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NONLINEAR VIBRATIONS IN AUTOMOTIVE SUSPENSIONS

Petre Stan[1], Marinică Stan[2]
[1]University of Pitesti , Romania, email: petre_stan_marian@yahoo.com
[2] University of Pitesti , Romania, email: stan_mrn@yahoo.com

Abstract *The object of the paper is to offer a brief survey of numerical simulations in nonlinear vibration systems. As nonlinear equations are difficult to solve, nonlinear systems are commonly approximated by linear equations Recently, considerable attention has been paid towards approximate solutions for analytically solving nonlinear differential equation.A new analytical method has been presented for approximating the stationary response statistics of a class of DDOF non-linear dynamical systems under stochastic white-noise excitation. This paper considers the response of a nonlinearizear string to random excitation.The developme:nt of computational methods provided an opportunity to create various kinds of software which are able to analyse the properties of the above mentioned systems.*
Keywords: *Nonlinear vibration systems, numerical simulation.*

1. INTRODUCTION

Recently, considerable attention has been paid towards approximate solutions for analytically solving nonlinear differential equation. A new analytical method has been presented for approximating the stationary response statistics of a class of DDOF non-linear dynamical systems under stochastic white-noise excitation. The most commonly applied and convenient procedure uses Yingfang, L. Zhao, Q. Chen suggestion to estimate the linearization coefficients in context with Wen's introduction of an analytical expression for the restoring force. In contrast to existing methods based on minimizing the error in the stochastic equation of motion, the proposed method minimizes a measure of the In particular, it gives considerably better estimates for the mean-square response than the partial linearization method. While some particular exact solutions are available for specific systems under white noise excitations, many practical problems have been handled by approximate approaches such as linearization and closure assumptions.

2 SYSTEM MODEL

This method has seen the broadest application because of their ability to accurately capture the response statistics [1,3,4] over a wide range of response levels while maintaining relatively light computational burden. The method will be briefly discussed in the following sections To illustrate the procedure of equivalent linearization theory, let us consider the following oscillator with a nonlinear damping force component. The excitation of the system assumed to be a Gaussian white noise with a constant spectrum S_0. The simplified 4-degree-of-freedom model [3] of a suspension system is shown in Fig.1. Model the transverse motion of a vehicle due to a bump in the road as the response due to an impulse of magnitude *I* applied to the front. Equations of motion [3,5,6], while neglecting very small terms we get

$$\begin{pmatrix} \frac{v^2M+I}{(u+v)^2} & \frac{uvM-I}{(u+v)^2} & 0 & 0 \\ \frac{uvM+I}{(u+v)^2} & \frac{v^2M+I}{(u+v)^2} & 0 & 0 \\ 0 & 0 & m & 0 \\ 0 & 0 & 0 & m \end{pmatrix} \begin{pmatrix} \ddot{x}_1 \\ \ddot{x}_2 \\ \ddot{x}_3 \\ \ddot{x}_4 \end{pmatrix} + \begin{pmatrix} c_1 & 0 & -c_1 & 0 \\ c_2 & 0 & -c_2 & 0 \\ -c_3 & 0 & c_1+c_3 & 0 \\ 0 & -c_2 & 0 & c_2+c_4 \end{pmatrix} \begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{pmatrix} +$$

$$+\begin{pmatrix} k_1 & 0 & -k_1 & 0 \\ 0 & k_2 & 0 & -k_2 \\ -k_1 & 0 & k_1+k_3 & 0 \\ 0 & -k_2 & 0 & k_2+k_4 \end{pmatrix}\begin{pmatrix} ¦_1 \\ ¦_2 \\ ¦_3 \\ ¦_4 \end{pmatrix} + \sim \begin{pmatrix} k_1 & 0 & -k_1 & 0 \\ 0 & k_2 & 0 & -k_2 \\ -k_1 & 0 & k_1+k_3 & 0 \\ 0 & -k_2 & 0 & k_2+k_4 \end{pmatrix}\begin{pmatrix} ¦_1^3 \\ ¦_2^3 \\ ¦_3^3 \\ ¦_4^3 \end{pmatrix} = \begin{pmatrix} \frac{W(t)}{4} \\ \frac{W(t)}{4} \\ \frac{W(t)}{4} \\ \frac{W(t)}{4} \end{pmatrix}. \qquad \textbf{(1)}$$

$W(t)$ is the external excitation signal with zero mean and $¦(t)$ is the displacement response of the system. The signal $¦(t)$ has a finite number of finite discontinuities [1,6,7] and contains a finite number of maxima and minima.

For large values of the $\dot{¦}$ damping force is positive, the curve has a positive slope and energy is removed from the system. On the other hand, for small values of the $\dot{¦}$ damping force is negative, the curve has a negative slope and energy is put into the system.

The stochastic linearization technique can be considered to be an extension of the equivalent linearization method for the treatment of nonlinear systems under deterministic excitations.

Based on the numerical studies of the rolling ship motion and the non-linearly damped Duffing oscillator [2,7,9,10], the proposed method is shown to be promising for providing computationally efficient and relatively accurate estimates of the stationary PDF, the mean-square response, and the out crossing rates.

The idea of linearization [13,14,15] is replacing the equation by a linear system. The linear equation can be write

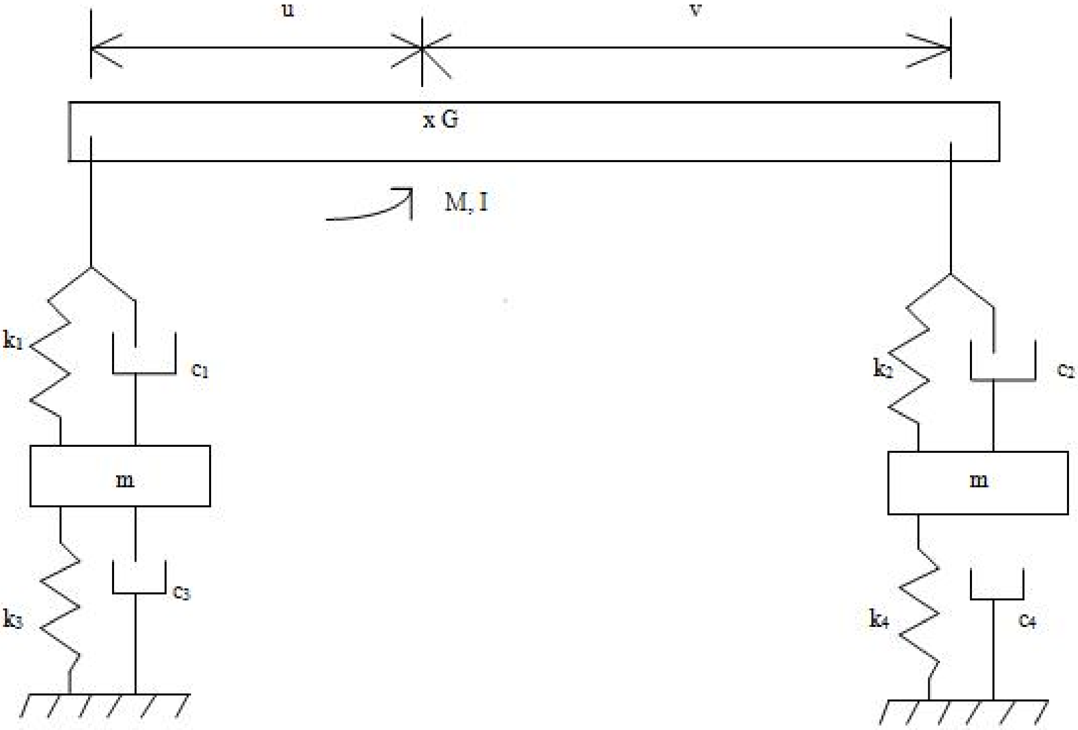


Figure. 1: 4-degree-of-freedom model of a suspension.

$$\begin{pmatrix} \frac{v^2M+I}{(u+v)^2} & \frac{uvM-I}{(u+v)^2} & 0 & 0 \\ \frac{uvM+I}{(u+v)^2} & \frac{v^2M+I}{(u+v)^2} & 0 & 0 \\ 0 & 0 & m & 0 \\ 0 & 0 & 0 & m \end{pmatrix} \begin{pmatrix} \ddot{¦}_1 \\ \ddot{¦}_2 \\ \ddot{¦}_3 \\ \ddot{¦}_4 \end{pmatrix} + \begin{pmatrix} c_1 & 0 & -c_1 & 0 \\ c_2 & 0 & -c_2 & 0 \\ -c_3 & 0 & c_1+c_3 & 0 \\ 0 & -c_2 & 0 & c_2+c_4 \end{pmatrix} \begin{pmatrix} \dot{¦}_1 \\ \dot{¦}_2 \\ \dot{¦}_3 \\ \dot{¦}_4 \end{pmatrix} +$$

$$+\begin{pmatrix} k_{1e} & 0 & -k_{1e} & 0 \\ 0 & k_{2e} & 0 & -k_{2e} \\ -k_{1e} & 0 & k_{1e}+k_{3e} & 0 \\ 0 & -k_{2e} & 0 & k_{2e}+k_{4e} \end{pmatrix} \begin{pmatrix} ¦_1 \\ ¦_2 \\ ¦_3 \\ ¦_4 \end{pmatrix} = \begin{pmatrix} \frac{W(t)}{4} \\ \frac{W(t)}{4} \\ \frac{W(t)}{4} \\ \frac{W(t)}{4} \end{pmatrix}. \qquad \textbf{(2)}$$

The difference between the nonlinear stiffness [7,8] and linear stiffness terms is

$$e=\begin{pmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \end{pmatrix}, \qquad \textbf{(3)}$$

where

$$e_1=k_1¦_1-k_1¦_3+\sim k_1¦_1^3-\sim k_1¦_3^3-k_{1e}¦_1+k_{1e}¦_3 \qquad \textbf{(4)}$$

$$e_2=k_2¦_2-k_2¦_4+\sim k_2¦_2^3-\sim k_2¦_4^3-k_{2e}¦_2+k_{2e}¦_4 \qquad \textbf{(5)}$$

$$e_3=-k_1¦_1+(k_1+k_3)¦_3-\sim k_1¦_1^3+\sim(k_1+k_3)¦_3^3+k_{1e}¦_1-(k_{1e}+k_{3e})¦_3 \qquad \textbf{(6)}$$

$$e_4=-k_2¦_2+(k_2+k_4)¦_4-\sim k_2¦_2^3+\sim(k_2+k_4)¦_4^3+k_{2e}¦_2-(k_{2e}+k_{4e})¦_4 \qquad \textbf{(7)}$$

The value of k_{1e}, k_{2e}, k_{3e}, k_{4e} can be obtained [8] by minimizing the expectation of the square error

$$\frac{dE\{e_1^2\}}{dk_{1e}}=0, \qquad \textbf{(8)}$$

$$\frac{dE\{e_2^2\}}{dk_{2e}}=0, \qquad \textbf{(9)}$$

$$\frac{dE\{e_3^2\}}{dk_{3e}}=0, \qquad \textbf{(10)}$$

$$\frac{dE\{e_4^2\}}{dk_{4e}}=0. \qquad \textbf{(11)}$$

Because

$$E\{¦^6\}=\frac{(\pm_{¦}\sqrt{2})^6}{\sqrt{®}}\left[-\frac{1}{2}e^{-u^2}u^5\Big|_{-\infty}^{\infty}+\frac{5}{2}\int_{-\infty}^{\infty}u^4e^{-u^2}du\right]=$$
$$=\frac{5(\pm_{¦}\sqrt{2})^6}{2\sqrt{®}}\int_{-\infty}^{\infty}u^4e^{-u^2}du=5\pm^2E\{¦^4\} \qquad \textbf{(12)}$$

$$E\{¦^4\}=3\pm^2E\{¦^2\}, \qquad \textbf{(13)}$$

$$E\{¦^2\}=\pm^2, \qquad \textbf{(14)}$$

obtain

$$k_{1e}=k_1+\frac{3\sim k_1(\pm_1^4+\pm_3^4)}{\pm_1^2+\pm_3^2} \qquad \textbf{(15)}$$

$$k_{2e}=k_2+\frac{3\sim k_2(\pm_2{}^4+\pm_4{}^4)}{\pm_2{}^2+\pm_4{}^2} \tag{16}$$

$$k_{3e}=k_3+3\sim\pm_3{}^2(k_1+k_3) \tag{17}$$

$$k_{4e}=k_4+3\sim\pm_4{}^2(k_2+k_4) \tag{18}$$

Using the Fourier transform [9,10] of equation (2.) and having the relations

$$\mathrm{F}\ (\dot{¦}_k(t))=i\mu\,\bar{¦}_k(\mu),\quad k=1,...,4. \tag{19}$$

$$\mathrm{F}\ (\ddot{¦}_k(t))=i\mu\ \mathrm{F}\ (\dot{¦}_k(t))=-\mu^2\,\bar{¦}_k(\mu) \tag{20}$$

$$\mathrm{F}\ ((F(t))=\bar{F}(\mu)\,, \tag{21}$$

obtain the system

$$\begin{cases} \bar{¦}_1\left(ic_1\mu+k_{1e}-\dfrac{v^2M+I}{(u+v)^2}\mu^2\right)-\dfrac{uvM-I}{(u+v)^2}\mu^2\,\bar{¦}_2-k_{1e}\,\bar{¦}_3=\dfrac{\bar{F}(\mu)}{4} \\ \bar{¦}_1\left(ic_2\mu-\dfrac{uvM+I}{(u+v)^2}\mu^2\right)-\left(\dfrac{v^2M+I}{(u+v)^2}\mu^2+k_{2e}\right)\bar{¦}_2-ic_3\mu\,\bar{¦}_3-k_{1e}\,\bar{¦}_3=\dfrac{\bar{F}(\mu)}{4} \\ \bar{¦}_1\left(-ic_3\mu+k_{1e}\right)+\left[-m\mu^2+(c_1+c_3)i\mu+(k_{1e}+k_{3e})\right]\bar{¦}_3=\dfrac{\bar{F}(\mu)}{4} \\ \bar{¦}_2\left(-ic_2\mu-k_{2e}\right)+\left[-m\mu^2+(c_2+c_4)i\mu+(k_{2e}+k_{4e})\right]\bar{¦}_4=\dfrac{\bar{F}(\mu)}{4} \end{cases} \tag{22}$$

We obtain the frequency response function [1,2] of the system.
The mean square value for the displacement [3,9,10] of the system is given by equation

$$\pm^2_{¦_k}=R_{¦_k}(0)=\int_{-\infty}^{\infty}\left|H_k(\mu)\right|^2 m_k{}^2S'_0d\mu=\frac{1}{m_k{}^2}\int_{-\infty}^{\infty}\left|H_k(\mu)\right|^2 m_k{}^2S'_0d\mu\,. \tag{23}$$

The power spectral density of response [13,14] for the first structure (in $m^2\cdot s$) is given by equation

$$S_k(\mu)=\left|H_k(\mu)\right|^2 S_F=\left|H_k(\mu)\right|^2 m_k{}^2S'_0(\mu)=\left|\frac{1}{m_k}H_k(\mu)\right|^2 m_k{}^2S'_0(\mu)=\left|H_1(\mu)\right|^2 S'_0(\mu) \tag{24}$$

3 NUMERICAL RESULTS

For example, we consider

$$u=2m,\quad v=2m,\ m=25kg,\ M=175kg,\ k_1=k_2=3\times10^5\frac{N}{m},\ k_3=k_4=1\times10^5\frac{N}{m}, \tag{25}$$

$$I=150kg\times m^2,\ c_1=c_2=3000\frac{N\times s}{m},\ c_3=c_4=750\frac{N\times s}{m},\ \sim=97,7m^{-2}. \tag{26}$$

Substituting given values, the mass, damping, and stiffness matrices are

$$\mathbf{M}=\begin{pmatrix} 53,125 & 34,375 & 0 & 0 \\ 53,125 & 53,125 & 0 & 0 \\ 0 & 0 & 25 & 0 \\ 0 & 0 & 0 & 25 \end{pmatrix} \tag{27}$$

$$\mathbf{C}=\begin{pmatrix} 3000 & 0 & -3000 & 0 \\ 3000 & 0 & -3000 & 0 \\ -3000 & 0 & 3750 & 0 \\ 0 & -3000 & 0 & 3750 \end{pmatrix} \tag{28}$$

$$\mathbf{K}=\begin{pmatrix} 3\times10^5 & 0 & -3\times10^5 & 0 \\ 0 & 3\times10^5 & 0 & -3\times10^5 \\ -3\times10^5 & 0 & 4\times10^5 & 0 \\ 0 & -k_2 & 0 & 4\times10^5 \end{pmatrix} \qquad \textbf{(29)}$$

The displacement variances [4,6] of the system under Gaussian white noise excitation can be expressed as,

$$\pm^2_{¦1} = 62,5\cdot10^{-5} m^2, \qquad \pm^2_{¦2} = 62,5\cdot10^{-5} m^2, \qquad \textbf{(30)}$$

$$\pm^2_{¦1} = 47\cdot10^{-5} m^2, \qquad \pm^2_{¦2} = 47\cdot10^{-5} m^2, \qquad \textbf{(31)}$$

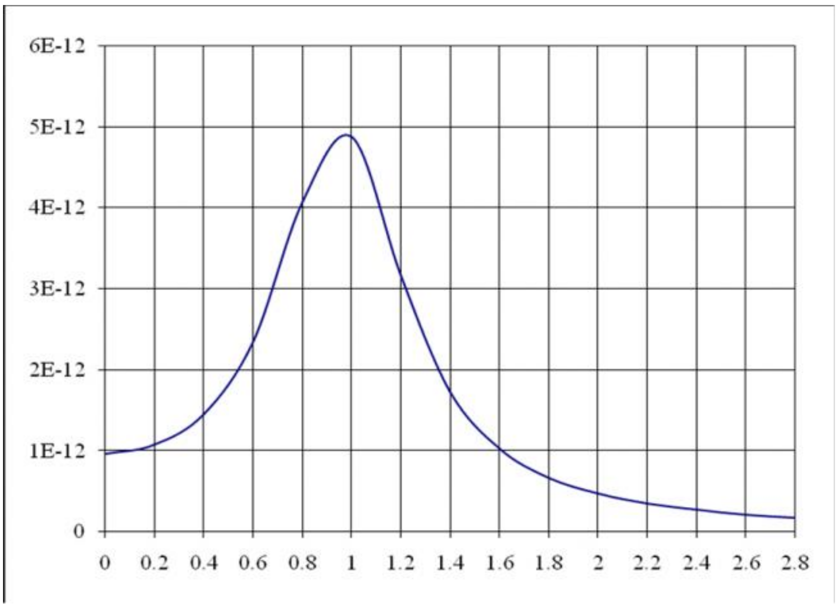
For the value of k_e, obtain

$$k_{1e} = k_1 + \frac{3\sim k_1(\pm_1{}^4 + \pm_3{}^4)}{\pm_1{}^2 + \pm_3{}^2} = 3,48\times10^5 \frac{N}{m}, \qquad \textbf{(32)}$$

$$k_{2e} = k_2 + \frac{3\sim k_2(\pm_2{}^4 + \pm_4{}^4)}{\pm_2{}^2 + \pm_4{}^2} = 3,48\times10^5 \frac{N}{m}, \qquad \textbf{(33)}$$

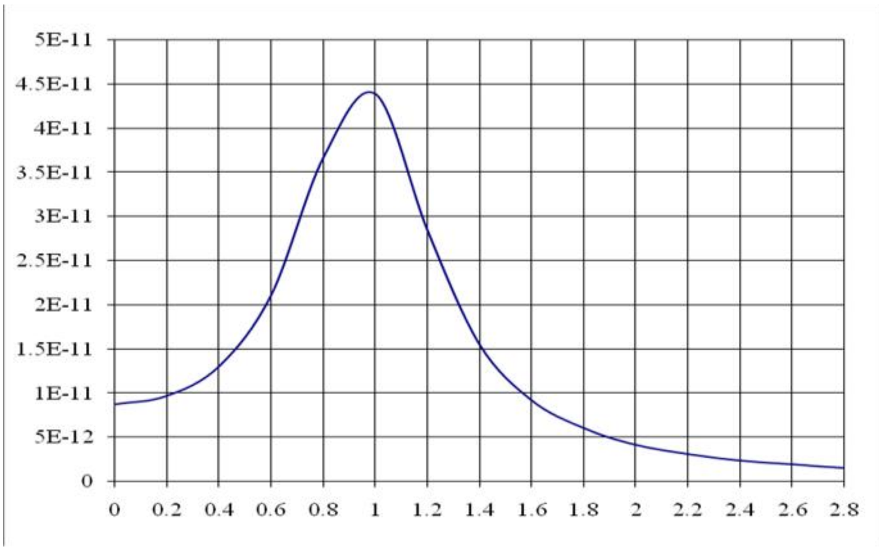
$$k_{3e} = k_3 + 3\sim \pm_3{}^2 (k_1 + k_3) = 1,23\times10^5 \frac{N}{m}, \qquad \textbf{(34)}$$

$$k_{4e} = k_4 + 3\sim \pm_4{}^2 (k_2 + k_4) = 1,23\times10^5 \frac{N}{m}. \qquad \textbf{(35)}$$



$$\frac{\mu}{2®}$$

Figure 2: The power spectral density of the response $S_k(\mu)\ [m^2 \cdot s], k = 1, k = 2.$



$$\frac{\mu}{2®}$$

Figure 3: The power spectral density of the response $S_k(\mu)\ [m^2 \cdot s], k = 3, k = 4.$

4. CONCLUSION

This nonlinearity is one of the reasons why accurate long-term forecasts are impossible with current technology. In research of vehicle system dynamics, investigation of parametral sensitivity of nonlinear systems is considered a very important question either in stability problems or in respect of chaotic behaviour. In studies of that purpose the computer aided numerical simulations are getting more and more important even if they consider relatively simple models.

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