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SPECTRAL RESPONSE FOR N RANDOM EXCITATIONS OF A NON-LINEAR OSCILLATOR WITH NON-LINEAR DAMPING CHARACTERISTIC IN THE FLUID MEDIUM

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Abstract: *In nuclear power plant reactors, the mechanical components are surrounded by heat-conducting fluid in steady motion. Some of them, such as fuel-rod assemblies, are separated by very small clearances. Non-linear vibratory behavoir can be induced either by the steady excitation or accidentally, due to seismic excitation for example. An approximate analytical procedure is presented to estimate the power spectral density of response of n random excitation of non-linear oscillator with non-linear damping characteristic in the fluid medium. A natural method of attacking non-linear problems ist a replace the governing set of non-linear differential equations by an equivalent set of linear equations. The method will be briefly discussed in the following sections. Fundamental theory of this linearization procedure can be found in Caughey, Roberts and Spanos.*
Keyword: *Non-linear oscillator, random vibration, fluid medium, response, equivalent linearization.*

1. SYSTEM MODEL

For the sferic body with a mass m, caught at end of a spring with a elastic constant k ,when the lenght of the undeformed spring is l_0, at random excitations in a liquid with the viscosity coefficient γ.
Consider a n-degree-of-freedom system, exposed to the simultaneous action of n forces: $W_1(t), W_2(t) \dots W_n(t)$ Let $\eta_{W_1}(t)$, $\eta_{W_2}(t), \dots \eta_{W_n}(t)$ denote the effect of the forces $W_1(t), W_2(t) \dots$and $W_n(t)$ on the system response, when the forces are applied separately. As the loads are applied on different degrees of freedom, the transfer of the excitations is described by two different mechanical transfer functions [1], $H_1(\omega), H_2(\omega), \dots H_n(\omega)$. To illustrate the procedure, let us consider the following oscillator with a nonlinear restoring force component. If the distance in the horizontal direction OA is equal with $d > l_0$, the motion equation is

$$m\ddot{\eta}(t)+c[\dot{\eta}(t)+\varepsilon r_3 \dot{\eta}^3(t)+\varepsilon r_5 \dot{\eta}^5(t)+...+\varepsilon r_{2n+1} \dot{\eta}^{2n+1}(t)]+F_r+F\cos\lambda=W(t)\,, \qquad (1)$$

where $W(t)$ is the external excitation signal with zero mean and $\eta(t)$ is the displacement response of the system, F_r is the resistence force met in its movement in the liquid, proportional with the liquid viscosity γ, with the representative lenght of the body l and its velocity v, $F_r = K\gamma l v$, $r_3, r_5, \dots, r_{2n+1}$ is the nonlinear factor to control the type and degree of nonlinearity in the system and $\eta(t)$ is the displacement response of the system. For a sferic body $K = 6\pi, l = r$, so $F_r = 6\pi r\gamma v$. If λ is the angle of inclination of the horizontal spring, and AB is deformed spring length at some time t, we write:

$$F = k(AB - l_0), \quad OB = AB\cos\lambda, \quad \cos\lambda = \frac{\eta}{\sqrt{d^2+\eta^2}},\ p^2 = \frac{k}{m}, \qquad (2)$$

result

$$\ddot{\eta}(\tau)+\left(2\xi p+6\pi r\frac{\gamma}{m}\right)[\dot{\eta}(t)+\varepsilon r_3 \dot{\eta}^3(t)+\varepsilon r_5 \dot{\eta}^5(t)+...+\varepsilon r_{2n+1} \dot{\eta}^{2n+1}(t)]+p^2\left(1-\frac{l_0}{d}\right)\eta(t)+p^2\frac{l_0}{2d^3}\eta^3(\tau)=w(t)\,, \qquad (3)$$

where ξ is the critical damping factor and p is the undamped natural frequency, for the linear system.

We aplicate

$$\frac{\eta}{\sqrt{d^2+\eta^2}}=\frac{1}{d}\left(1+\frac{\eta^2}{d^2}\right)^{-\frac{1}{2}}+\frac{1}{d}\left(1-\frac{\eta^2}{2d^2}+\frac{3\eta^4}{8d^4}+...\right). \quad (4)$$

Using the notation

$$h(\eta(t),\dot{\eta}(t))=\left(2\xi p+6\pi r\frac{\gamma}{m}\right)[\dot{\eta}(t)+\varepsilon r_3\dot{\eta}^3(t)+\varepsilon r_5\dot{\eta}^5(t)+...+ +\varepsilon r_{2n+1}\dot{\eta}^{2n+1}(t)]+p^2\left(1-\frac{l_0}{d}\right)\eta(t)+p^2\frac{l_0}{2d^3}\eta^3(\tau) \quad (5)$$

the equation of motion can be rewritten as:

$$\ddot{\eta}(t)+h(\eta(t),\dot{\eta}(t))=w(t). \quad (6)$$

The idea of linearization [2,3] is replacing the equation by a linear system:

$$\ddot{\eta}(t)+\beta_{ech}\dot{\eta}(t)+\gamma_{ech}\eta(t)=w(t) \quad (7)$$

It is necessary to minimize the expected value [1,2] of the difference between equations (2) and (3) in a least square sense.

Now, the difference is the difference between the nonlinear stiffness and linear stiffness terms , which is

$$\varepsilon=\left(2\xi p+6\pi r\frac{\gamma}{m}\right)[\dot{\eta}(t)+\varepsilon r_3\dot{\eta}^3(t)+\varepsilon r_5\dot{\eta}^5(t)+ +...+\varepsilon r_{2n+1}\dot{\eta}^{2n+1}(t)]-\beta_{ech}\dot{\eta}(t)-\gamma_{ech}\eta(t) \quad (8)$$

The value of β_{ech} and γ_{ech} can be obtained by minimizing [3] the expectation of the square error.

The expression of β_{ech} and γ_{ech} can be obtained as:

$$\beta_{ech}=\frac{E\{\eta^2\}E\{\dot{\eta}h\}-E\{\eta\dot{\eta}\}E\{\eta h\}}{E\{\eta^2\}E\{\dot{\eta}^2\}-(E\{\eta\dot{\eta}\})^2}, \quad (9)$$

$$\gamma_{ech}=\frac{E\{\dot{\eta}^2\}E\{\eta h\}-E\{\eta\dot{\eta}\}E\{\dot{\eta}h\}}{E\{\eta^2\}E\{\dot{\eta}^2\}-(E\{\eta\dot{\eta}\})^2}. \quad (10)$$

The linear equation [2,3] for the random excitation is

$$\ddot{\eta}(t)+\frac{E\{\dot{\eta}h\}}{E\{\dot{\eta}^2\}}\dot{\eta}(t)+\frac{E\{\eta h\}}{E\{\eta^2\}}\eta(t)=w(t). \quad (11)$$

The displacement variance [1,2] of the system under Gaussian white noise excitation can be expressed as

$$\sigma_\eta^2=\frac{1}{m}\int_{-\infty}^{\infty}\frac{S_0}{\left(\frac{E\{\eta h\}}{E\{\eta^2\}}-\omega^2\right)^2+\omega^2\left(\frac{E\{\dot{\eta}h\}}{E\{\dot{\eta}^2\}}\right)^2}d\omega, \quad (12)$$

where the frequency response function [4] of the single degree of freedom system is

$$H(\omega)=\frac{1}{m\left(-\omega^2+i\omega\frac{E\{\dot{\eta}h\}}{E\{\dot{\eta}^2\}}+\frac{E\{\eta h\}}{E\{\eta^2\}}\right)}. \quad (13)$$

From the Fourier-transforms [1] of the response, results:

$$\bar{\eta}_1(\omega)=H(\omega)F_{W_1}(\omega),\bar{\eta}_2(\omega)=H(\omega)F_{W_2}(\omega),...\bar{\eta}_n(\omega)=H(\omega)F_{W_n}(\omega) \quad (14)$$

and the complex conjugates of these expressions are

$$\bar{\eta}^*_1(\omega) = H^*(\omega)F^*_{W_1}(\omega)\,,\ \bar{\eta}^*_2(\omega) = H^*(\omega)F^*_{W_2}(\omega)\,,\ldots \bar{\eta}^*_n(\omega) = H^*(\omega)F^*_{W_n}(\omega) \tag{15}$$

The response spectrum is real:

$$S_\eta(\omega) = \sum_{j=1}^{n} \left|H_j(\omega)\right|^2 S_{W_j}(\omega) + 2\mathrm{Re}[\sum_{\substack{r=1\\s=1\\r<s}}^{n} H^*_r(\omega)H_s(\omega)S_{W_rW_s}(\omega)]. \tag{16}$$

For completely uncorrelated processes we have

$$S_{W_rW_s}(\omega) = S_{W_sW_r}(\omega) = 0,\ r \neq s \tag{17}$$

and the power spectral density of the response is

$$S_\eta(\omega) = \sum_{j=1}^{n} \left|H_j(\omega)\right|^2 S_{W_j}(\omega). \tag{18}$$

For completely uncorrelated processes we have

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$$S_\eta(\omega) = \sum_{j=1}^{n} \left|H_j(\omega)\right|^2 S_{W_j}(\omega). \tag{20}$$

The variance of the process $W(t)$ is [4,5]

$$\sigma^2_\eta = R_\eta(0) = \int_{-\infty}^{\infty} \left|H(\omega)\right|^2 S_W d\omega = \int_{-\infty}^{\infty} \left|H(\omega)\right|^2 [S_{W_1}(\omega) + S_{W_2}(\omega) + \ldots S_{W_n}(\omega) +$$

$$2\mathrm{Re}\sum_{\substack{r=1\\s=1\\r<s}}^{n} S_{W_rW_s}(\omega)]d\omega = \int_{-\infty}^{\infty} \frac{S_{W_1}(\omega)+S_{W_2}(\omega)+\ldots S_{W_n}(\omega)+2\mathrm{Re}\sum_{\substack{r=1\\s=1\\r<s}}^{n} S_{W_rW_s}(\omega)}{m^2\left[\left(\frac{E\{\eta h\}}{E\{\eta^2\}}-\omega^2\right)^2+\omega^2\left(\frac{E\{\dot{\eta}h\}}{E\{\dot{\eta}^2\}}\right)^2\right]} d\omega. \tag{21}$$

The velocity variance [4,5] is given by:

$$\sigma^2_{\dot{\eta}} = \int_{-\infty}^{\infty} \omega^2 \left|H(\omega)\right|^2 S_W d\omega = \int_{-\infty}^{\infty} \omega^2 \left|H(\omega)\right|^2 [S_{W_1}(\omega) + S_{W_2}(\omega) + \ldots S_{W_n}(\omega) + 2\mathrm{Re}[\sum_{\substack{r=1\\s=1\\r<s}}^{n} S_{W_rW_s}(\omega)]d\omega. \tag{22}$$

The power spectral density of response [5,6] is

$$S_\eta(\omega) = \frac{S_{W_1}(\omega)+S_{W_2}(\omega)+\ldots S_{W_n}(\omega)+2\mathrm{Re}[\sum_{\substack{r=1\\s=1\\r<s}}^{n} S_{W_rW_s}(\omega)]}{m^2\left[\left(\frac{E\{\eta h\}}{E\{\eta^2\}}-\omega^2\right)^2+\omega^2\left(\frac{E\{\dot{\eta}h\}}{E\{\dot{\eta}^2\}}\right)^2\right]} \tag{23}$$

2. NUMERICAL RESULTS:

For $m = 1kg,\ n = 2,\ d = 1m,\ k = 36\frac{N}{m},\ c = 4\frac{Ns}{m},\ l_0 = 0{,}5m,\ r = 6\cdot 10^{-2}m\,,\alpha = 3m^{-2}\ S_{W_1} = S_{W_2} \overset{not}{=} S_0 = 2\ N^2 \cdot s\,,$ $r_3 = 4\cdot 10^3 s^2/m^2,\ r_5 = 2{,}74\cdot 10^3 s^4/m^4,\ \varepsilon = 0{,}01$ can by written:

$$p = \sqrt{\frac{k}{m}} = 6s^{-1},\ \frac{c}{m} = 2\xi p \Rightarrow \xi = 0{,}33\,, \tag{24}$$

$$\Gamma(\frac{3}{2}) = \frac{1}{2}\Gamma(\frac{1}{2}) = \frac{\sqrt{\pi}}{2}. \tag{25}$$

By adding two important formulas

$$E\{\dot{\eta}^6\} = \frac{63}{4}\sigma^2_{\eta} E\{\dot{\eta}^4\} \tag{26}$$

$$E\{\dot{\eta}^4\} = \frac{45}{4}\sigma^2_{\eta} E\{\dot{\eta}^2\} \tag{27}$$

$$\sigma^2_{\eta} = \frac{\sqrt{\left[(c+6\pi r\gamma)\left(1-\frac{l_0}{d}\right)\right]^2 + \frac{12 l_0}{d^3}\left(\frac{1}{2}+3\pi r\gamma\right)\frac{\pi S_0}{p^2}} - (c+6\pi r\gamma)\left(1-\frac{l_0}{d}\right)}{\frac{6 l_0}{d^3}\left(\frac{1}{2}+3\pi r\gamma\right)}. \tag{28}$$

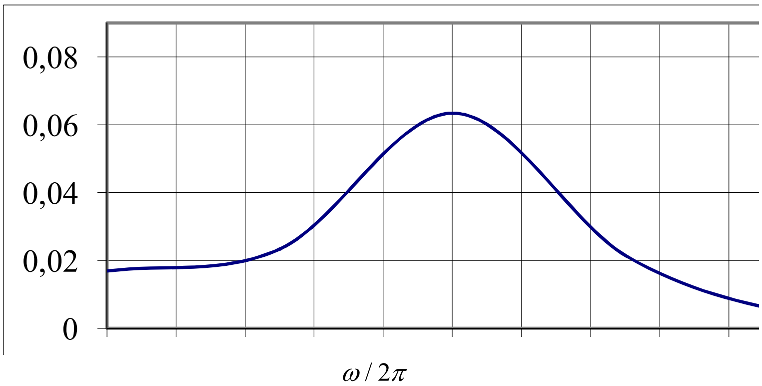

Fig. 1. The power spectral density of response $S_{\eta}\ [m^2 \cdot s]$ for the power spectral density of excitation $S_0 = 2N^2 \cdot s$ and $\gamma = 1,393 \cdot 10^{-2} \frac{kg}{m \cdot s}$ (glycerin)

Obtain

$$\sigma^2_{\eta} = 0,24 m^2. \tag{29}$$

The standard deviation of vellocity is

$$\sigma^2_{\dot{\eta}} = \frac{2\pi S_0}{m\left(2\xi p + 6\pi r \mu\right)} = \frac{2\pi S_0}{c + 6\pi r\gamma}, \tag{30}$$

or

$$\sigma^2_{\dot{\eta}} = 2,16 \frac{m^2}{s^2}. \tag{31}$$

3. CONCLUSION

It observes the modus in witch the medium values are decreasing the standard deviation and velocity variance of the moment and for speed at the grow of fluid viscosity. The decrise tens to a form approximately linear . Also observing the decrise of pulsation at the rize of viscosity. Efficient equivalent linear systems with random coefficients for approximating the power spectral density can be deduced. The resonant peak is described very satisfactorily by the approximate solution.

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