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STUDIES AND COMPARATIVE ANALYZES REGARDING USUAL BEAMS UNDER STATIC AND DYNAMIC LOADINGS BY SHOCK USING ANALYTICAL AND NUMERICAL METHODS

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Abstract: *The literature reflected that the beams response under static and dynamic is done taking into account the shape of the median axes, boundary conditions, loading mode and solving method adopted. The paper presents the studies and analyzes who followed up four distinct components starting with static analysis and ending with the representation of percentage deviations of efforts and displacements. Dynamic analysis will not take into account the issues determined by free and forced vibration of the beams.*

Key words: *beam, static, dynamic, shock, method.*

1. Introduction

The beams studied in this paper have the same length, they are made of the same material, but have different sections, circular and rectangular.

Studies and static and dynamic analysis by shock was performed using analytical and numerical methods.

Static analysis was done considering the beams loading by a uniformly distributed forces using the analytical method and finite element method. Dynamic analysis by shock was performed using analytical method for two cases:

- neglecting the beams weight;
- by considering the beams weight.

2. Objectives

Studies and analyzes followed up 4 distinct components. These are:

- research and analysis of static beam loaded by external forces;
- Research and analysis of dynamic beam loaded by external forces;
- Comparative analysis of sectional efforts which appear in dangerous sections as well as the displacements in the same sections.
- Presentation of percentage deviations which appear in the case of dynamical load compared to static load

3. Materials and Methods

Dynamic analysis will not take into account the issues determined by free and

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forced vibration of the beam. They are considered as a separate chapter of the dynamics of construction. The study and subsequent analysis of the results was performed for dynamical shock loads when the acceleration varies very fast in time, suddenly the case of contact of the construction elements with another body [7], [9]. In these situations it is not known acceleration variation law and therefore can not enter the inertial forces from the principle of D'Alembert [4], [8].

The study and theoretical analysis of the proposed work was done for the case of a beam made of fir, use as a constructive component of a structure familiar household type. To simplify the calculations, we taking into account the elasticity theory of simplifying assumptions, the beams shall be considered only required bending over fiber, disregarding the deformations in the perpendicular plane of fibers. Also the material is considered as working in elastic behavior, so the Hooke's law is valid [3], [5], [6] The geometric characteristics of the material is considered as the same, therefore, and dimensional characteristics are identical.The study and analysis was made for the two types of beams of the same material (fir) with rectangular and circular sections with the following dimensions:

Rectangular section Fig. 1,3

$L = 200\,[cm]$, the length of the beam;he

$b = 25\,[cm]$, the width of the beam;

$h = 30\,[cm]$, the height of the beam.

Circular section Fig. 2

$L = 200\,[cm]$,the length of the beam;

$D = 25\,[cm]$, the diameter of the beam.

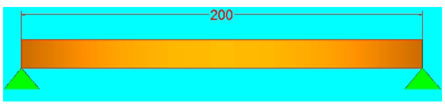

Fig. 3. *Beams length*

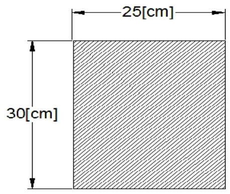

Fig. 1. *Rectangular section*

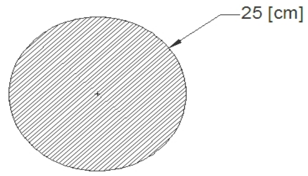

Fig. 2. *Circular section*

Static analysis was conducted both by analytical and finite element method.

To correct solution are considered beams through static and dynamic loads applied evenly distributed with the same intensity as:

$$q = 10\left[\frac{daN}{cm}\right]$$

The modulus of elasticity in bending parallel of fibers for static and dynamic loadings are considered as:

$$E_{II} = 113000\left[\frac{daN}{cm^2}\right]$$

For static analysis, the study of beams having circular and rectangular sections starts from the resistance conditions as:

$$\sigma_{\max} = \frac{M_{\max}}{W_{ef}} \le \sigma_{adm} \qquad (1)$$

Rectangular sections

Calculating resistance standards, R, of the various species of wood in different applications, depending on the operating conditions of the construction elements which are designed [3], [6].

$$\sigma_{adm} = m_{ui} \cdot m_{di} \cdot R_i \cdot \gamma_i \qquad (2)$$

m_{ui} – coefficient that takes account of the humidity of the wood.

m_{di} – coefficient that takes into account the loading time.

R_i – characteristic resistance of the different species.

γ_i – partial safety coefficients, define by the type of loadings.

$$m_{ui} = 1$$
$$m_{di} = 0.55$$
$$R_i = 240\left[\frac{daN}{cm^2}\right]$$
$$\gamma_i = 1{,}10$$
$$\sigma_{adm} = m_{ui} \cdot m_{di} \cdot R_i \cdot \gamma_i = 145.2\left[\frac{daN}{cm^2}\right]$$

Using (1), we check the most dangerous section of the beam [3], [6].

$$\sigma_{\max} = \frac{M_{\max}}{W_{ef}} = \frac{10^5}{3750} = 26.66\left[\frac{daN}{cm^2}\right] \le \sigma_{adm}$$

Also, using direct integration method is determining the displacements in the middle section of the beam (section C).

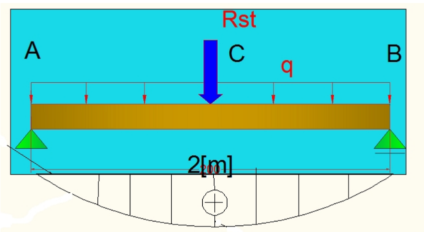

Fig. 4. *Beams deflections*

Using differential equation [3], [6]:

$$E \cdot I_z \frac{d^2 v}{dx^2} = -M_x \qquad (3)$$

we obtain the maximum displacement on the middle of the beam:

$$v_{ST} = 0.032\,[cm].$$

Circular Sections

On beam with circular section, considering the geometrical characteristics of section and follow the same algorithm are obtained [3], [5], [6]:

$$\sigma_{\max} = \frac{M_{\max}}{W_{ef}} = \frac{10^5}{1533.203} = 65.35\left[\frac{daN}{cm^2}\right] \le \sigma_{adm}$$

$$v_{ST} = 0.0961\ [cm]$$

By applying the finite element method [10], were obtained the following results. Fig. 5, 6, 7.

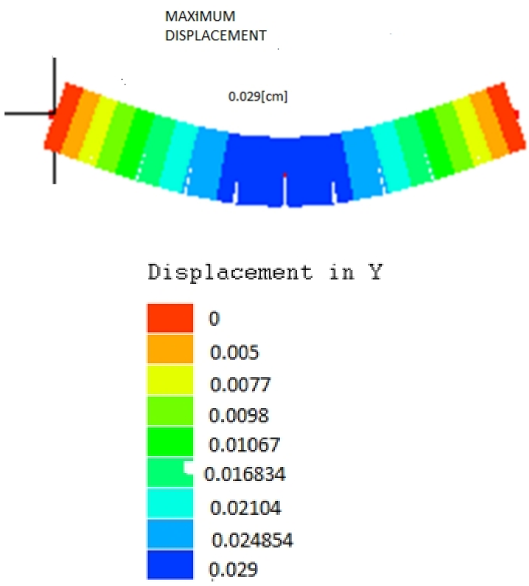

Fig. 5. *Transversal displacement - rectangular section*

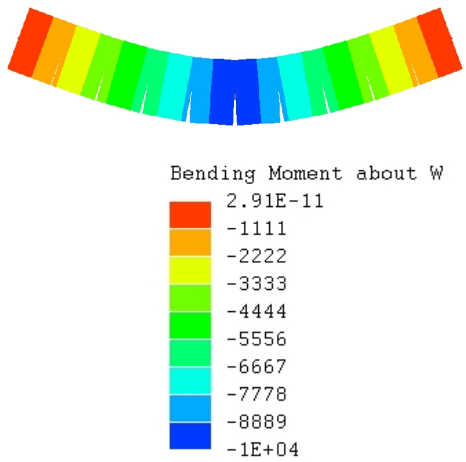

Fig. 6. *Deformed beam - bending moment values*

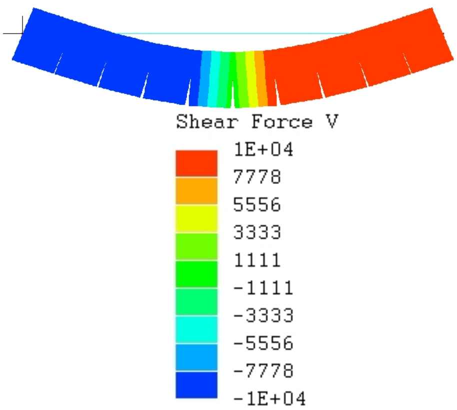

Fig. 7. *Deformed beam - shear forces values*

For circular section using the finite element method we obtained the following valuues. Fig. 8, 9,10

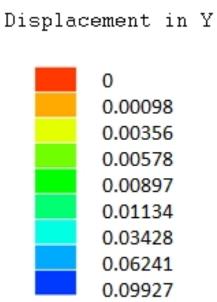

Fig. 8. *Displacements values circular sections*

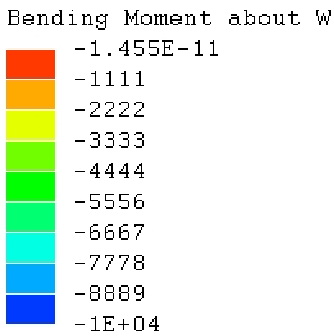

Fig. 9. *Bending moments values circular section.*

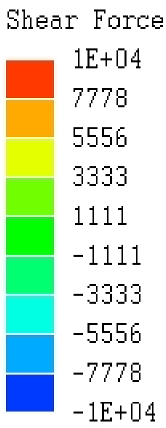

Fig. 10. *Shear forces circular section*

Rectangular section

Dynamic analysis by shock neglecting the beams weight [1], [2].

The study was conducted considering the impact abrupt application of uniformly distributed load at height

$$h = 0.1\,[m] = 10\,[cm]$$

The calculation algorithm that has been followed in paper is:

1. Determination of dynamic multiplier expressed by the relationship:

$$\Psi = 1 + \sqrt{1 + \frac{2 \cdot h}{v_{st}^{\max}}} \qquad (4)$$

$$\Psi = 27$$

2. Maximum bending moment in the middle of the beam. Dynamic moment is a function of the static moment and the dynamic multiplier determined [1], [2], [9].

$$M_{din}^{\max} = \Psi \cdot \frac{R \cdot l}{4} \quad (5)$$

$$M_{din}^{\max} = \Psi \cdot \frac{R \cdot l}{4} = 2.7 \cdot 10^6 \,[daN \cdot cm]$$

3. Determination of maximum shear force. It is expressed according to the shear force determined in static and dynamic multiplier [9].

$$T_{din}^{\max} = \frac{R}{2} \cdot \Psi \,[daN] \quad (5)$$

$$T_{din}^{\max} = \frac{R}{2} \cdot \Psi \cong 27000 \,[daN]$$

4. The maximum displacement in in dynamic conditions is determined using the formula

$$v_{din}^{\max} = v_{st}^{\max} \cdot \Psi] \quad (6)$$

$$v_{din}^{\max} = v_{st}^{\max} \cdot \Psi = 0.864 \,[cm]$$

The admissible displacement for these structural element is given by the formula:

$$v_{adm} = \frac{L}{150} [cm] \quad (7)$$

$$v_{adm} = \frac{L}{150} = \frac{200}{150} = 1.33 \,[cm]$$

Rectangular sections

Dynamic analysis by shock by considering the beams weight.

The calculation algorithm that has been followed in paper is:

1. It is calculated the weight beam (like a uniform loading)

$$Q = mg = \rho V g$$
$$Q = 7950 \,[daN]$$

where:

$\rho -$ density;

$m -$ mass;

$V -$ volume;

$g -$ gravitational acceleration.

2. Dynamic multiplication coefficient [9]

$$\Psi = 1 + \sqrt{1 + \frac{2 \cdot h}{v_{st}^{\max}} \cdot \frac{1 + K_2 \cdot \frac{Q}{R_{din}}}{\left(1 + K_1 \frac{Q}{R_{din}}\right)^2}} \quad (8)$$

$$\Psi = 13$$

Where:

$K_2 = \frac{17}{35}$, $K_1 = \frac{5}{8}$, weight reduction coefficients.

3. Maximum bending moment will appear at the middle of the beam will be given depending by the determined static moment and dynamic multiplication coefficient [9].

$$M_{din}^{\max} = \Psi \cdot \frac{(R+Q) \cdot l}{2} [daN \cdot cm] \quad (9)$$

$$M_{din}^{\max} = 6{,}4675 \cdot 10^6 \,[daN \cdot cm]$$

4. The maximum shear forces will be given by the relation [9]:

$$T_{din}^{\max} = \frac{(R+Q)}{2} \cdot \Psi \,[daN] \quad (10)$$

$$T_{din}^{\max} \cong 6.4675 \cdot 10^4 \,[daN]$$

5. It is determined the maximum displacement in dynamical loading conditions.

$$v_{din}^{\max} = v_{st}^{\max} \cdot \Psi \,[cm] \qquad (11)$$

$$v_{din}^{\max} = v_{st}^{\max} \cdot \Psi = 13 \cdot 0.032 = 0.416 \,[cm]$$

The admissible displacement for these structural element is:

$$v_{adm} = \frac{L}{150} = \frac{200}{150} = 1.33 \,[cm]$$

Circular sections

Dynamic analysis by shock neglecting the beams weight.

Following the same calculation algorithm that has been done in paper we obtained:

$$\Psi = 1 + \sqrt{1 + \frac{2 \cdot h}{v_{st}^{\max}}} = 12.24 \approx 12$$

$$M_{din}^{\max} = \Psi \cdot \frac{R \cdot l}{4} = 1.2 \cdot 10^6 [daN \cdot cm]$$

$$T_{din}^{\max} = \frac{R}{2} \cdot \Psi \cong 12000 \,[daN]$$

$$v_{din}^{\max} = v_{st}^{\max} \cdot \Psi = 12 \cdot 0.099 = 1.18 \,[cm]$$

$$v_{adm} = \frac{L}{150} = \frac{200}{150} = 1.33 \,[cm].$$

Circular section

Dynamic analysis by shock by considering the beams weight.

Following the same steps it obtained:

$$Q = mg = \rho V g$$

$$Q = 5294 \,[daN]$$

$$\Psi = 1 + \sqrt{1 + \frac{2 \cdot h}{v_{st}^{\max}}} \cdot \sqrt{\frac{1 + K_2 \cdot \frac{Q}{R_{din}}}{\left(1 + K_1 \frac{Q}{R_{din}}\right)^2}} \approx 8$$

$$M_{din}^{\max} = \Psi \cdot \frac{(R+Q) \cdot l}{2} = 2.9176 \cdot 10^6 \,[daN \cdot cm]$$

$$T_{din}^{\max} = \frac{(R+Q)}{2} \cdot \Psi \cong 4.2352 \cdot 10^4 \,[daN]$$

$$v_{din}^{\max} = v_{st}^{\max} \cdot \Psi = 8 \cdot 0.0961 = 0.768 \,[cm]$$

$$v_{adm} = \frac{L}{150} = \frac{200}{150} = 1.33 \,[cm]$$

4. Results and Discussions

Analysis of the efforts, displacements and deformations that occur in structural wood beams has considered for the static and dynamic actions by shock loads.

Watching the results obtained can be expressed some conclusions with theoretical and practical aspects.

Maximum bending moments for static and dynamic loadings are presented in table 1 for rectangular sections.

Bendind moments Table 1

Rectangular Section	x=100 [cm]
Static Analyses	10^5
Dynamical Analyses neglecting the beams weight.	$2.7 \cdot 10^6$
Dynamical Analyses considering the beams weight	$6.4675 \cdot 10^6$

Bendind moments Table 2

Circular Section	x=100 [cm]
Static Analyses	10^5
Dynamical Analyses neglecting the beams weight.	$1{,}2 \cdot 10^6$
Dynamical Analyses considering the beams weight	$2.9176 \cdot 10^6$

For circular sections the results are presented in table 2.

Maximum shear forces are presented in table 3 for rectangular sections:

Shear forces Table 3

Rectangular Section	x=100 [cm]
Static Analyses	10^3
Dynamical Analyses neglecting the beams weight.	$2.7 \cdot 10^4$
Dynamical Analyses considering the beams weight	$6.4675 \cdot 10^4$

For circular sections the results are presented in table 4.

Shear forces Table 4

Circular Section	x=100 [cm]
Static Analyses	10^3
Dynamical Analyses neglecting the beams weight.	$1.2 \cdot 10^4$
Dynamical Analyses considering the beams weight	$4.2352 \cdot 10^4$

Maximum displacements for static and dynamic loadings are presented in table 5 for rectangular section.

Displacements Table 5

Rectangular Section	x=100 [cm]
Static Displacement	0.032
Dynamical displacement neglecting the beams weight.	0.864
Dynamical displacement considering the beams weight	0.416

Maximum displacements for static and dynamic loadings for circular sections are presented in table 6.

Displacements Table 6

Circular Section	x=100 [cm]
Static Displacement	0.0961
Dynamical displacement neglecting the beams weight.	1.18
Dynamical displacement considering the beams weight	0.768

5. Conclusions

The analysis of obtained results regarding the maximum bending moments, shear forces and displacements by static and dynamic shock loadings, can be conclude the following.

- at the wooden beams with circular sections the value of the bending moment represents 44.4% percent of the bending moment obtained from rectangular section beams when they neglected their own weight and 45.04% percent when is not neglected its own weight.

Also, the maximum bending moments under static loads represent approximately 3.7% percent of the bending moment under dynamic load when it neglects its own weight.

- at the wooden beams with circular sections the value of the shear forces represents 44.88% percent of the shear forces obtained from rectangular section beams when they neglected their own weight and 65.48% percent when is not neglected its own weight. The maximum static shear forces represents about 8.33% of the dynamic shear force when neglecting its own weight.

- at the wooden beams with rectangular sections the value of the maximum displacements represents 73.22 % percent of the maximum displacements obtained from circular sections beams when they neglected their own weight.
- at the wooden beams with circular sections the value of the maximum displacements represents 54.16 % percent of the maximum displacements obtained from rectangular sections beams when is not neglected its own weight.
- the maximum transversal displaceme nt is recorded for the circular section when is neglect its own weight and is approximately 88.72% from the maximum allowable displacement.

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