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RANDOM VIBRATION OF DUFFING OSCILLATOR FOR N CORRELATION FUNCTIONS

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Abstract: *The response of a Duffing oscillator to narrow band random excitation is considered. Results obtained by applying the method linearization statistiques to random vibration problems are discussed. The equivalent linearization are found to give reasonable results only for very small non-linearities. This method is applicable to a variety of problems involving the response of lightly damped systems to broad-band random excitations. The theoretical analyses are verified by numerical results. Theoretical analyses and numerical simulations show that when the intensity of the random excitation increases.*
Keywords: *Duffing oscillator, random excitation, the power spectral density*

1. INTRODUCTION

The present approximate representation of the spectrum is applied to a nonlinear oscillator in wich the non-linearity has pronounced on the response spectrum. The effect of non-linearities on the response power spectral density has been studied by a number of investigators This method is applicable to a variety of problems involving the response of lightly damped systems to broad-band random excitations. If however, any of the basic components behave nonlinearly, the vibration is called nonlinear vibration. The differential equations that govern the behaviour of vibratory non-linear systems are non-linear.

2 SYSTEM MODEL

Consider a Duffing oscillator of which the equation is

$$m\ddot{\xi}(t)+c\dot{\xi}(t)+k\xi(t)+\varepsilon k\xi^{3}(t)=W(t) \quad \textbf{(1)}$$

where *m* is the mass, *c* is the viscous damping coefficient, *W*(*t*) is the external excitation signal with zero mean, ε is the nonlinear factor to control the type and degree of nonlinearity in the system and $\xi(t)$ is the displacement response of the system.
Dividing the equation by *m*, the equation of motion can be rewritten as:

$$\ddot{\xi}(t)+2\zeta p\dot{\xi}(t)+p^{2}\xi(t)+\varepsilon p^{2}\xi^{3}(t)=f(t), \quad \textbf{(2)}$$

where ζ is the critical damping factor and p is the undamped natural frequency, for the system.
We want to act on this oscillator random excitations narrowband random excitations products through a number of n correlation functions containing it will introduce parameters $A_1,...A_n,\varepsilon_1,...\varepsilon_n,\omega_1,...\omega_n$ real, strictly positive.

$$R_F(\tau)=A_1e^{-\varepsilon_1\tau^2}\cos\omega_1\tau+A_2e^{-\varepsilon_2\tau^2}\cos\omega_2\tau+...A_ne^{-\varepsilon_n\tau^2}\cos\omega_n\tau . \quad \textbf{(3)}$$

The parameter A_k influences directly proportional to the spectral density of initial excitation intensity and moderate printing relatively rapid variations. Increasing parameter ε_k produces excitations with increases and decreases slow spectral density. Increasing parameter ε_k widens excitation power and the drop was performed narrowing the spectrum. Excitation control parameters ω_k contribute to the excitation spectral density leves peak delayed. We say that the parameter maximum spectral density peaks moves to the right

The power spectral density of excitation W (t) is determined using the relationship

$$S_F(\mu)=\frac{1}{2®}\int_{-\infty}^{\infty}R_F(\text{²})e^{-i\mu\text{²}}d\text{²}\ . \tag{4}$$

Solving this integral the relation sends us

$$S_F(\mu)=\frac{A_1\sim_1}{®}\frac{\mu^2+\sim_1^2+¥_1^2}{\left|(i\mu)^2+2ª(i\mu)+\sim_1^2+¥_1^2\right|^2}+\frac{A_2\sim_1}{®}\frac{\mu^2+\sim_2^2+¥_2^2}{\left|(i\mu)^2+2ª(i\mu)+\sim_2^2+¥_2^2\right|^2}+...+\frac{A_n\sim_n}{®}\frac{\mu^2+\sim_n^2+¥_n^2}{\left|(i\mu)^2+2ª(i\mu)+\sim_n^2+¥_n^2\right|^2} \tag{5}$$

The power spectral density of response is

$$S_{¦}(\mu)=\left|H(\mu)\right|^2S_F(\mu)=\frac{S_F(\mu)}{(k_e-m\mu^2)^2+c^2\mu^2}=\frac{1}{m^2}\frac{S_F(\mu)}{(p_e^2-\mu^2)^2+4¶^2p^2\mu^2}\ , \tag{6}$$

Substituting equation (5) into (6), obtain

$$S_{¦}(\mu)=\left\{\frac{1}{®m^2}\frac{A_1\sim_1(\mu^2+\sim_1^2+¥_1^2)}{\left[\left(p^2-\mu^2+3\sim p^2±^2_{¦}\right)^2+4¶^2p^2\mu^2\right]\left[\left(\sim_1^2+¥_1^2-\mu^2\right)^2+4\sim_1^2\mu^2\right]}+\right.$$

$$+\frac{A_2\sim_2(\mu^2+\sim_2^2+¥_2^2)}{\left[\left(p^2-\mu^2+3\sim p^2±^2_{¦}\right)^2+4¶^2p^2\mu^2\right]\left[\left(\sim_2^2+¥_2^2-\mu^2\right)^2+4\sim_2^2\mu^2\right]}+...+$$

$$\left.+\frac{A_n\sim_n(\mu^2+\sim_n^2+¥_n^2)}{\left[\left(p^2-\mu^2+3\sim p^2±^2_{¦}\right)^2+4¶^2p^2\mu^2\right]\left[\left(\sim_n^2+¥_n^2-\mu^2\right)^2+4\sim_n^2\mu^2\right]}\right\} \tag{7}$$

We start from the known formula

$$±^2_{¦}=\int_{-\infty}^{\infty}\left|H(\mu)\right|^2S_Fd\mu. \tag{8}$$

We obtain

$$±^2_{¦}=\sum_{k=1}^{n}\frac{A_k\sim_k}{m®}\int_{-\infty}^{\infty}\frac{(\mu^2+\sim_k^2+¥_k^2)}{\left\{\left[p^2-\mu^2+3\sim p^2±^2_{¦}\right]^2+4¶^2p^2\mu^2\right\}\left[\left(\sim_k^2+¥_k^2-\mu^2\right)^2+4\sim_k^2\mu^2\right]}d\mu\ . \tag{9}$$

This formula has a integral type

$$\int_{-\infty}^{\infty}\frac{\mu^2+d}{\left|(i\mu)^2+2ª(i\mu)+d\right|^2\left|(i\mu)^2+b_1(i\mu)+b_0\right|^2}d\mu=\frac{®(b_oh_1+h_1h_2-h_3)}{b_0(h_1h_2h_3-b_oh_1^2d-h_3^3)} \tag{10}$$

where

$$h_1=b_1+2ª,\quad h_2=b_0+2ªb_1+d,\quad h_3=2ªb_0+db_1\ . \tag{11}$$

We finally get a 4 degree equation with unknown $±^2_{¦}$

$$l±^8_{¦}+n±^6_{¦}+r±^4_{¦}+s±^2_{¦}+q=0\ . \tag{12}$$

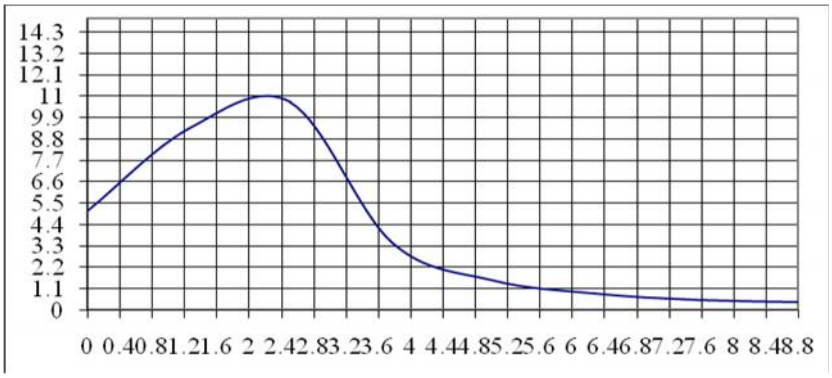
We can find a solution of the equation as a boundary of the next string

$$x_n=x_{n-1}-\frac{f(x_{n-1})}{f'(x_{n-1})},\quad \forall n\geq 1. \tag{13}$$

3. NUMERICAL RESULTS

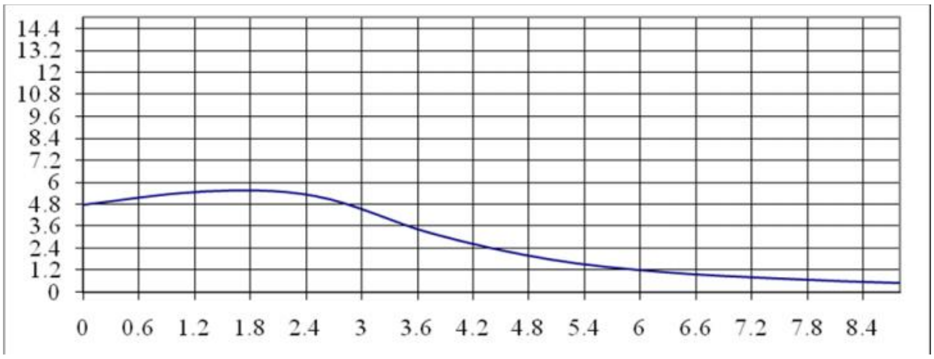
We consider a two component excitation and

$$m = 1kg, k = 30\frac{N}{m}, c = 3£\frac{Ns}{m}, £ = 3m^{-2},$$
$$A_1 = 50N^2, A_2 = 50N^2, \sim_1 = 1s^{-1}, \sim_2 = 1{,}5s^{-1}, ¥_1 = ¥_2 = 3s^{-1}. \qquad \textbf{(14)}$$



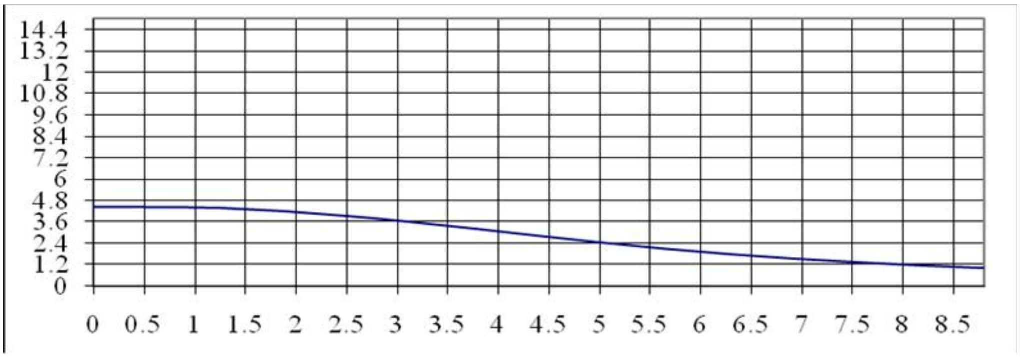
$\frac{μ}{2®}$

Figure 1: The power spectral density of excitation $S_F\ [N^2 \cdot s]$ for $A_1 = A_2 = 40\ N^2, \sim_1 = \sim_2 = 1s^{-1}, ¥_1 = ¥_2 = 2s^{-1}, n = 2.$



$\frac{μ}{2®}$

Figure 2: The power spectral density of excitation $S_F\ [N^2 \cdot s]$ for $A_1 = A_2 = 30\ N^2, \sim_1 = \sim_2 = 2s^{-1}, ¥_1 = ¥_2 = 2s^{-1}, n = 2.$



$\frac{μ}{2®}$

Figure 3: The power spectral density of excitation $S_F\ [N^2 \cdot s]$ for $A_1 = A_2 = 20\ N^2, \sim_1 = \sim_2 = 3s^{-1}, ¥_1 = ¥_2 = 1s^{-1}, n = 2.$

We obtain

$$1458\cdot10^{3}\pm^{8}_{¦} \quad +6231{,}91\cdot10^{4}\pm^{6}_{¦} \quad +7254{,}52\cdot10^{3}\pm^{4}_{¦} \quad +1250\pm^{2}_{¦} \quad -143=0 \qquad \mathbf{(15)}$$

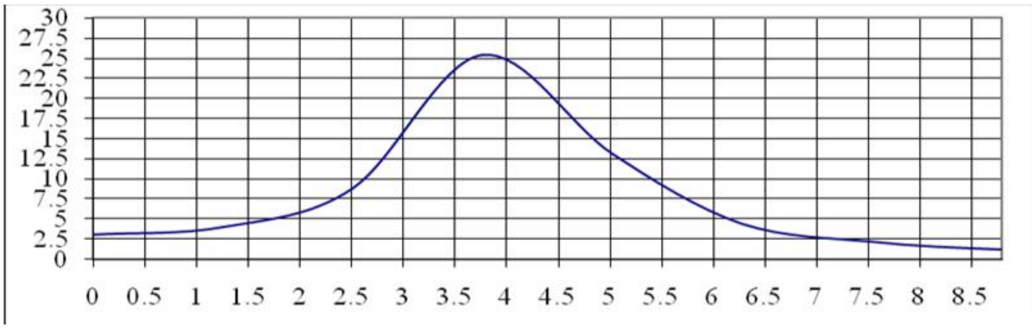
which has the solution

$$\pm^{2}_{¦} = 0{,}041m^{2}. \qquad \mathbf{(16)}$$

$\frac{\mu}{2®}$

Figure 4: The power spectral density of excitation $S_F\ [N^2\cdot s]$

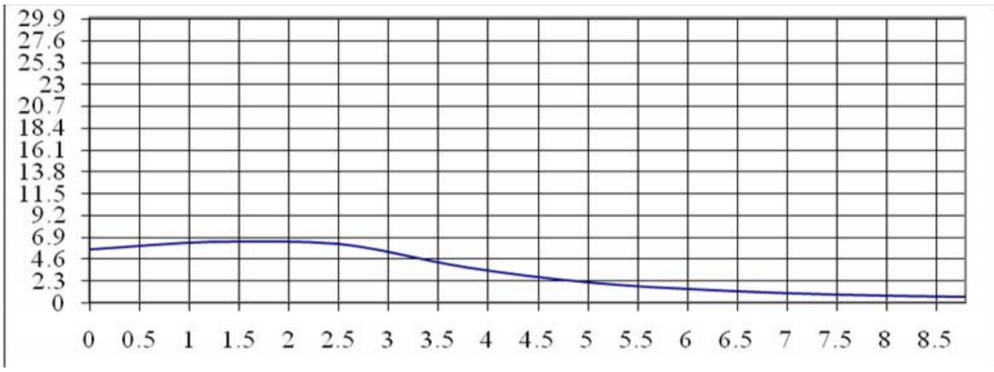
for $A_1 = A_2 = 20\ N^2, \sim_1 = \sim_2 = 2s^{-1}, ¥_1 = ¥_2 = 2s^{-1}, n = 3.$



$\frac{\mu}{2®}$

Figure 5:.The power spectral density of excitation $S_F\ [N^2\cdot s]$

for $A_1 = A_2 = 55\ N^2, \sim_1 = \sim_2 = 1s^{-1}, ¥_1 = ¥_2 = 4s^{-1}, n = 3.$



$\frac{\mu}{2®}$

Figure 6: The power spectral density of excitation $S_F\ [N^2\cdot s]$

for $A_1 = A_2 = 70\ N^2, \sim_1 = \sim_2 = 2s^{-1}, ¥_1 = ¥_2 = 2s^{-1}, n = 1.$

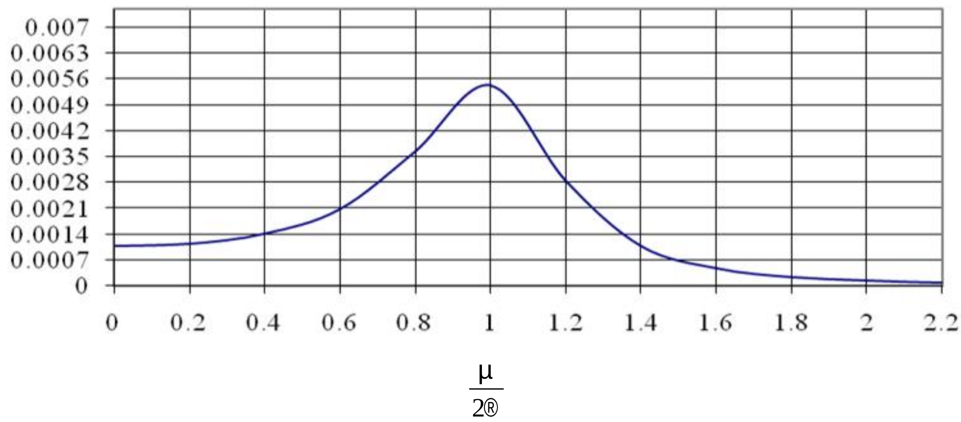


Figure 7: The power spectral density of response $S_{ı}\,[m^2 \cdot s]$ for $m = 1kg, k = 30\frac{N}{m}, c = 3\frac{Ns}{m}, \sim\, = 3m^{-2}$.

4. CONCLUSION

The theoretical analyses are verified by numerical results. Theoretical analyses and numerical simulations show that when the intensity of the random excitation increases. A second-order closure method is presented for determining the response of non-linear systems to random excitations. The random excitation is taken to be the sum of a deterministic harmonic component and a random component. The presence of the nonlinearity causes multi-valued regions where more than one mean-square value of the response is possible. Various applications of the theory to engineering problems are outlined.

Using computer diagrams below its trend highlighted how the power spectral density given by equation (6). The parameter $A_k\left[N^2\right]$ influences directly proportional to the spectral density of initial excitation intensity and moderate printing relatively rapid variations (fig. 1, 2, 3). Increasing parameter $\sim_k\left[s^{-1}\right]$ produces excitations with increases and decreases slow spectral density. Increasing parameter $\sim_k$ widens excitation power (fig. 4, 5, 6), and the drop was performed narrowing the spectrum. Excitation control parameters $¥_k\left[s^{-1}\right]$ contribute to the excitation spectral density levels peak delayed. (fig. 3, 4).

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