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THE MODAL ANALYSIS OF 3D COMPLEX BODIES

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Abstract: This article describes a simple method for the modal analysis of 3D complex bodies using Matlab program package, because in practice there are cases where the development of some computer programs for the modal analysis of 3D complex bodies is required, especially with new materials. This paper describes the way to create such a program. The modal analysis represents the first step in the dynamic analysis of a structure, in the case that for the dynamic analysis the modal superposition is used. In this article the tetrahedron element having 3 dynamic degrees of freedom on the node and 12 dynamic degrees of freedom on the element is used for the modeling of 3D complex bodies.

Key words: finite element method, modal analysis.

1. GENERAL ASPECTS

In order to perform a modal analysis, three-dimensional bodies are subdivided into tetrahedral finite elements for which the mass and stiffness matrix are being determined. The tetrahedral finite element is the simplest finite element of the three-dimensional solid bodies that can be used in the displacement and strain field analysis. The element has four triangular sides and can be used in modeling complex-shaped bodies. In the four nodes, which represent the tetrahedron apexes, each three unknown displacements $((u_1, v_1, w_1), (u_2, v_2, w_2), (u_3, v_3, w_3), (u_4, v_4, w_4))$ are being considered. The total number of dynamic degrees of freedom is 12.

If linear equations in x y and z coordinates, are being considered for the hypothetic displacement field $u(x, y, z)$, $v(x, y, z)$ and $w(x, y, z)$, it yields:

$$\begin{aligned} u(x,y,z) &= a_1 + a_2 x + a_3 y + a_4 z; \\ v(x,y,z) &= a_5 + a_6 x + a_7 y + a_8 z; \\ w(x,y,z) &= a_9 + a_{10} x + a_{11} y + a_{12} z. \end{aligned} \tag{1}$$

After calculations, the relation between the displacement field and nodal displacements is obtained:

$$\begin{aligned} u &= N_1(x,y,z)u_1 + N_2(x,y,z)u_2 + N_3(x,y,z)u_3 + N_4(x,y,z)u_4; \\ v &= N_1(x,y,z)v_1 + N_2(x,y,z)v_2 + N_3(x,y,z)v_3 + N_4(x,y,z)v_4; \\ w &= N_1(x,y,z)w_1 + N_2(x,y,z)w_2 + N_3(x,y,z)w_3 + N_4(x,y,z)w_4. \end{aligned} \tag{2}$$

Equation (2) can be expressed under a matrix form, as following:

$$\begin{Bmatrix} u \\ v \\ w \end{Bmatrix} = \begin{bmatrix} N_1 & 0 & 0 & N_2 & 0 & 0 & N_3 & 0 & 0 & N_4 & 0 & 0 \\ 0 & N_1 & 0 & 0 & N_2 & 0 & 0 & N_3 & 0 & 0 & N_4 & 0 \\ 0 & 0 & N_1 & 0 & 0 & N_2 & 0 & 0 & N_3 & 0 & 0 & N_4 \end{bmatrix} \begin{Bmatrix} u_1 \\ v_1 \\ w_1 \\ u_2 \\ v_2 \\ w_2 \\ u_3 \\ v_3 \\ w_3 \\ u_4 \\ v_4 \\ w_4 \end{Bmatrix} = [N] \begin{Bmatrix} u_1 \\ v_1 \\ w_1 \\ u_2 \\ v_2 \\ w_2 \\ u_3 \\ v_3 \\ w_3 \\ u_4 \\ v_4 \\ w_4 \end{Bmatrix}, \tag{3}$$

where,

$[N] = \begin{bmatrix} N_1 & 0 & 0 & N_2 & 0 & 0 & N_3 & 0 & 0 & N_4 & 0 & 0 \\ 0 & N_1 & 0 & 0 & N_2 & 0 & 0 & N_3 & 0 & 0 & N_4 & 0 \\ 0 & 0 & N_1 & 0 & 0 & N_2 & 0 & 0 & N_3 & 0 & 0 & N_4 \end{bmatrix}$ is the interpolation or form functions matrix;

$\{d\}^T = \{u_1\, v_1\;\, w_1\, u_2\, v_2\;\, w_2\, u_3\, v_3\;\, w_3\, u_4\, v_4\;\, w_4\}^T$ - is the nodal displacements vector, corresponding to the finite element.

In order to determine the natural angular frequencies, the solutions of the characteristic polynom must be calculated:

$$\det([K] - \omega^2 [M]) = 0\,. \tag{4}$$

The distributed mass matrix for a tetrahedral element is:

$$[M]^{(e)} = \iiint_V [N]^T [N] \rho dV = \frac{\rho V}{20} \begin{bmatrix} 2 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 2 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 2 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 2 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 2 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 2 & 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 0 & 2 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 2 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 2 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 2 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 2 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 2 \end{bmatrix}, \tag{5}$$

where

ρ - is the material's density;

V - the tetrahedron's volume.

The consistent mass matrix of the tetrahedral element is being determined as following:

$$[M]^{(e)} = \frac{\rho V}{4}\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}. \qquad (6)$$

The way to determine the stiffness matrix is presented in [1]. After solving the characteristic equation, can be represented: natural frequencies, fig.1 and eigenmodes (fig.2 and fig.3).

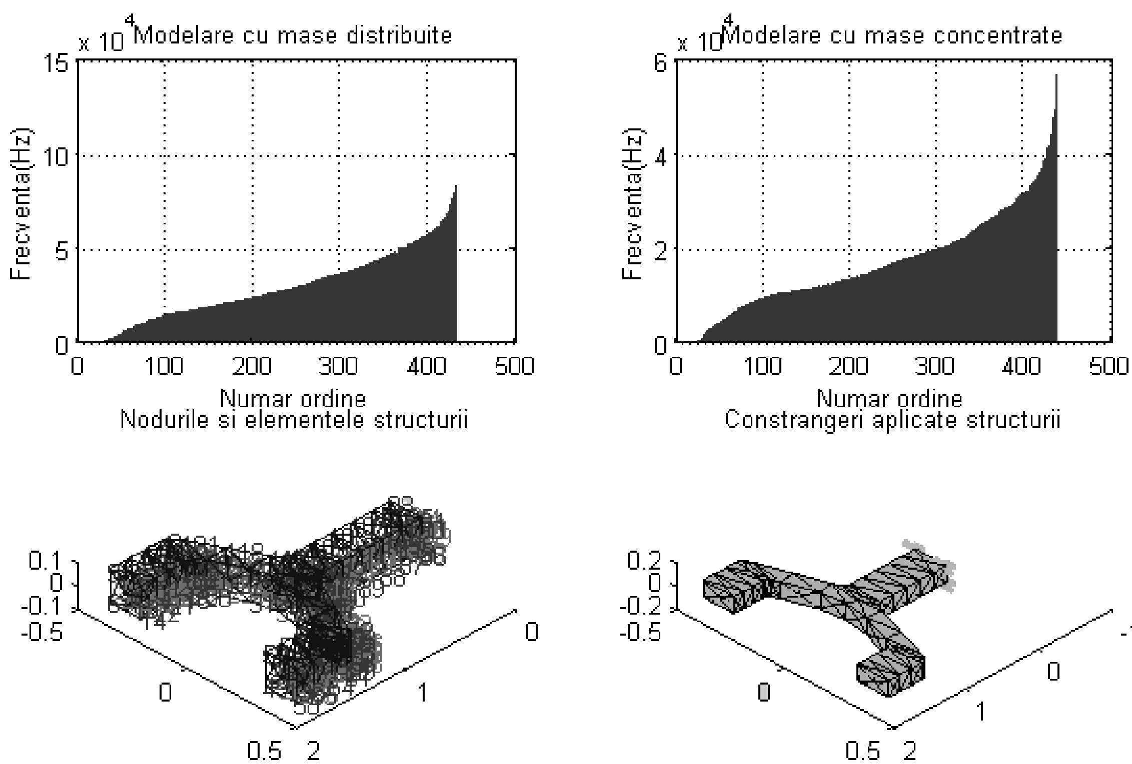

Fig.1 Representation of natural frequencies (structure modeling with concentrated and distributed mass)

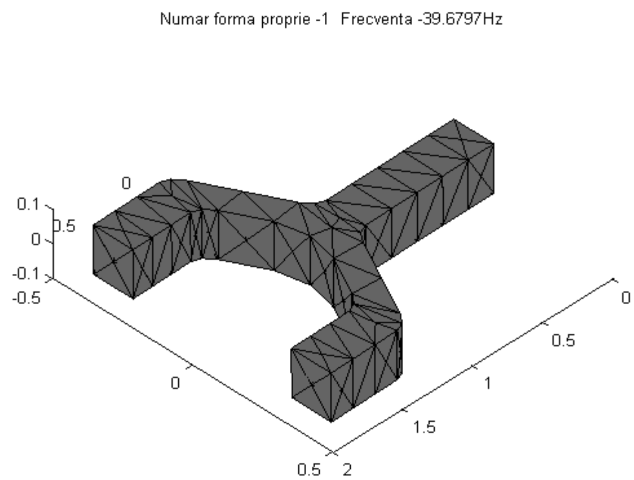

Fig.2 Eigenmode 1, structure's deformed shape

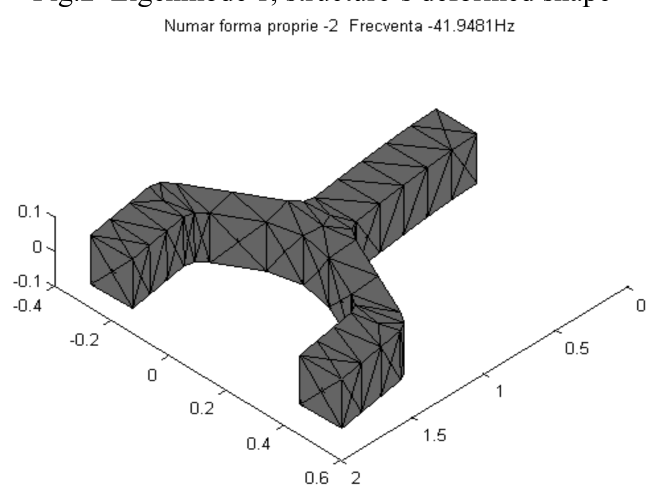

Fig.3 Eigenmode 2, structure's deformed shape

2. CONCLUSIONS AND RESULTS

The calculus procedure for the modal analysis of bodies having a complex configuration is simple and admits the modal analysis of the three-dimensional bodies having complex shapes and a large number of dynamic degrees of freedom;

For meshing three-dimensional bodies into tetrahedral finite elements, a commercial package of finite elements like ABAQUS, ANSYS or SAP2000 can be used;

The obtained results after the modal analysis can be employed in the dynamic analysis of three-dimensional bodies having a complex configuration.

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