Geometric Constraints at the Valve Actuation Mechanism with Spherical Contact between the Lever and the Head of the Valve

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Abstract. In our paper we establish the geometric relations that have to hold true due to the spherical contact between different geometric elements of a valve actuation mechanism. Using these relations one may determine the rotation angle of the lever as function of the rest of the parameters, and the maximum rotation angle as function of the contact position between the lever and the valve. Numerical applications and different diagrams of variation highlight the theory.

Keywords: valve-lever contact, geometric relations, rotation angle.

1 Introduction

The problem of the valve actuation mechanism with different type of contact between the cam and tappet, and between the lever and the head of the valve is of great importance in the field of automotive. Different types of cam-follower mechanisms are studied in the literature [1]. Some of the modern automobiles use now roller tappet mechanism and spherical contact between the lever and the head of the valve. The general synthesis of a distribution mechanism with general contact curve is described in [2]. The problem of a continuously variable valve lift mechanism from the point of view of the analytical synthesis and kinematic analysis is discussed in [3]. The general method used in the cam synthesis may lead to singularities which may cause failures in functioning. A new method to obtain convex cam is to use the Jarvis march which assures the convexity of the cam [4, 5].

The study of such mechanism leads to complicate formulae and the determination of different parameters that appear in these formulae cannot be made in an analytical way. For these reasons a numerical solution must be given. In addition, the accuracy of the results is obtained using a very small scale (in our paper we used a precision of

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10^{-13}, that is, the solving of the non-linear systems obtained in the paper is performed until the absolute value of the function is less than this value).

Moreover, the derivatives of the rotations angle in function of the rotation angle of the crankshaft have to be determined using the theory of the implicit functions. These derivatives will be developed in another paper in which we will discuss the synthesis of the cam mechanism.

Based on the previous considerations, we have drawn some diagrams which present the variations of the rotation angle as function of different other parameters.

2 Description of the system

The considered system (Fig. 1) consists in the bar (which symbolizes the lever) OC_2 having the length equal to l, and having at its end a roll of radius R_2. In the initial position the angle between the bar and the horizontal direction is equal to β_0, which is known. The rotation of the valve about its own axis of symmetry is a redundant degree of freedom which is not important in our analysis. For this reason, the problem may be considered a planar one and the sphere-sphere contact is presented as a contact between two circles situated in the same plan. The physical realization of the contact uses two spheres because the elimination of the rotation of the valve is a complicate task, and this rotation is wished from the point of an uniform wear of the lever and valve.

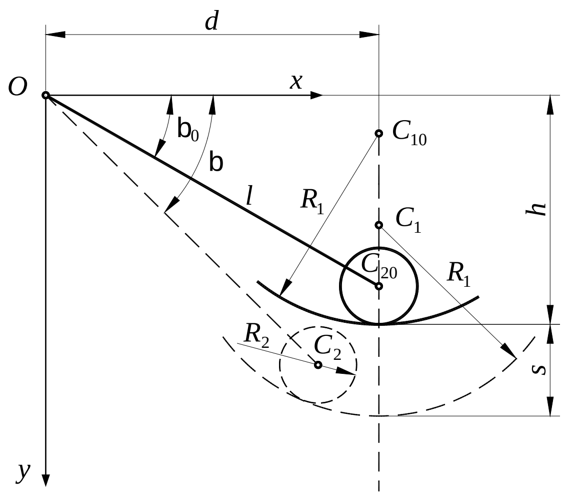

Fig. 1. The mechanical system

The roll of radius R_2, supports at any moment of the motion, on an arc of circle of radius R_1; the center of this circular arc is situated at the distance d from the vertical axis O_y.

We may write the relation

$$d = l\cos\beta_0 . \tag{1}$$

The displacement of the valve in vertical direction with the distance s leads to the displacement of the circular arc of radius R_1, so that the center C_1 of this arc moves from the position C_{10} to the new position C_1, but remaining situated at the distance d from the axis O_y.

The bar of length l rotates such that the roll of radius R_2 remains tangent to the circular arc of radius R_1, while the angle between the bar OC_2 and the axis O_x takes the value β.

The systems must assure a required maximum displacement of the valve, s_{max}. This value is necessary for a good intake of the fuel in the cylinders.

3 Geometric considerations

The coordinates of the point C_2 (the center of the roll) read

$$x_2 = l\cos\beta, \quad y_2 = l\sin\beta . \tag{2}$$

The circle of center C_1 has the equation

$$(x - x_1)^2 + (y - y_1)^2 = R_1^2 , \tag{3}$$

where x_1 and y_1 are the coordinates of the center C_1,

$$x_1 = d , \; y_1 = h + s - R_1 . \tag{4}$$

The equation of the circle of center C_2 and radius R_2 has the form

$$(x - x_2)^2 + (y - y_2)^2 = R_2^2 . \tag{5}$$

The intersection point of the two circles is obtained as the solution of the following system

$$\begin{cases} (x - x_1)^2 + (y - y_1)^2 = R_1^2 , \\ (x - x_2)^2 + (y - y_2)^2 = R_2^2 . \end{cases} \tag{6}$$

Subtracting the two equations (6), term by term, one obtains the relation

$$(2x - x_1 - x_2)(x_2 - x_1) + (2y - y_1 - y_2)(y_2 - y_1) = R_1^2 - R_2^2 , \tag{7}$$

where from it results the expression

$$2(x_2 - x_1)x + 2(y_2 - y_1)y = R_1^2 - R_2^2 + x_2^2 - x_1^2 + y_2^2 - y_1^2 . \tag{8}$$

Denoting

$$A_1 = 2(x_2 - x_1),\ B_1 = 2(y_2 - y_1),\ C_1 = R_1^2 - R_2^2 + x_2^2 - x_1^2 + y_2^2 - y_1^2 , \tag{9}$$

the expression (8) becomes

$$A_1 x + B_1 y = C_1 , \tag{10}$$

where from

$$y = \frac{C_1}{B_1} - \frac{A_1}{B_1} x . \tag{11}$$

Replacing now in the first relation (6), one gets

$$(x - x_1)^2 + \left(\frac{C_1 - A_1 x}{B_1} - y_1\right)^2 = R_1^2 , \tag{12}$$

expression which leads to the equation

$$x^2 + \frac{A_1^2}{B_1^2} x^2 - 2xx_1 - \frac{2A_1(C_1 - B_1 y_1)}{B_1^2} x + x_1^2 + \left(\frac{C_1 - B_1 y_1}{B_1}\right)^2 - R_1^2 = 0 . \tag{13}$$

With the aid of the notations

$$A_2 = 1 + \frac{A_1^2}{B_1^2} ,\ B_2 = -2x_1 - \frac{2A_1(C_1 - B_1 y_1)}{B_1^2} ,\ C_2 = x_1^2 + \left(\frac{C_1 - B_1 y_1^2}{B_1}\right)^2 - R_1^2 , \tag{14}$$

the expression (13) may be put in the form of a second degree equation in the unknown x,

$$A_2 x^2 + B_2 x + C_2 = 0 . \tag{15}$$

The tangency condition of the two circles implies that the equation (15) has a unique solution in the unknown x, that is, its discriminant vanishes,

$$\Delta = B_2^2 - 4A_2 C_2 = 0 . \tag{16}$$

Keeping into account the relations (2), one successively obtains

$$A_1 = 2(l \cos\beta - x_1),\ B_1 = 2(l \sin\beta - y_1),\ C_1 = R_1^2 - R_2^2 + l^2 - \left(x_1^2 + y_1^2\right), \tag{17}$$

$$A_2 = 1 + \left(\frac{l\cos\beta - x_1}{l\sin\beta - y_1}\right)^2,$$

$$B_2 = -2x_1 - \frac{(l\cos\beta - x_1)\left[R_1^2 - R_2^2 + l^2 - \left(x_1^2 - x_2^2\right) - 2(l\sin\beta - y_1)y_1\right]}{(l\sin\beta - y_1)^2},$$

$$C_2 = x_1^2 + \left[\frac{R_1^2 - R_2^2 + l^2 - \left(x_1^2 + x_2^2\right) - 2(l\sin\beta - y_1)y_1}{2(l\sin\beta - y_1)}\right]^2 - R_1^2 . \tag{18}$$

We denote

$$A_3 = R_1^2 - R_2^2 + l^2 - \left(x_1^2 + y_1^2\right) \tag{19}$$

and we get

$$A_2 = \frac{l^2 + x_1^2 + y_1^2 - 2lx_1\cos\beta - 2ly_1\sin\beta}{(l\sin\beta - y_1)2},$$

$$B_2 = \frac{-2x_1(l\sin\beta - y_1)^2 - (l\cos\beta - x_1)[A_3 - 2(l\sin\beta - y_1)y_1]}{(l\sin\beta - y_1)^2},$$

$$C_2 = \frac{4x_1^2(l\sin\beta - y_1)^2 + [A_3 - 2(l\sin\beta - y_1)y_1]^2 - 4R_1^2(l\sin\beta - y_1)^2}{4(l\sin\beta - y_1)^2}. \tag{20}$$

The equation (16) becomes now

$$\begin{aligned} &\left\{2x_1(l\sin\beta - y_1)^2 + (l\cos\beta - x_1)[A_3 - 2y_1(l\sin\beta - y_1)]\right\}^2 \\ &\quad - \left(l^2 + x_1^2 + y_1^2 - lx_1\cos\beta - 2ly_1\sin\beta\right) \\ &\times \left\{4x_1^2(l\sin\beta - y_1)^2 + [A_3 - 2y_1(l\sin\beta) - y_1]^2 - 4R_1^2(l\sin\beta - y_1)^2\right\} = 0, \end{aligned} \tag{21}$$

from which one determines the angle β.

Obviously, this method is not the only one which determines the angle β. All the methods lead to a non-linear equation which has to be solved by numerical methods. We preferred to use this method for the simplicity of the partial derivatives of the function described in equation (21).

The partial derivatives of this function are used to determine the derivative of the angle β with respect to the parameter s (the displacement of the valve) and, consequently, the derivative of the same angle with respect to the rotation angle of the crankshaft. These derivatives can be obtained using the theory of the implicit functions.

If the head of the valve is a planar one, then one has to consider in expression (21) that $R_1 \to \infty$, that is, $R_2/R_1 \to 0$.

4 Numerical example

Let us consider as known values the following data: β_0=30º, l=50 mm, R_2=5 mm, R_1=25 mm, s=12 mm.

It successively results

$$x_1 = d = l\cos\beta_0 = 43.30127\,\text{mm}\,,\; h = l\sin\beta_0 + R_2 = 30\,\text{mm}\,,$$
$$y_1 = h + s - R_1 = 7\,\text{mm}\,,\; A_3 = R_1^2 - R_2^2 + l^2 - \left(x_1^2 + y_1^2\right) = 1451\,\text{mm} \tag{22}$$

The solutions of the equation (21) are

$$\beta_1 = 45.0°\,,\; \beta_2 = 57.4°\,. \tag{23}$$

Obviously, only the value β_1 will be kept.

The diagrams of variation of the angle β in function of the parameters R_1, R_2 and s are captured in Figs. 2, 3, and 4.

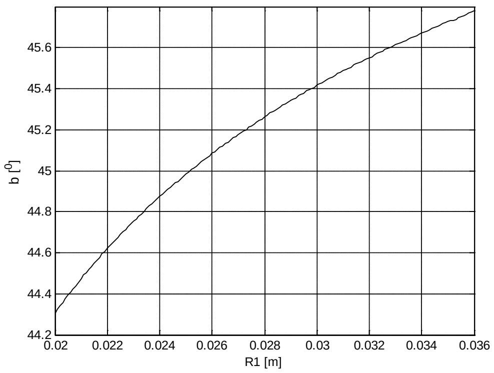

Fig. 2. The variation $\beta = \beta(R_1)$

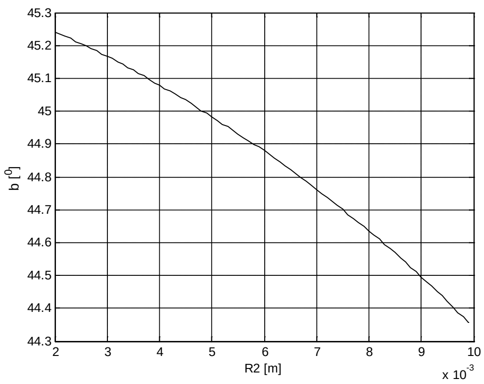

Fig. 3. The variation $\beta = \beta(R_2)$

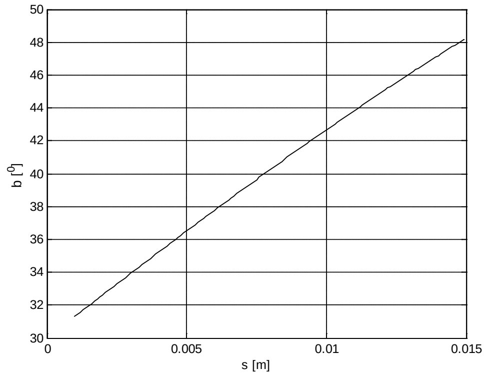

Fig. 4. The variation $\beta = \beta(s)$

Analyzing these figures one may conclude that the angle β increases when the radius R_1 increases, and it decreases when the radius R_2 increases. These variations are non-linear ones, and the influence of the radius R_1 is greater than that of the radius R_2. The variation of the angle β in function of the valve's displacement s is a quasi-linear one.

5 Determination of the possible values

Using the schema presented in Fig. 5, one may write

$$C_2A = \sqrt{l^2 + d^2 - 2ld\cos\beta}\,, \tag{24}$$

$$C_1A = h + s - h_1 = l\sin\beta_0 + R_2 - R_1 + s = l\sin\beta\,, \tag{25}$$

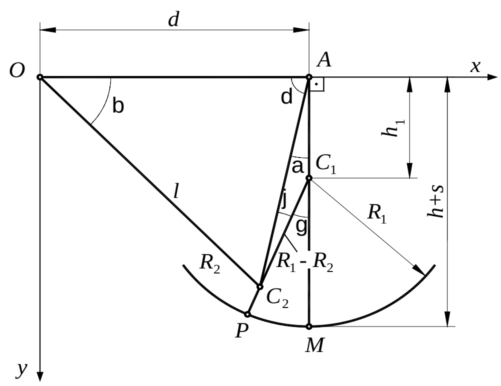

Fig. 5. The geometric schema

$$\frac{l}{\sin\delta}=\frac{C_2A}{\sin\beta}, \tag{26}$$

$$\sin\delta=\frac{l\sin\beta}{\sqrt{l^2+d^2-2ld\cos\beta}}. \tag{27}$$

$$C_2C_1=R_1-R_2, \tag{28}$$

$$\frac{h_1}{\sin\varphi}=\frac{C_2C_1}{\sin\alpha}, \tag{29}$$

$$\sin\varphi=\frac{h_1\cos\delta}{R_1-R_2}, \tag{30}$$

$$\begin{aligned}\gamma=\alpha+\varphi&=90-\arcsin\left(\frac{l\sin\beta}{\sqrt{l^2+d^2-2ld\cos\beta}}\right)\\&+\arcsin\left(\frac{h_1}{R_1-R_2}\sqrt{1-\frac{l^2\sin^2\beta}{l^2+d^2-2ld\cos\beta}}\right).\end{aligned} \tag{31}$$

From the bending condition of the valve (due to the eccentricity of the contact point, the valve is acted by an eccentric force during its operation cycle, that is, this force has a maximum value obtained from the theory of the strength of materials), the angle γ is limited to a maximum value

$$\gamma\le\gamma_{\max} \tag{32}$$

and from the formula (31) we get

$$\begin{aligned}&\arcsin\left(\frac{l\sin\beta}{\sqrt{l^2+d^2-2ld\cos\beta}}\right)\\&-\arcsin\left(\frac{h_1}{R_1-R_2}\sqrt{1-\frac{l^2\sin^2\beta}{l^2+d^2-2ld\cos\beta}}\right)\ge 90-\gamma_{\max},\end{aligned} \tag{33}$$

relation from which one determines the maximum value $\beta_{\max}$.

In the case of the considered numerical example, taking $\gamma_{\max}=60^{\circ}$, one obtains the value

$$\beta_{\max}=55.1^{0}. \tag{34}$$

6 Conclusion

In this paper we performed a geometric study of the contact between the lever and the head of the valve for a spherical contact. We determined the rotation angle of the lever and its maximum values resulted from the bending condition of the valve.

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