.647	2.8
1.299	1.267	-1.2
0.47	0.508	3.8
0.111	0.137	2.6

Table 6: *(* $p = 0.004 MPa$, $E_1 = 22164 MPa$, $E_2 = 18057 MPa$)

ANSYS (mm)	EXPERIMENTALLY (mm)	Difference (%)
0.019	0.000	1.8
0.617	0.619	-0.2
1.453	1.423	3.0
2.123	2.119	0.4
2.514	2.501	1.3
2.615	2.607	0.8
2.417	2.44	-2.3
1.929	1.921	0.8
1.181	1.198	-1.7
0.357	0.328	2.9

2.3. Figures

Figure.1(a)

Figure.1(b)

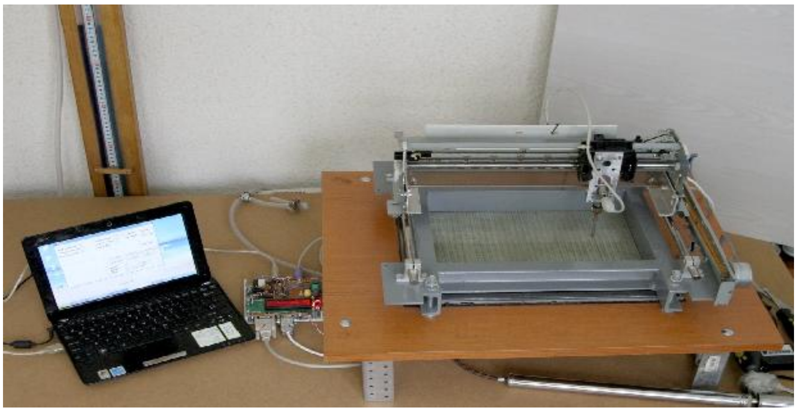
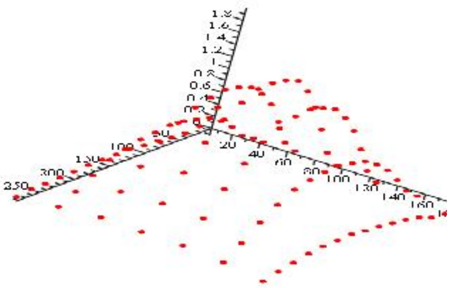
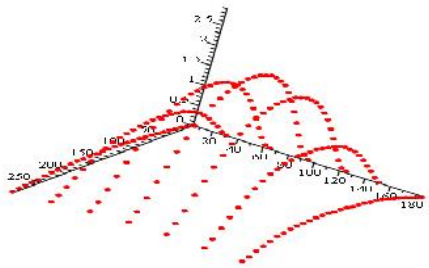


Figure.1(c)



a



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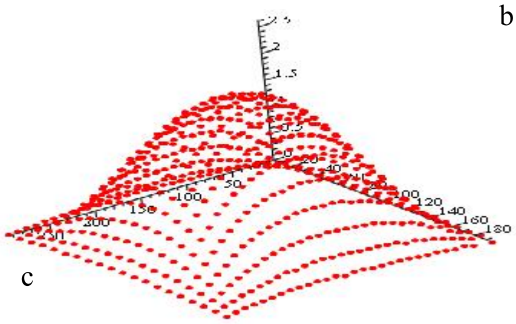

c

Figure. 2

3. CONCLUSION

This paper is an analysis of orthotropic plates, based on CLPT, characterizing thin plates, assuming small deformations. I proposed an analytical solution for case of clamped edge. After comparing the results with those treated in the literature, as well as the results from finite element, and experimentally analysis, we conclude that the solution presented in this paper approximates well plate subjected to compressive deformation.
Mechanical device allows measurements for other conditions imposed on the outline of the rectangular plate, simply supported, articulated and free.

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