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MATLAB PROGRAM FOR STATIONARY FLOW SIMULATION IN THE CASE OF AN IDEAL FLUID

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Abstract: In this article a procedure for a MATLAB program is presented for determining the velocity distribution in the case of a stationary flow. The program can be used for determining the velocity amplifications that appear in a fluid that flows around obstacles having different geometric shapes.

Key words: finite element method in flow simulation, stationary flow, incompressible and inviscid fluid.

1. INTRODUCTION

The main purpose of this paper is to determine the velocity distribution in the case of a stationary flow. The two-dimensional potential flow problems can be formulated in terms of a velocity potential function Φ. In terms of the velocity potential, the equation for a two-dimensional problem is given by the following:

$$\frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial y^2} = 0 \tag{1}$$

Where the velocity components are given by:

$$u = \frac{\partial \Phi}{\partial x}, \ v = \frac{\partial \Phi}{\partial y} \tag{2}$$

If an ideal fluid is considered, its motion does not penetrate into the surrounding body or separate from the surface. This gives the boundary condition that the component of the fluid velocity normal to the surface must be equal to the component of the velocity of the surface in the same direction. Therefore, the following equation yields:

$$\vec{V}\cdot\vec{n}=\vec{V}_B\cdot\vec{n} \quad \text{or} \quad ul_x+vl_y=u_Bl_x+v_Bl_y \tag{3}$$

where,

$\vec{V}$ - is the velocity of the fluid

$\vec{V}_B$ - is the velocity of the boundary

$\vec{n}$ - is the outward drawn normal to the boundary

l_x, l_y - boundary direction cosines

If the boundary is fixed ($\vec{V}_B$=0), there will be no flow and therefore no velocity perpendicular to the boundary. For that cause, all fixed boundaries can be considered as streamlines because there will be no fluid velocity perpendicular to a streamline. If $\vec{V}_B$=0, form (2) and (3) the following conditions yield:

$$\frac{\partial\Phi}{\partial n}=\frac{\partial\Phi}{\partial y}l_x+\frac{\partial\Phi}{\partial y}l_y=0 \tag{4}$$

Equation (4) indicates that the normal derivative of the potential function is zero.

The boundary value problem for potential flows can be stated as follows. In order to find the velocity potential $\Phi(x,y)$ in a given region S surrounded by the curve Γ:

$$\nabla^2\Phi=\frac{\partial^2\Phi}{\partial x^2}+\frac{\partial^2\Phi}{\partial y^2}=0 \text{ in } \Gamma \tag{5}$$

with the following boundary conditions:

- Dirichlet condition: $\Phi=\Phi_0$ on Γ_1 (6)
- Neumann condition: $V_n=\frac{\partial\Phi}{\partial n}=\frac{\partial\Phi}{\partial x}l_x+\frac{\partial\Phi}{\partial y}l_y=V_0$ on Γ_2 (7)

where: $\Gamma=\Gamma_1+\Gamma_2$

V_0 - is the prescribed value of the velocity normal to the boundary surface

In order to find the velocity potential $\Phi(x,y)$ that minimizes the functional

$$I=\frac{1}{2}\iint_S\left[\left(\frac{\partial\Phi}{\partial x}\right)^2+\left(\frac{\partial\Phi}{\partial x}\right)^2\right]\cdot dS-\int_{\Gamma_2}V_0\Phi d\Gamma_2 \tag{8}$$

we use the boundary condition:

$$\Phi=\Phi_0 \text{ on } \Gamma_1 \tag{9}$$

2. THE FINITE ELEMENT SOLUTION USING THE GARLEKIN APPROACH

The finite element analysis using the Garlekin method assumes a suitable interpolation model for $\Phi^{(e)}$ in element e as following:

$$\Phi^{(e)}(x,y)=\left[N(x,y)\right]\vec{\Phi}^{(e)}=\sum_{i=1}^{n}N_i(x,y)\vec{\Phi}_i^{(e)} \qquad (10)$$

Set the integral of the weighted residue over the region of the element equal to zero by taking the weights same as the interpolation functions N_i. This yields:

$$\iint_{S^{(e)}} N_i\left[\frac{\partial^2\Phi^{(e)}}{\partial x^2}+\frac{\partial^2\Phi^{(e)}}{\partial y^2}\right]\cdot dS=0 \quad \text{, i=1,2,...,n} \qquad (11)$$

Equation (11) can be expressed in a matrix form as following:

$$\left[K^{(e)}\right]\{\Phi^{(e)}\}=\{P^{(e)}\} \qquad (12)$$

where:

$$\left[K^{(e)}\right]=\iint_{S^{(e)}}[B]^T[D][B]\cdot dS \qquad (13)$$

$$\{P^{(e)}\}=-\int_{\Gamma_2^{(e)}}V_0[N]^T d\Gamma_2 \qquad (14)$$

$$[B]=\begin{bmatrix}\frac{\partial N_1}{\partial x} & \frac{\partial N_2}{\partial x} & \cdots & \frac{\partial N_n}{\partial x}\\ \frac{\partial N_1}{\partial y} & \frac{\partial N_2}{\partial y} & \cdots & \frac{\partial N_n}{\partial y}\end{bmatrix} \qquad (15)$$

$$\left[D\right]=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} \qquad (16)$$

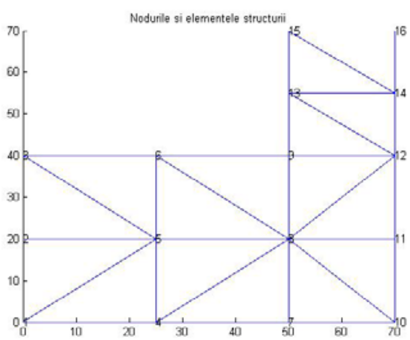

Fig.1 Finite element mesh

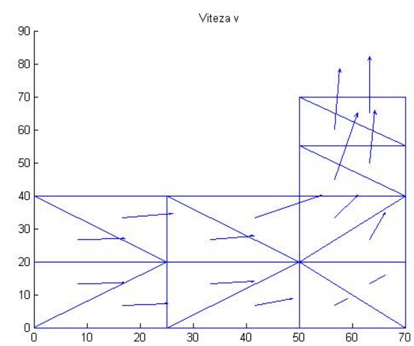

Fig.4 Resultant velocity

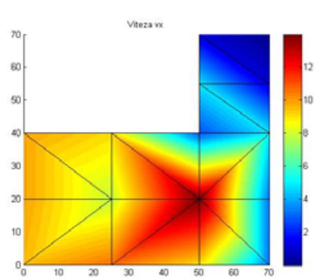

Fig.2 Velocity u

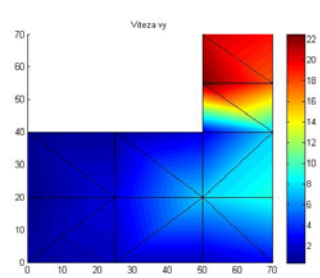

Fig.3 Velocity v

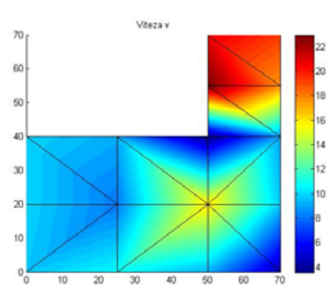

Fig.3 Resultant velocity

Assemble the element equations (12) to obtain the overall equations as:

$$[K]\{\Phi\} = \{P\} \qquad (17)$$

From figure 1 to 8, velocity distributions are presented for two flow domains obtained with the Matlab program developed by the authors.

After determining the values of the potential function Φ, one can calculate velocities in every point of the fluid flow domain.

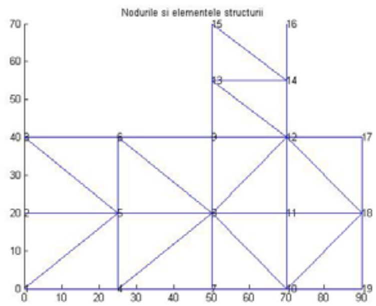

Fig.5 Finite element mesh

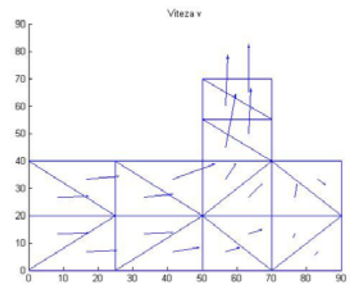

Fig.9 Resultant velocity

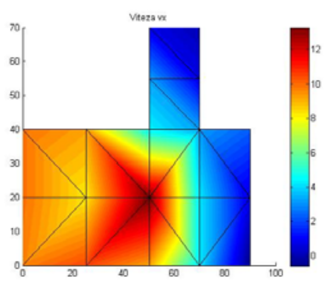

Fig.6 Velocity u

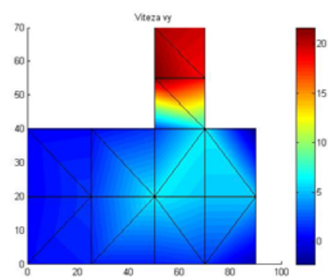

Fig.7 Velocity v

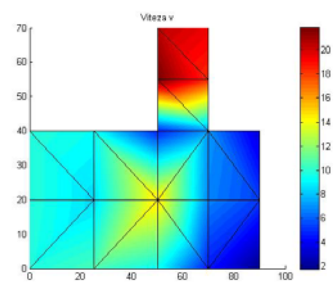

Fig.8 Resultant velocity

3. CONCLUSIONS

-The flow analysis around bodies having different geometric shapes can be done with the Matlab program developed by the authors.

-With this program, global velocity distribution in the field domain as well as local velocity amplifications can be determined.

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