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THE ANALYSIS OF AN ANALOGOUS HUYGENS PENDULUM CONNECTED WITH I.T.C. USING FLEXIBLE MULTIBODY DYNAMIC SIMULATIONS

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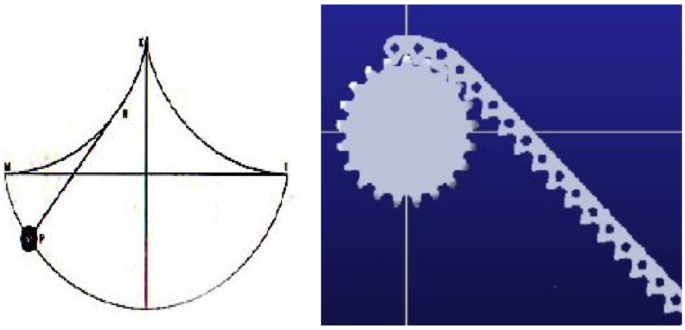
Abstract:*This paper discusses the use of the Analogous Huygens Pendulum model used to investigate the dynamic behavior of the Inverted Tooth Chain (I.T.C.), where the I.T.C.is extensively applied in automotive and conveyor industries. The analysis is based on the flexible multibody dynamics approach and is expected to give a whole new perspective in evaluating the Analogous Huygens Pendulum system in order to find the different modes and natural frequencies of the system.*
The relevance of using flexible multibody dynamics analysis is that it would simply show the most affected zones subjected to wear and deterioration of the I.T.C. in the Analogous Huygens Pendulum system due to different types of stresses whether on the sprocket, or on the plates, or on the contacts between them.The model developed and presented in the paper is made of flexible bodies and allows investigating the system behavior from the point of view of contacts and stresses.The model developed and presented in the paper is made of flexible bodies and allows investigating the system behavior from the point of view of contacts and stresses.
Keywords: *Flexible Bodies, Contacts, Kinetic Energy, Damping Form*

1. INTRODUCTION

Some common problems in chain drive systems, not related to the chain architecture (bush, roller or I.T.Chain) appear due to the contact between the chain elements and the sprocket teeth. Therefore, high contact forces are created that produce vibrations and noise.
In order to understand more profoundly the contacts between the plates of the I.T. Chain and the sprocket, a multibody dynamics simulation software MSC ADAMS and Nastran will be used for simulating the flexible plates dynamic behaviour using the multi-body theory. The general purpose is to understand the behaviour in different situations of an I.T. Chain by applying different conditions such as changing the geometries of the plates, which can be symmetric or asymmetric,oraligning the plates to the sprocket. This study is based on the Analogue Huygens Pendulum effect[1],which is caused by a fixed centroid around which the chain oscilates against the sprocket. A mono-involute motion is being created. In general, the tangent to an involute motion is normal to the evolution of the intersections of the curves made by the motion of the chain[2].
Most of the analysis will be made on flexible bodies which are in contact to make the simulations as real as possible. After knowing what forms will the contact havedifferent positions would be analized. It becomes easier through flexible body analysis to know what are the general mass, general stiffness and the damping ratio of the system. Also to understand the different distribution of the stresses that occur during free oscillation and contacts.



a) b)

Figure 1: Huygens Pendulum [2] (a); the model of the studied analogue Huygens Pendulum (b)

2. THEORETICAL BACKGROUND

In order to understand the flexible multibody methodology used it is important to define the flexible bodies. There are many approaches to define flexible bodies and maybe one of the most important definition from the point of view of multibody dynamics is that of Shabana [3].

2.1 Flexible Body Definition

Shabana defined flexible bodies from the moving frame approach within the setting provided by the reference point coordinates, in 1989. The natural coordinates of the bodyisuniquivocally used to define the large overall rigid bodies motion to which the elastic deformations variables are referred. Overall one can conclude that flexible bodies can be defined as rigid bodies multiplied by the coefficient of the elasticity of the body to express the deformation caused on the body [3].

The natural coordinates of a body do not include relative translation or rotations and are subjected to the corresponding rigid body constraints. The constraints in flexible bodies are different than that of rigid bodies as points and vectors can't be shared at the joints because the elastic deformations should be included.

Floating frame reference formulation suggested by Shabana [4] is presented below.

In general the global position coordinates of an arbitrary point "p" on a body "i" can be donated by q^i, which can be represented as $q^i = \begin{bmatrix} R^i \\ \theta^i \\ q_f^i \end{bmatrix}$, where Ri and θ^i are reference coordinates and q_f^i represents the elasticcoordinates.

One can write the general equation of a point displacement of a body as shown in the equation below [4]:

$$\overline{r_i} = R^i + A(\overline{U}^i), \tag{1}$$

where:

A is the transformation matrix

R^i is the reference coordinate

$\overline{U}^i$ is general displacement of the body "i"

If one will add the element of deformability of the body, then the equation can be writtenby adding the vector of elasticity of the flexible body,knowing in general that the modal matrix for a point "i" has translational (S_T^i) and rotational (S_R^i) modes meaning that [4]:

$$S^i = [S_T^i \quad S_R^i], \tag{2}$$

where $\overline{U}^i$ can be (written in) expressed by the following equation[4]:

$$\overline{U}^i = S^i q_f^i. \tag{3}$$

$S^i q_f^i$ is the vector of elastic generalized displacement of body "i". By replacing equation 3 in equation, the general displacement of deformed bodies is obtained, as shown in the equation below [4]:

$$\overline{r_i} = R^i + A^i(\overline{U}_0^i + S^i q_f^i). \tag{4}$$

The Kinetic energy can be written in the following form [4]:

$$T^i = \frac{1}{2}\dot{q}^{iT} M^i \dot{q}^i, \tag{5}$$

where $M^i$is the symmetric mass matrix of the body "i" in the MultibodySystem.

2.2 Theoretical approach of contact modeling

Regarding contact modeling, there are many theories about how to obtain reasonable results of modeling contacts betweenbodies. This is mainlybecause more complicated systems have a huge number of DOFs due to many bodies that can fall in contact with one another and are in motion relatively with each other. Literature has assumed three main types of contacts according to the rigidity or flexibility of the parts in contacts such as:rigid-rigid bodies, flexible-rigid bodies and flexible-flexible bodies. One main and important reason to study contacts is to see the propagation of stresses and strains affecting the parts during impacts or contacts. This can only appear when one of the bodies is considered flexible or deformable.

A contact between rigid bodies can only approximate the values and shape of a contact adequate to the system. Since parts are deformable, the contacts of rigid bodies are clearly not completely accurate to represent the deformation occurred during contacts and can't find the propagation of impact stresses in the bodies.

Many theoretical approached for modelingcontacts can be found in the literature. Perhaps the most commonly used is the Hertezian theory for elastic contacts [5];itis often the basic law for direct and central impacts between rigid bodies having locally a contact surface which can be described by a quadratic function. The Hertezian law represents the contact force magnitude as a nonlinear function of the normal penetration "l" with a contact stiffness "k_p" where the contact force "f" can be represented as following [5]:

$$f = k_p l^n, \tag{6}$$

where"n" is the exponent of penetration between materials of different penalization having elliptical contacts.

The contact stiffness parameter depends on material properties as for the shape of the contact surface. For example if we take two spheres in contact, the contact stiffness can be given as [5]:

$$K_p = \frac{4}{3(\sigma_1+\sigma_2)}\sqrt{\frac{R_1 R_2}{R_1+R_2}} \ , \tag{7}$$

whereR_1 and R_2 are radii of the spheres, the material properties σ_i are computed from Young's modulus "E_i" and the Poisson's ratio "υ_i" of the body materials through the equation below [5]:

$$\sigma_i = \frac{1-\gamma^2}{E_i}. \tag{8}$$

Hertz's laws are limited to isotropic elastic bodies and donot account for kinetic energy losses. In order to include the energy dissipation during the contact process, the Kelvin Voigt model was created,statingthat the model consists of a linear spring to represent the elastic force and a linear damper to model the energy loss, [5,6]:

$$f = k_p l \quad + c\dot{l} \ , \tag{9}$$

where

c is the damping coefficient or hysteresis coefficient,

$\dot{l}$is the relative normal contact velocity.

The major drawback in the Kelvin Voigt contact model is the linearity of the contact force with respect to the load indentation; therefore the nonlinear nature of deformation is not correctly represented.

Hunt and Crossley [6]proposed to combine Hertz's laws and the Kelvin Voigt [5,6] model in order to extend the nonlinear Hertz's law to account for the kinetic energy losses during contacts. Also in order to avoid a jump at the beginning of the impact and tension forces at the end they multiplied the classical viscous damping term $c\dot{l}$withl^n. The internal damping contribution depends on the penetration velocity as that of the penetration length. The final model appeared in the form of the following equation [6]:

$$f = k_p l^{\,n} + c\dot{l}l^n. \tag{10}$$

They proposed to express the damping coefficient as a function of restitution e in the following form [6]:

$$C = \frac{3(1-e)}{2}\frac{k_p}{\dot{l}_s} \ , \tag{11}$$

where$\dot{l}_s$ is the initial relative normal velocity between impacting bodies.

The dependency of the damping coefficient with respect to the restitution coefficient allows controlling the amount of energy dissipated by each impact.Many have adjusted the Hunt and Crossley impact laws maybe the two most important contributions were Lankarini, Nikraveshi [7] and Flores, Machado[8]. They have developed the coefficients for the damping coefficient calculation.

3. NUMERICAL MODELING IN MSC ADAMS

All flexible bodies need to have a finite element model structure for simulating their behavior. Due to this structure the DOFs which are infinite become finite, yet with very large number of DOFs.Each part has its own local reference frame (coordinate system) that is defined by a position vector of a global reference frame.

3.1 Flexible Bodies in MSC ADAMS

ADAMS Flex-Bodiesconsiders a small linear deformation at the local form reference frame[9]. The small linear deformations can be approximated as superposition of a number of shape vectors. The shape vectors can be determined with a modal frequency analysis called modal superposition. This can be done by ADAMS/FLEX or any FEA program. In this study *Patran* and *Nastran* were used to perform modal analysis[10]. The natural frequencies and their corresponding mode shapes are determined .The results of the analysis are stored as binary files or modal neutral files (.mnf) that ADAMS can import and represent the flexible bodies.

3.2The Modal Superposition Theory

The location of a node "P" is defined by the vector from the ground origin to the origin of the local body reference frame "B". This is quite similar to that of Shabana's Floating Frame Theory, where$\vec{S}_p$is the position vector from the load body reference frame of B to the un-deformed position of point "P" and $\vec{l}_p$ is the translational deformation vector of point "P". The translational deformation vector $\vec{l}_p$ is a modal superposition and can also be written in matrix form [9]

$$U_p] = [\emptyset_p] * q,$$

where (12)

$\emptyset_p$is a 3*M matrix and q is the amplitude of each shape.

The three rows are for the corresponding translational DOFs of node "P".The same will be for the rotational DOFs.The point of derivations for the flexible body generalized coordinates is introduced as [9]:

$$\xi = \{X \quad Y \quad Z \quad \psi \quad \theta \quad \phi \quad i = 1 - N\}, \tag{13}$$

where {X,Y, Z} are the generalized coordinates, $\{\psi, \theta, \phi\}$ are the Euler angles of the flexible bodies and the modal coordinates are represented by q_i (i=1-N).

A simplified equation of motion in MSC ADAMS is represented below [9]:

$$[M]\ddot{\xi} + K\xi = Q, \tag{14}$$

whereQ is the generalized force applied.

The simplified equation of Kinetic Energy "T" [9]:

$$T = \frac{1}{2}\xi^T * M(\xi) * \xi \quad . \tag{15}$$

This conveys an understanding of the kinematics created in MSC ADAMS when equations for flexible bodies are to be resolved. The importance of such data is that it illustrates how the chain plate structure is affected during contacts. This leads to the next topic which is: how does MSC ADAMS calculate contacts?

3.3Contacts in MSC ADAMS

ADAMS does not totally base the calculations on the Hertz theory for calculating contact forces and stiffness. The geometrical shape of the plate and the geometrical shape of the sprocket are also important so to give ADAMS the ability to detect the bodies entering contacts. The stiffness of the bodies that are subjected to contacts is also important. The stiffness is calculated by the equation:

$$K = F_N / r_{p_c}^i \quad , \tag{16}$$

where F_N is the normal force acting at the contact point and$r_{p_c}^i$is the displacement of the point during contact.

An assumption has been created that only the plate links of the chain are flexible and subjected to deformations and the sprocket is considered as rigid body. This is in order to simplify the calculations and to find out how the contact affects the chains at low speeds and frequencies.

Animportant factor to calculate the contacts is the penetration depth [9]. The penetration depth is a value that defines at what amount of penetration the solver will use full damping. For example at 0 penetration 0 damping occurs. MSC ADAMS suggests 0.01 mm is a reasonable value but the value also depends on the complexity of the model one is analyzing. In this case the penetration depth is estimated at 0.05 mm for calculation reduction.

The next important factor is the damping values [9] that define the properties of the material with which bodies are in contact. MSC ADAMS suggests a damping coefficient that is about 1 % of the stiffness coefficient.

One of the important factors is the force exponent which is a parameter of the spring force of the impact function which has recommended value >1. For example in this case hard metals, such as steel to steel, the force exponent is 2.2.

4. RESULTS AND DISCUSSIONS

The importance of studying the model from the flexible multibody dynamics point of view is to see the effect of the plates colliding with the sprocket from the kinetic energy, deformation, frequency response point of views. As mentioned before for reducing the magnitude of simulation only the plates are considered flexible bodies. It can be said that the motion of the dead body weight in time can represent the total amount of kinetic energy lost as shown in figure 2 during motion. The kinetic energy lost is due to contacts and friction of the plates with the sprocket and the friction of the plates with the pins represented in the model as revolute joints. According to the factors mentioned above it is safe to say that the (I.T.C.) looses energy in an exponential form giving an understanding that the (I.T.C.) has a healthy (good) damping behavior in its natural form and to multiple shocks without adding external excitation forces. The exponential damping of the system gives the opportunity for one to understand why (I.T.C.) can also be called “silent chains”. Also the total angular displacement of the dead weight of the body shows how the system is simply damped having an exponential curve. The cause is the friction between the joints connecting the plates of the chain and the contacts between the chain plates and the sprocket.

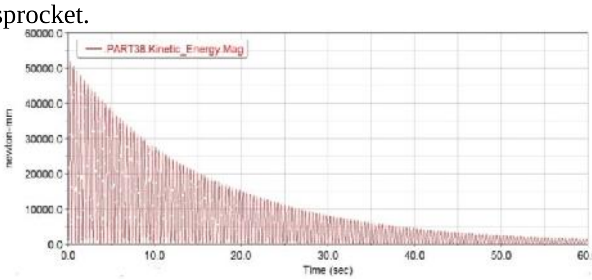


Figure 2: The kinetic energy of the deadweight

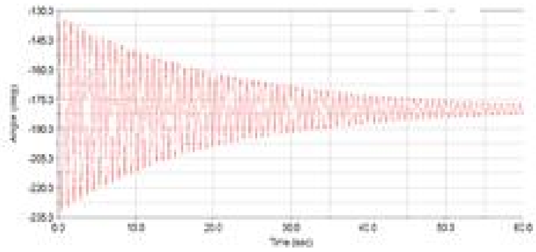


Figure 3: Angular displacement of the deadweight

We can simply see how the clean accelerations help in understanding the increase or decreaseof the amount of vibrations induced due to the contacts of the moving plates with the fixed sprocket. This can be seen in the figures below by following the development of bodyaccelerations and their corresponding frequencies responses.Figure 4 illustrates the acceleration of the last body in contact during oscillation and how the frequencies dissipate when the system decelerates.

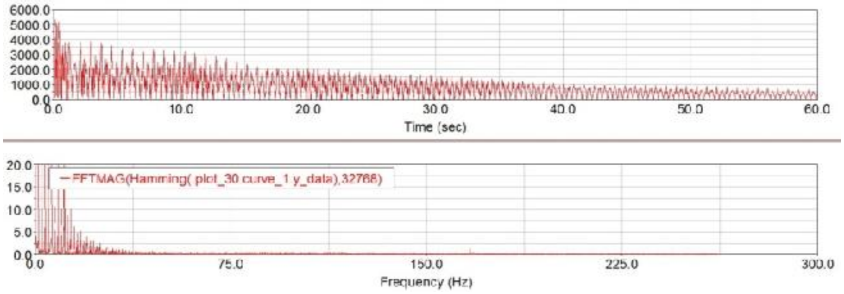


Figure 4: The acceleration of the last plate in contact in time and frequency domains

The highest contacts of pair plates of the chain can be simply subdivided to main intervals. One can notice the engagement, friction and disengagement impacts to form a total contact of the body as shown in figure 5.The second intervalillustrates the creation of the impulsive impacts due to the total loss of the kinetic energy in the chain during motion causing the chain to simply reach the rest position.The contact will simply dissipate and will take the form of the last body in contact with the sprocket in the form of the second interval of a normal plate position in contact, meaning the impulsive contacts as shown in figure 6.

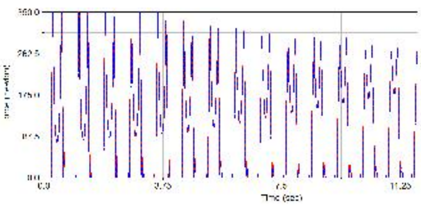


Figure 5: Contact Forces during the first interval

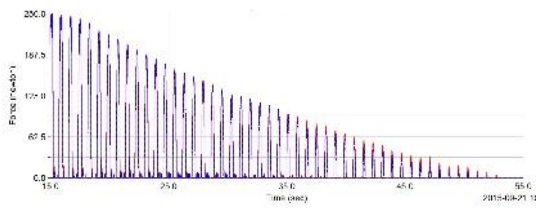


Figure 6: Contact forces dissipating till reaching impulsive contacts or the second interval of contacts

As the kinetic energy decreases the contact's usual form disappearsfrom the point of view of engagement, friction and disengagement, giving the contacts of small friction interval and producing impulsive contacts. The birth of the impulsive contacts simply illustrates that the oscillation of the pendulum is damped and that the oscillation motion simply dissipates and disappears.Figure 7 indicates the impulsive contacts with very low bandwidth and the magnitude of the contact forces that simply disappears till no impulsive contact is observed meaning that the oscillation doesnot have the sufficient power toraise the plate to contact the sprocket.

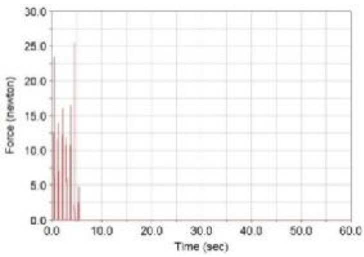


Figure 7: Impulsive contact forces

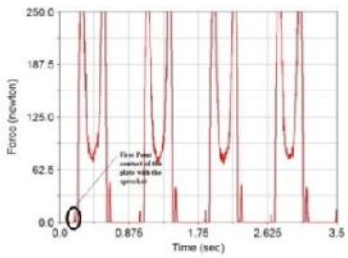


Figure 8: First contact location of a plate with the sprocket

One can approximately indicate the period and location of the first contact of the plate with the sprocket and the small spikes of pre total contacts. The impulsive spike contact as shown in figure 8 simply decreases the magnitude force of the total contact of the plate. The orientation of the plate towards the sprocket for a first impulsive sprocket is indicated in figure 9. One can deduce approximately the location of the first impulsive contact as shown in figure 10.

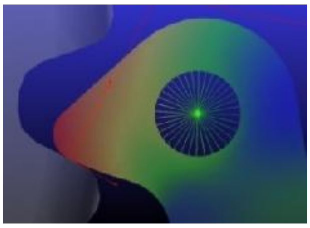


Figure 9: First contact partial view with deformation spectra due to contact

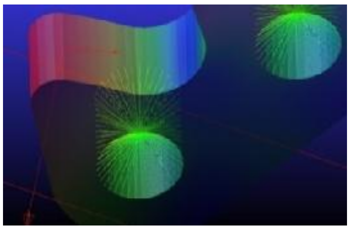


Figure 10: Estimated location of the first line of impulsive (spike) pre-contact

5. CONCLUSIONS

In this paper theflexible plates in an Analogous Huygens Pendulum were analyzed in Flexible Multi-Body Dynamic software. This type of analysis gives us a better perception of the Analogous Huygens Pendulum motion from the time and frequency points of view. It also gives the prospect of how does the system decelerates in an exponential form.

Along all the results of the simulations, one can realize and conclude the following:

1. As the angular gap between the plates and the sprocket is larger, the vibration produced becomes smaller giving an impression that the length is an inverse proportion to the frequency.
2. The single connecting plate suffers much more contact than the 2 plates in a link as the surface area is much bigger.
3. There is a first contact on the plate with the sprocket which simply gives a reduction of contact acting like a first aid damping contact even if the values are small. The first point contact simply gives a sudden deceleration so not to have a full contact at a single step.
4. One can realize if there is no first contact at the inner side of the plates during oscillation and contacts, the magnitude of the contacts is much higher and gives a higher distortion or vibration of the system. To have a first contact this also depends on the angle of the plates coming in contact with the sprocket.The double contact of the I.T. Chains gives an exponential deceleration as shown clearly at the displacements of the system and the deceleration curve.
5. The first contact assures a better positioning and assures a better engagement to reduce the total deformation of the chain plates during contacts.
6. The system in itself suffers during low frequencies and as the frequency increases the system gives a more stable behavior

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