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THEORETICAL METHODS FOR DETERMINING THE PERMEABILITY COEFFICIENT

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Abstract: *Stability of embankment and construction foundations of any kind, on sites that presents pronounced moisture, requires accurate knowledge of the direction and speed of the water flow trough soil, as well as how it influences the bearing capacity of ground. To determine the water velocity through the soil, it is necessary to know the permeability coefficient. The present paper aims to present the main empirical relationships that can be used to determinate soils permeability, presenting for each one the validity domain.*

Key words: *permeability coefficient, porosity, equivalent diameter, granulometric analysis, voids ratio, empirical theoretical relations.*

1. Introduction

Water movement into the ground is one of the fundamental problems of geotechnical engineering. If we look carefully at the previous statement and analyse all unfavourable situations encountered in practice due to the presence of water in the foundation soil, we can conclude that the lack of water in the ground would cancel, in most cases, the utility of geotechnical engineering. It should however be kept in mind that the previous statement is exacerbated, the purpose of this exaggeration is to highlight the importance played by water in the analysis of soils and rocks.

The presence of groundwater on a site where a building will be constructed is a complex problem and difficult to solve, because is important to know how to approach the foundation works and how to maintain the stability for the surrounding constructions.

Knowing the phenomena of water movement through the soil is mandatory for the problems encountered by the civil engineer.

2. Soil permeability

2.1. Generalities

Water movement through soil can define an important property that it is to permit a fluid to pass through its pores, property called permeability.

The permeability depends on porosity, so it is more pronounced at soils with higher porosity, where the spaces between particles are larger, favouring the continuous circulation of water. It is

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believed that the water flow regime, for the vast majority of soils, is laminar, water molecules describing continuous and parallel current lines.

Permeability (defined according to *STAS 3950-81*) is evaluated through the coefficient of permeability, *k*.

The permeability coefficient can be expressed depending on grain size and porosity of the soil which is crossed by the fluid (water), according to equation 1 [3] or according to equation 2 [11].

$$k = \frac{15 \cdot 10^4}{f^2 \cdot A_{sp}^2} - \frac{n^3}{(1-\eta)^2}. \qquad (1)$$

k − permeability coefficient [*L/T; L – length units; T – time units*];
f − coefficient depending on the shape of mineral grains that form the soil skeleton (table *1*) [-];
A_{sp} – specific surface area [L^2/L^3];
n – porosity [*%*];
η – dynamic (absolute) viscosity of the liquid [$kg \cdot m^{-1} \cdot s^{-1} = P$].

Values of "f" coefficient for different forms of mineral grains that compose the soil skeleton Table 1

Grains shape	Spherical	Rounded	Sharp	Very sharp
f [-]	1	1.1	1.25	1.40

$$k = \frac{v}{i}. \qquad (2)$$

$$i = \frac{\partial h}{\partial l}. \qquad (3)$$

k − permeability coefficient [*L/T*];
v − the apparent filtration speed which is the ratio between the flow rate of water passing through a flat surface, from the porous body, perpendicular to the direction of fluid flow and on the respective surface area [*L/T*];
i – the hydraulic gradient or hydraulic slope (relation 3), which is the ratio between the difference of piezometric level *h*, for two equipotential surfaces, and current line length *l* [-].

The permeability coefficient is influenced by a number of factors that depend on the characteristics of the mineral skeleton of the soil, as well as the characteristics of the water flowing through its pores. The main factors that dictate the size of the permeability coefficient are [7], [9], [11]:

- porosity and geometrical characteristics of the pores;
- mineralogical nature of the soils;
- granulometric distribution of the soil;
- degree of saturation;
- the specific unit weight of water;
- size of hydraulic gradient *i* (at clays);
- water viscosity;
- stratification (for determination of the permeability coefficient it must be mentioned the sampling direction from the layer).

Figure 1 illustrates the soil permeability variation. The arrow indicates the decreasing direction for the permeability coefficient.

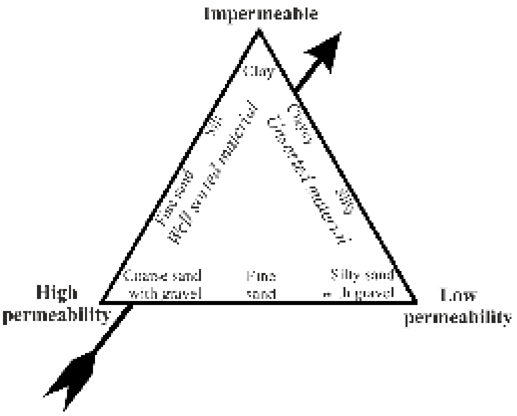

Fig. 1. *Variation of soil permeability [1]*

A classification of soils can be made based on permeability coefficient values, as shown in the table *2* [4].

Analyzing the specialized literature it can be notice that the value of permeability presents multiple intervals in which can vary, depending on soil nature. Thus, in table 3 are summarized the values of permeability coefficient according to some authors.

Soil classification after permeability coefficient Table 2

Soil	**k [cm/s]**
Very permeable	$> 10^{-1}$
Permeable	$10^{-1} \div 10^{-4}$
Less permeable	$10^{-4} \div 10^{-7}$
Impermeable	$< 10^{-7}$

Permeability values for different types of soils Table 3

<table>
<tr><td colspan="11"><i>Boţu and Muşat, 2003</i></td></tr>
<tr><td>Soil type</td><td>Gravel</td><td colspan="3">Sand</td><td colspan="5">Silt</td><td>Clay</td></tr>
<tr><td><i>k</i> [m/s]</td><td>$\geq 10^{-2}$</td><td colspan="3">$10^{-2} \div 10^{-4}$</td><td colspan="5">$10^{-4} \div 10^{-6}$</td><td>$10^{-6} \div 10^{-8}$</td></tr>
<tr><td colspan="11"><i>Manea et al., 2003 / STAS 1913/6-76</i></td></tr>
<tr><td>Soil type</td><td>Gravel</td><td colspan="3">Sand and gravel,
coarser sand,
medium sand</td><td colspan="3">Fine sand,
silty sand,
sandy silt</td><td colspan="2">Loamy silt,
silty clay,
sandy clay</td><td>Clay</td></tr>
<tr><td><i>k</i> [m/s]</td><td>$10^{-1} \div 10^{-3}$</td><td colspan="3">$10^{-3} \div 10^{-5}$</td><td colspan="3">$10^{-5} \div 10^{-7}$</td><td colspan="2">$10^{-7} \div 10^{-9}$</td><td>$< 10^{-9}$</td></tr>
<tr><td colspan="11"><i>Sarsby, 2000</i></td></tr>
<tr><td>Soil type</td><td>Gravel</td><td colspan="3">Sand</td><td colspan="5">Silt</td><td>Clay</td></tr>
<tr><td><i>k</i> [m/s]</td><td>$1 \div 10^{-2}$</td><td colspan="3">$10^{-2} \div 10^{-5}$</td><td colspan="5">$10^{-5} \div 10^{-8}$</td><td>$\leq 10^{-8}$</td></tr>
<tr><td colspan="11"><i>Stanciu and Lungu, 2006</i></td></tr>
<tr><td>Soil type</td><td>Gravel</td><td>Coarser sand</td><td>Medium sand</td><td>Fine sand</td><td>Silty sand</td><td>Fine sand (U_n= 2÷5)</td><td>Dunes sand</td><td>Loess</td><td>Silt</td><td>Clay</td></tr>
<tr><td><i>k</i> [m/s]</td><td>10^{-2}</td><td>$10^{-2} \div 10^{-4}$</td><td>$10^{-4} \div 5 \cdot 10^{-5}$</td><td>$5 \cdot 10^{-4} \div 10^{-5}$</td><td>$2 \cdot 10^{-5} \div 10^{-6}$</td><td>$6 \cdot 10^{-5} \div 10^{-6}$</td><td>$10^{-3} \div 3 \cdot 10^{-3}$</td><td>$10^{-5} \div 10^{-6}$</td><td>$5 \cdot 10^{-6} \div 10^{-7}$</td><td>$< 10^{-7}$</td></tr>
</table>

2.2. Methods for determining the permeability

Over time there have been developed a number of methods for determining the permeability coefficient of soils, these being based on research, design and execution needs that occurred in the construction process.

These methods can be classified as following [5], [8], [10]:

- laboratory methods:
  - methods based on granulometric analysis results that allow the use of empirical formulas - are used to determine the permeability coefficient having in common the presence of information obtained from granulometric analysis of the aquifer material and are based on the Darcy's law of linear filtration;
  - methods that use water or air permeameters.
- *in situ* tests (site methods):
  - debit tests through pumping;
  - punctual tests;
  - tests with special devices (Paker);
  - Lafrane method;
  - permeameter tests (with Brillant vacuum);
  - experimental water castings;
  - determinations by measuring the actual speed of groundwater flow.

It is important to note that, from all hydraulic parameters characterizing the aquifers, only the permeability coefficient can be determined by laboratory methods specific to geotechnics practice. Transmissivity, storage coefficient and drainage process parameters can be evaluated only based on field data obtained from experimental pumping.

It is also very important to pay attention to the interpretation and validity of the results, detailing the technical conditions for sampling and analysis, in order to link correctly the adopted assumptions from the analytical models with the natural conditions.

3. Methods that allow the use of empirical formulas that are based on granulometric analysis results

This paper aims to present the main empirical relationships that can be used to calculate the permeability coefficient of soils. Further, there will be verify the validity scopes of these relations by making some simple calculations using actual data for certain types of soils.

Empirical formulas are used to determine the permeability coefficient having as common element the presence of information from granulometric analysis of the aquifer material and there are based on Darcy's law for linear filtration. In generally these relations have the following form:

$$k = C \cdot d^2. \tag{4}$$

C − coefficient which varies depending on the formula;
d − particle diameter; usually it represents d_{10} from the grading curve

The results obtained using these formulas are approximate and are generally used in preliminary stages of the hydraulic calculations.

The most used formulas are [2], [5], [6], [8], [9], [10]:

- **Allen Hazen**

$$k = C \cdot d_e^2 (0.7 + 0.03t). \tag{5}$$

C − coefficient which depends on the porosity and homogeneity of the material;

$$C = 400 + 40 \cdot (n - 26). \tag{6}$$

n − porosity [%];
d_e − effective diameter [*mm*];

$$d_e = d_{10}. \tag{7}$$

d_{10} − particle diameter which represents *10%* of the sample mass [*mm*];
t − water temperature [°*C*].

This formula can be used to determine the permeability coefficient of uniform sands that simultaneously fulfil the following conditions:

$$\begin{cases} \dfrac{d_{60}}{d_{10}} < 5 \\ 0.01mm < d_e < 3mm \end{cases}$$

- **Slichter**

This formula is applicable for sedimentary rocks with effective diameter ranging between *0.01* mm and *5* mm.

$$k = A \cdot d_e^2 \cdot m \cdot \frac{1}{\eta}. \tag{8}$$

A − dimensional homogenization coefficient; $A = 88.3$;
d_e − equivalent diameter [*mm*];

$$d_e = \frac{\sum_{i=1}^{N} g_i \cdot d_i}{\sum_{i=1}^{N} g_i}. \tag{9}$$

N − number of granulometric fractions;
g_i − fraction with d_i diameter [%];
d_i − medium diameter [*mm*];

$$d_i = \frac{d_i + d_{i+1}}{2}. \tag{10}$$

d_i, d_{i+1} − upper and lower diameter for g_i fraction [*mm*];
m − Slichter's number that depends on

the porosity (figure *2*);

η − dynamic viscosity of water [*P*].

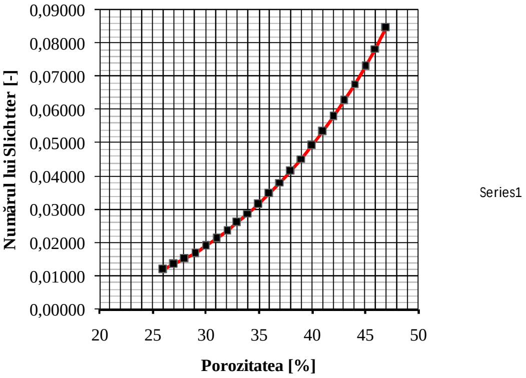

Fig. 2. *Variation of Slichter's number with porosity*

❖ Creager

This formula is valid for determining the permeability coefficient at a temperature of *10˚C*.

$$k = 322\cdot\frac{n}{(1-n)^2}\cdot d_e^2. \tag{11}$$

n − permeability [*%*];

d_e − equivalent diameter [*cm*]:

$$d_e = 100\cdot\left(\sum_{i=1}^{N}\frac{g_i}{d_i}\right)^{-1}. \tag{12}$$

N − number of granulometric fractions;

g_i − fraction with d_i diameter [*%*];

d_i diameter of medium granulometric fraction, calculated according to equation *9* [*cm*];

Good results are obtained with this formula for medium sands; not so accurate are the results for clayey sands.

❖ Zamarin

The formula can be used for determining the permeability coefficient at all sandy soils.

$$k_{10} = 5572\cdot\left[\frac{n}{(1-n)^2}\right]\cdot\alpha_1^2\cdot d_e^2. \tag{13}$$

α_1 − coefficient that consider the linked water:

$$\alpha_1 = 1.275 - 1.5\cdot n. \tag{14}$$

n − porosity [*%*];

d_e − equivalent diameter, calculated according to the relation proposed by Zamarin [*mm*]:

$$d_e = \frac{100}{\frac{2}{3}\cdot\frac{g_1}{d_1} + \sum_{2}^{N}\left(\frac{g_i}{d_{i+1}-d_i}\cdot\ln\frac{d_{i+1}}{d_i}\right)}. \tag{15}$$

g_1 − the quantity of granulometric fractions with the diameter ranging between *0* and d_1 (the finest fraction) [*%*];

d_1 − upper limit of the smallest granulometric fraction (fraction with $d < 0.0025$ *mm*) [*mm*];

N − number of granulometric fractions in which is divided the soil for calculation;

d_i, d_{i+1} − lower and upper diameter for the particles corresponding to "*i*" granulometric fraction [*mm*].

Expression of Zamarin's formula for a water temperature of *18˚C* is:

$$k = 7.94\cdot\frac{n^3\cdot C_1\cdot\tau\cdot d_e^2}{1-n^2}. \tag{16}$$

C_i − coefficient that depends on the soil porosity;

τ − correction coefficient for the water temperature;

d_e − effective diameter of the particles calculated according to equation *15* [*mm*].

To determine the permeability coefficient for clays with undisturbed and disturbed structure, Zamarin proposed the following formulas:

$$k = 3.63\cdot 10^6\,\frac{n}{\theta^2}. \tag{17}$$

$$k_{10} = 1.55 \cdot 10^{6} \frac{n}{\theta^{2}}. \tag{18}$$

θ − sum of the areas for the soil particles [cm^2]:

$$\theta = 6 \cdot (1-n) \cdot \frac{1}{100} \cdot \sum_{i=1}^{N} \frac{g_i}{d_i}. \tag{19}$$

τ − correction coefficient for water temperature.

❖ Terzaghi

$$k = \frac{6 \cdot C}{\eta} \cdot \left(\frac{n - 0.13}{\sqrt[3]{1-n}} \right)^{2} \cdot (1 + 0.034 \cdot t) \cdot d_{10}^{2}. \tag{20}$$

C − coefficient that take into consideration the characteristics of the particle surfaces; $C=10.48$ for smooth round particles and $C=6.02$ for the rough or sharp ones;
η − dynamic viscosity of water [P];
n − porosity [%];
t − water temperature [°C];
d_{10} − particle diameter which represents *10%* of the sample mass [*mm*].

This relationship can be applied to sands.

❖ Jaky

$$k = 100 \cdot d_m^{2}. \tag{21}$$

d_m − diameter with the highest frequency in the granulometric curve [*mm*].

❖ Taylor

$$k = d_e^{2} \cdot \frac{\gamma_w}{\eta} \cdot \frac{e^{3}}{1+e} \cdot C. \tag{22}$$

d_e − equivalent diameter [L];
γ_w − unit weight of water [F/L^3];
η − dynamic viscosity of water [P];
e − void ratio [-];
C − coefficient that takes into consideration the cross-section through which flow occurs.

This relation can be used for all types of soils.

❖ Shahabi – Das - Tarquin

$$k = 1.2 \cdot C^{0.735} \cdot d_{10}^{0.88} \cdot \frac{e^{3}}{1+e}. \tag{23}$$

C − coefficient depending on the particles shape [-];
d_{10} − particle diameter which represents *10%* of the sample mass [L];
e − void ratio [-].

This formula can be used for medium and fine sands.

❖ Carrier - Beckman

$$k = \frac{1}{1+e} \cdot \left(\frac{I_L + 0.242}{95.21} \right)^{4.29}. \tag{24}$$

e − void ratio [-].
I_L − liquidity index [-].

The relation can be used for soft clays.

❖ Amer - Awad

$$k = C \cdot d_{10}^{2.32} \cdot U_n^{0.6} \frac{e^{3}}{1+e}. \tag{25}$$

C − coefficient depending on the particle shape [-];
d_{10} − particle diameter which represents *10%* of the sample mass [*mm*];
U_n − coefficient of uniformity [-];
e − void ratio [-].

This method of determine the permeability coefficient may be applied to sands.

3. Results obtained for the permeability coefficient using empirical relations

This study aims to determine the precision of empirical relations that are based on granulometric analysis of different types of soils.

From the previous section it can be seen that the different equations used to determine the permeability coefficient can be applied especially for sands and clays.

To evaluate the accuracy of those formulas, the permeability for a specific sand and clay was determined. The calculations are based on laboratory tests that were made on these soils.

Thus, the analyzed sand has the following characteristics:

- granulometric composition - clay *4.5*%, silt *4.7*% and sand *90.8*%;
- water content - w = *23.13*%;
- unit weight in natural state - γ = *17.8* kN/m^3;
- unit weight in dry state - γ_d = *14.46* kN/m^3;
- skeleton unit weight - γ_s = *25.97* kN/m^3;
- porosity - n = *44.33*%;
- void ratio - e = *0.80*.

For the clay were taken into consideration the following physical characteristics:

- granulometric composition - clay *60.27*%, silt *29.10*% and sand *10.63*%;
- water content - w = *19.58*%;
- unit weight in natural state - γ = *18.90* kN/m^3;
- unit weight in dry state - γ_d = *15.79* kN/m^3;
- skeleton unit weight - γ_s = *26.19* kN/m^3;
- porosity - n = *39.75*%;
- void ratio - e = *0.66*.

For all calculations, the water temperature that flows through the soil had been considered *10*°C.

For the considered sand, several relations have been used, with the following values for the permeability coefficient:

- ❖ Allen Hazen - $k = 1.65\cdot10^{-4}$ m/s;
- ❖ Slichter - $k = 2.7\cdot10^{-4}$ m/s;
- ❖ Cre]ager - $k = 5.18\cdot10^{-4}$ m/s;
- ❖ Zamarin - $k = 3.69\cdot10^{-4}$ m/s;
- ❖ Terzaghi - $k = 1.36\cdot10^{-4}$ m/s;
- ❖ Shahabi-Das-Tarquin-$k = 1.45\cdot10^{-4}$ m/s;
- ❖ Amer-Awad - $k = 5.41\cdot10^{-4}$ m/s.

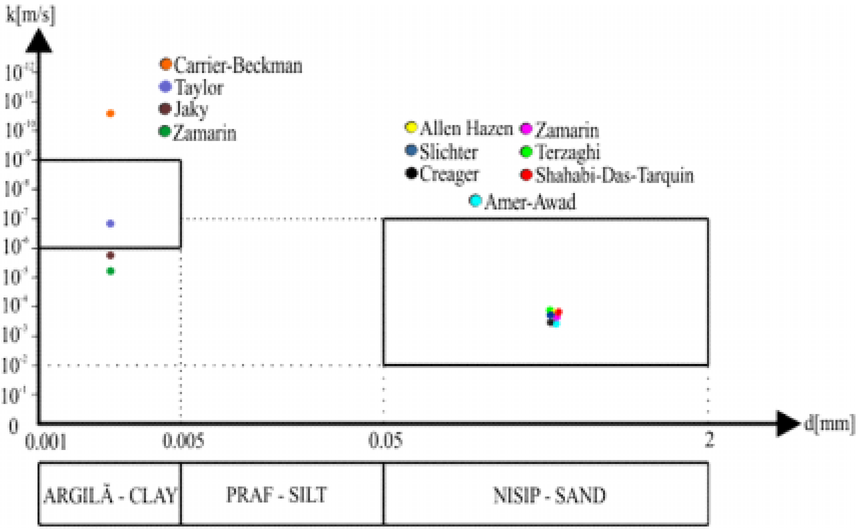

Fig. 3. *Accuracy of empirical relations used to calculate the permeability coefficient*

For the clay characterized above, the permeability coefficient values, obtained with empirical formulas, are:

- ❖ Zamarin - $k = 7.19\cdot10^{-6}$ m/s;
- ❖ Jaky - $k = 3.25\cdot10^{-6}$ m/s;
- ❖ Taylor - $k = 1.1\cdot10^{-7}$ m/s;
- ❖ Carrier-Beckman - $k = 4.36\cdot10^{-11}$ m/s.

Figure *3* illustrates the results obtained for sand and clay, in relation with the variation range of permeability coefficient for each type of soil. This variation range was established based on the studied literature.

3. Conclusions

Empirical formulas, for calculating the permeability coefficient, can help the civil engineer to achieve a fast and efficient calculation, providing valuable information about the approximate size of this parameter.

However, must be taken into consideration that the results obtained in this way don't offer a satisfying precision for significant works and the values thus obtained serve just for guidance.

It is true that this approach greatly simplifies the permeability calculation for the designer and leads to reduced costs for investigating the properties of the foundation soil, compared to *in situ* methods. However, the large number of variables involved in the calculation can lead to erroneous results. Also the precision of these results depends on the accuracy of laboratory analysis and the way in which the samples were handled.

The results presented in this paper for sand are satisfactory, falling in the variation range of permeability coefficient. The values obtained for clay are not so accurate since they do not fall in the permeability characteristic range for this type of soil.

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