6th International Conference
ðComputational Mechanics and Virtual Engineering ð
COMEC 2015
15-16 October 2015, Braşov, Romania

THE INFLUENCE OF TEMPERATURE'S CHANGE ALONG THE RADIUS ON THE MEMBRANE STRESSES FIELD AT ROTATING DISKS

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Abstract: *The rotation of a disk with constant angular velocity* µ *leads to a membrane stresses field; traction stresses which can be solved with the equations below [4]:*

$$\begin{cases} \pm_r = \dfrac{EA}{1-\neg} - \dfrac{EB}{r^2(1+\neg)} - \dfrac{3+\neg}{8}\cdot\dfrac{¥}{g}\cdot µ^2 r^2 \\ \pm_- = \dfrac{EA}{1-\neg} + \dfrac{EB}{r^2(1+\neg)} - \dfrac{1+3\neg}{8}\cdot\dfrac{¥}{g}\cdot µ^2 r^2 \end{cases} \qquad (19)$$

where - the constant values A and B are determined as a function depending are imposed support condition, for each case partly;

E – is the material's longitudinal modulus of elasticity;

¬ - is Poisson's ratio;

$\dfrac{¥}{g}$ *- is the specific mass of the disk's material;*

In the present paper, for the temperature's distribution along the disk radius, one uses the following law:

$$T = T_0\left(\frac{r}{b}\right)^{¥} \quad or \quad T_0 = \frac{\Delta T}{1-\left(\dfrac{a}{b}\right)^{¥}} \qquad (20)$$

¥ = 1/3, ½, 1, 2 … 30, a and b being the inner and the outer radius of the disk, respectively.

Keywords: *Disk, eigenvalues problems, membrane stresses, critical stresses, stability.*

This stresses field introduces a great influence upon the membrane stresses field given by the change of temperature along the radius and one established that its implication on the appearance of elastic stability loss at disks is very important.

The above mentioned influence is put on evidence on basis of the exposed theory, meaning equations (4) and (5) from part I, which are eigenvalues problems which lead to the determination of critical membrane stresses, where the stability loss may occur and to the determination of vibration eigenfrequencies and eigenmodes, respectively.

Based on the known solutions of the two mentioned problems, on their interdependence, one sets the problem of finding the solution of the eigenvalues problem given by equation (6) – (21)

$$\left([K] + ª\left[K_G\right] - µ^2[M]\right)\{u\} = 0 \qquad (21)$$

In part I the parabolic dependence between the eigenvalues Ω_i and ω_i was established and one puts in evidence that in the case of more existing membrane stresses fields which work simultaneously, the elastic loss of stability at disks occurs when the following equation is accomplished:

$$\sum \frac{\lambda^n}{\lambda_i^n} = 1 \tag{22}$$

One assumes that the critical stresses given by the two studied membrane stresses fields are known (rotational motion and temperature).
One puts in evidence the linear dependence of the critical fields mentioned above, given by equation:

$$\frac{\lambda^T}{\lambda_{cr}^T} + \frac{\lambda^R}{\lambda_{cr}^R} = 1 \tag{23}$$

where λ^T represents the eigenvalues due to the membrane stresses field given by the change of temperature along the radius; λ^R – represents the eigenvalues due to the membrane stresses field given by the rotational motion.
If one makes in equation (23) the following replacements:

$$T_{cr} = \lambda_{cr}^T \cdot T_0,$$

$$\mu_{cr}^2 = \lambda_{cr}^2 \cdot \mu_0^2, \qquad \text{we obtain} \tag{24}$$

$$\frac{\lambda^T \cdot T_0}{T_{cr}} + \frac{\lambda^R \cdot \mu_0^2}{\mu_{cr}^2} = 1$$

If the eigenvalues $\lambda^T \cong \lambda^R \cong \lambda$, than from equation (24) yields:

$$\lambda = \frac{1}{\dfrac{T_0}{T_{cr}} + \dfrac{\mu_0^2}{\mu_{cr}^2}} \tag{25}$$

One reaches to a very important conclusion, that $\lambda = 1$ at the superposition of the effects of the two membrane stresses fields only if the following condition is accomplished:

$$\frac{T_0}{T_{cr}} + \frac{\mu_0^2}{\mu_{cr}^2} = 1 \tag{26}$$

getting finally the equation :

$$T_0 = \left(1 - \frac{\mu_0^2}{\mu_{cr}^2}\right) T_{cr} \tag{27}$$

where T_0 represents the change of temperature along the radius due to which the elastic stability loss by the disk branching off (buckling) occurs, in the presence of the two membrane stresses fields .
Fig. 2 shows the change of critical temperatures for stability loss as a function of the disk radius ratio, for different combinations of the number of nodal diameters and nodal circles.
Based on the conclusions yielded from equation (23) one calculated T_{cr}, μ_{cr}^2 and T_0, the results being tabular presented.

One mentions that in calculations we considered more ratios $\sim = \dfrac{a}{b}$, and also more types of occurring the loss of stability, which is put in practice by: zero or more nodal circles joined with zero or more modal diameters.

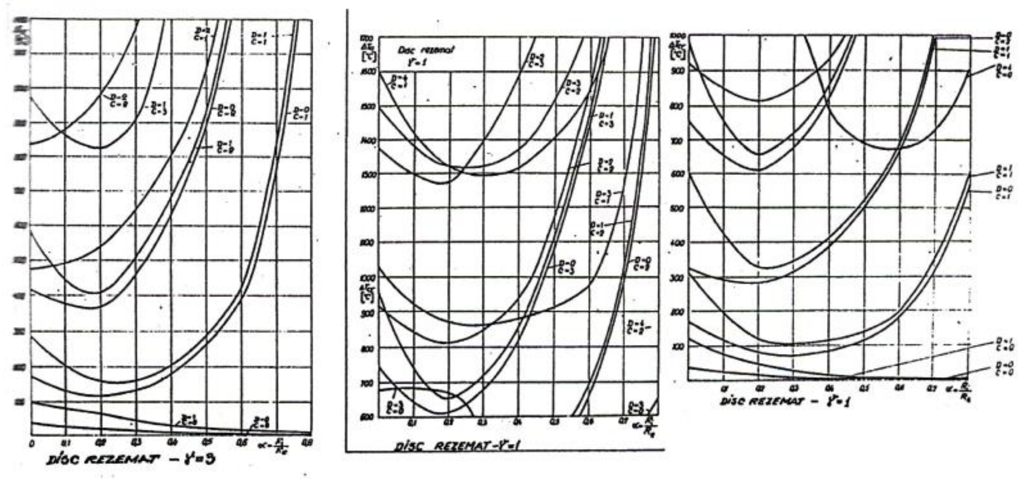


Fig. 2

The solutions of the elastic loss of stability at disks subjected to different combinations based on computational methods, put in evidence that the condition λ = 1 is accomplished not only for zero nodal circles and zero nodal diameters but for other situations too. One enumerates some of them:

$$\begin{cases} i=0 \\ n=1, \end{cases} \begin{cases} i=1 \\ n=0, \end{cases} \begin{cases} i=1 \\ n=1, \end{cases} \begin{cases} i=2 \\ n=1, \end{cases} \begin{cases} i=2 \\ n=0. \end{cases}$$

The other studied situations are not for practical importance because it increase very much the temperature difference and these cases are not met in practice.
By *i* and *n* are denoted the number of nodal circles and of nodal diameters, respectively.
In the tables met in the present paper one presents the critical temperatures of stability loss, and also the interdependence of the membrane stresses fields.

Tabel 3.1.
Plate on mandrel

- The table contains the values on ΔT_{cr}
- γ = 30 (coefficient from the temperature's law of distribution)
- ΔT = 10°C

Table 3.2
- Disk on mandrel
- Number of nodal circles: C = 0
- γ = 1 (coefficient from the temperature's law of distribution)
- ΔT = 10°C; n = 1000 rot/min

Table 3.3.
- Disk on mandrel
- Number of nodal circles: C = 1

- γ = 1 (coefficient from the temperature's law of distribution)
- - ΔT = 10°C; n = 1000 rot/min

Table 3.4
- Disk on mandrel
- Number of nodal circles: C = 2

- γ = 1 (coefficient from the temperature's law of distribution
- ΔT = 10°C; n = 1000 rot/min

- Table 3.1.

Table 3.2.

	Nr. cercuri nodale \ Nr. diametre nodale	d=0	d=1	d=2	d=3	d=4
$\frac{R_i}{R_e}=0$	C=0	37,92	41,522	79,99	127,118	184,61
	C=1	90,22	147,78	219,78	300	392,156
	C=2	227,70	311,688	422,535	533,33	659,34
	C=3	427,04	533,333	685,714	833,33	991,73
	C=4	689,655	810,81	1016,949	1188,118	1384,08
$\frac{R_i}{R_e}=0,2$	C=0	42,44	63,707	89,55	134,378	194,174
	C=1	62,49	82,19	247,933	318,302	410,95
	C=2	192,926	209,05	483,87	571,42	693,64
	C=3	413,793	431,65	799,99	902,25	1043,47
	C=4	727,27	745,34	1212,12	1307,189	1458,08
$\frac{R_i}{R_e}=0,4$	C=0	66,2	70,28	118,8	162,16	224,71
	C=1	86,33	95,23	363,63	416,66	493,827
	C=2	322,58	332,4	759,49	821,91	902,25
	C=3	714,28	727,27	1318,68	1372,99	1463,41
	C=4	1263,157	1276,59	2023,6	2205,88	2185,79
$\frac{R_i}{R_e}=0,6$	C=0	78,9	128,7	226,41	269,05	332,409
	C=1	193,86	202,02	794,7	851,06	923,076
	C=2	754,716	764,33	1726,61	1788,37	1877,934
	C=3	1678,32	1690,14	3030,3	3092,78	3199,99
	C=4	2970,29	2985,07	4743,08	4780,87	4897,95
$\frac{R_i}{R_e}=0,8$	C=0	160,8	260,52	307,9	1034,48	1142,85
	C=1	916,03	930,23	967,74	3703,7	3870,96
	C=2	3519,06	3539,82	3603,6	8003,69	8275,86
	C=3	7843,13	7843,13	7947,01	14051,52	14302,74
	C=4	13793,103	13808,97	13904,98	21857,92	22140,22

C=0	DIAMETRE NODALE	$T_{cr}=\frac{\Delta T}{\lambda}$	$\omega_{cr}^2=-\frac{1}{\lambda}\cdot 10^6$	$\dot{T}_0$ (calculat)
$\frac{R_i}{R_e}=0$	d=0	35,211	$+13,03\cdot 10^6$	59,531
	d=1	112,032	$+35,001\cdot 10^6$	140,84
	d=2	316,555	$+6,706\cdot 10^6$	741,39
	d=3	683,15	$+20,66\cdot 10^6$	980,789
	d=4	1154,25	$+41,684\cdot 10^6$	1403,464
$\frac{R_i}{R_e}=0,2$	d0	25,846	$+11,38\cdot 10^6$	46,286
	d1	92,592	$+4,325\cdot 10^6$	285,268
	d2	301,93	$+6,958\cdot 10^6$	692,468
	d3	467,726	$+18,69\cdot 10^6$	692,955
	d4	672,992	$+37,03\cdot 10^6$	836,560
$\frac{R_i}{R_e}=0,4$	d0	23,207	$+12,007\cdot 10^6$	40,602
	d1	70,62	$+7,209\cdot 10^6$	158,784
	d2	526,37	$+8,598\cdot 10^6$	1077,22
	d3	633,77	$+16,44\cdot 10^6$	980,63
	d4	739,86	$+28,97\cdot 10^6$	969,709
$\frac{R_i}{R_e}=0,6$	d0	27,247	$+18,737\cdot 10^6$	40,33
	d1	74,294	$+14,93\cdot 10^6$	119,079
	d2	1594,13	$+15,535\cdot 10^6$	2517,46
	d3	1714,97	$+20,53\cdot 10^6$	2466,78
	d4	1772,04	$+28,571\cdot 10^6$	2330,24
$\frac{R_i}{R_e}=0,8$	d0	46,772	$+56,915\cdot 10^6$	54,168
	d1	120,26	$+55,61\cdot 10^6$	139,723
	d2	1312,33	$+56,18\cdot 10^6$	1522,56
	d3	[illegible]	$+60,42\cdot 10^6$	15021,398

- Table 3.3.

C=1	DIAMETRE NODALE	$T_{cr}\left(\frac{10}{\lambda}\right)$	$\omega_{cr}^2=\left(-\frac{1}{\lambda}\cdot 1000^2\right)$	T_0 (calculat)
$\frac{R_i}{R_e}=0$	d=0	169,692	$+69,252\cdot 10^6$	191,745
	d=1	300,12	$+110,399\cdot 10^6$	324,586
	d=2	559,28	$+66,44\cdot 10^6$	635,04
	d=3	1025,85	$+104,36\cdot 10^6$	1114,31
	d=4	1593,168	$+149,58\cdot 10^6$	1689,026
$\frac{R_i}{R_e}=0,2$	d=0	187,65	$+85,105\cdot 10^6$	207,494
	d=1	266,737	$+70,323\cdot 10^6$	300,874
	d=2	680,272	$+77,579\cdot 10^6$	759,19
	d=3	1018,453	$+100,59\cdot 10^6$	1109,55
	d=4	1395,069	$+134,64\cdot 10^6$	1488,34
$\frac{R_i}{R_e}=0,4$	d=0	437,828	$+227,324\cdot 10^6$	455,162
	d=1	492,125	$+181,06\cdot 10^6$	516,587
	d=2	1489,86	$+156,10\cdot 10^6$	1575,75
	d=3	1830,59	$+151,60\cdot 10^6$	1939,26
	d=4	2380,95	$+159,31\cdot 10^6$	2515,45
$\frac{R_i}{R_e}=0,6$	d=0	1602,564	$+1093\cdot 10^6$	1615,759
	d=1	1663,89	$+835,42\cdot 10^6$	1681,815
	d=2	5162,62	$+616,52\cdot 10^6$	5237,98
	d=3	5577,24	$+485,67\cdot 10^6$	5680,59
	d=4	6426,735	$+410,34\cdot 10^6$	6567,69
$\frac{R_i}{R_e}=0,8$	d=0	1879,78	$+61625,28\cdot 10^6$	14136,33
	d=1	14240,95	$+12881,6\cdot 10^6$	14250,89
	d=2	16186,46	$+8733,6\cdot 10^6$	16203,14
	d=3	45269,35	$+6013,23\cdot 10^6$	45337,10
	[illegible]	[illegible]	[illegible]	[illegible]

Table 3.4.

C=2	DIAMETRE NODALE	$T_{cr}=\frac{\Delta T}{\lambda}$	$\omega_{cr}^2=-\frac{1}{\lambda}\cdot 10^6$	$\dot{T}_0$ (calculat)
$\frac{R_i}{R_e}=0$	d=0	405,56	$-168,01\cdot 10^6$	427,253
	d=1	576,76	$+224,87\cdot 10^6$	599,843
	d=2	913,24	$+165,43\cdot 10^6$	962,923
	d=3	1485,35	$+224,06\cdot 10^6$	1545,013
	d=4	2182,92	$+290,52\cdot 10^6$	2250,544
$\frac{R_i}{R_e}=0,2$	d=0	571,42	$+258,131\cdot 10^6$	591,343
	d=1	640,61	$+232,34\cdot 10^6$	665,424
	d=2	1271,61	$+230,62\cdot 10^6$	1321,234
	d=3	1650,16	$+249,31\cdot 10^6$	1709,73
	d=4	2209,94	$+284,33\cdot 10^6$	2279,89
$\frac{R_i}{R_e}=0,4$	d=0	1491,84	$+774,59\cdot 10^6$	1509,173
	d=1	1542,02	$+687,28\cdot 10^6$	1562,2129
	d=2	3132,83	$+607,53\cdot 10^6$	3179,24
	d=3	3460,20	$+550,66\cdot 10^6$	3516,75
	d=4	4338,39	$+516,25\cdot 10^6$	4414,02
$\frac{R_i}{R_e}=0,6$	d=0	5656,10	$+3866,97\cdot 10^6$	5678,28
	d=1	5714,28	$+3414,13\cdot 10^6$	5729,343
	d=2	11.539,349	$+2873,56\cdot 10^6$	11.575,49
	d=3	11.900,51	$+2388,34\cdot 10^6$	11.945,344
	d=4	12.841,91	$+1997,80\cdot 10^6$	12.899,769
$\frac{R_i}{R_e}=0,8$	d=0	50.607,28	$+61.652,28\cdot 10^6$	50.614,667
	d=1	50.709,93	$+54.555,37\cdot 10^6$	50.718,296
	d=2	58.761,31	$+44.923,62\cdot 10^6$	58.773,082
	d=3	102.880,65	$+35.893,76\cdot 10^6$	102.906,446
	d=4		$+28.344,67\cdot 10^6$	

Based on the exposed theory and on the FEM study the way of stability loss for different combinations of nodal diameters and nodal circles is shown in Fig. 3. One establishes that the condition $\lambda = 1$ is accomplished and so the solutions are in concordance with the theory.

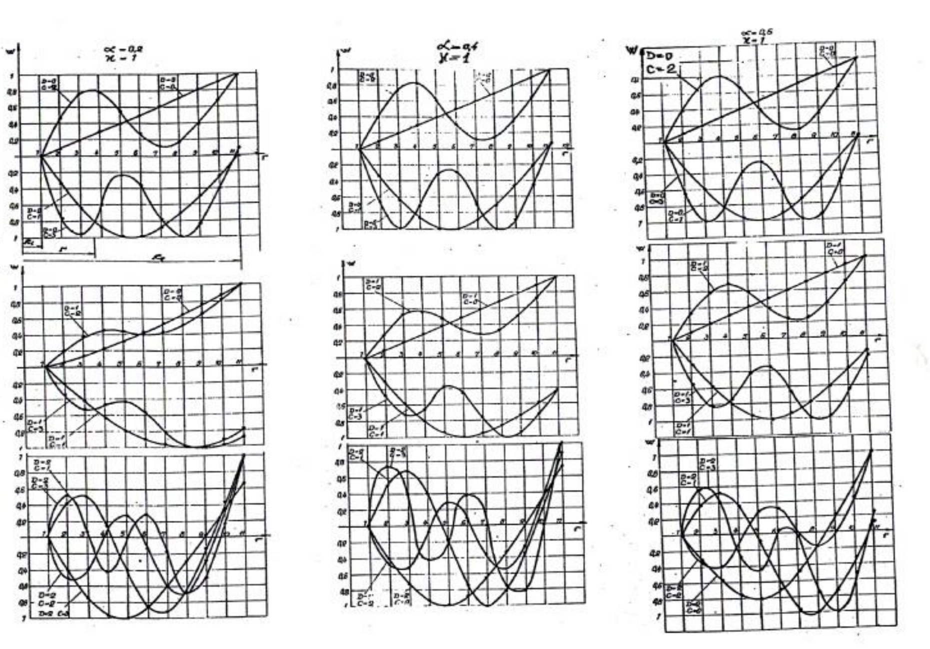
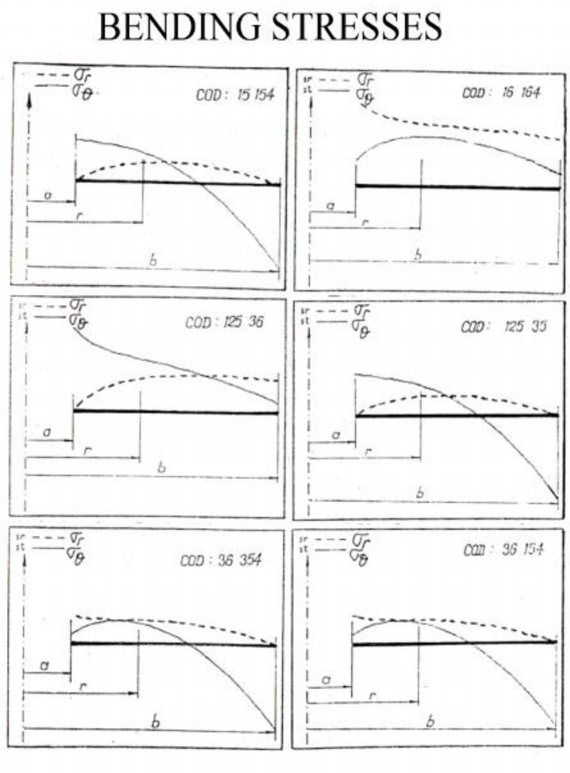


Fig. 3



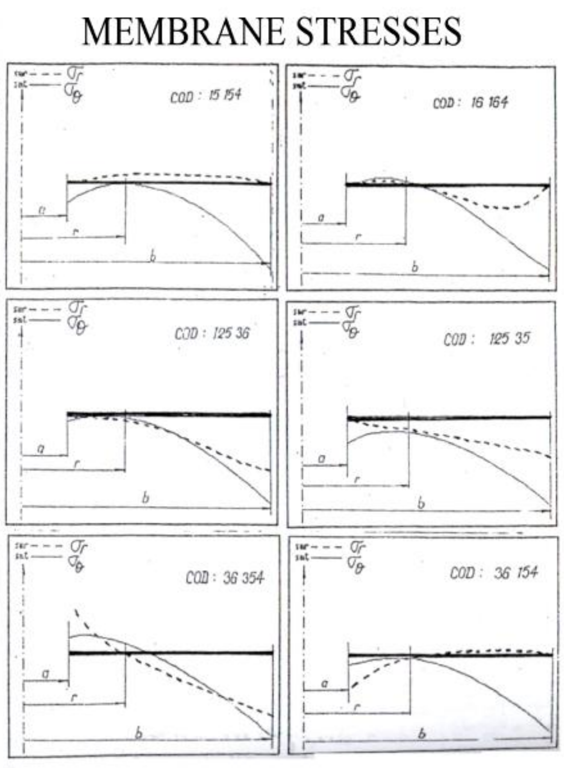


Fig.4 Fig.5.

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