

Tribosystems Modeling with Reciprocating Sliding

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Abstract. The paper addresses the issue of the influence that the law of friction has on the dynamic behavior of the mechanical system that interacts with a tribosystem. The emergence of certain nonlinearities of higher order into the law of friction leads to an intensification of the dissipative process and to tribosystem destabilization. Consequently, friction excited self-oscillations are generated into the elements of the mechanical system with a wide spectrum of frequencies, sustained from the external source of energy. The theoretical and experimental modeling of the dissipative process and of the generation of is based on the frictional harmonic oscillator that interacts with the tribosystem. The oscillator is used as a sensitive element to the fluctuations of the frictional force and as a measure of the dissipated energy. Starting from the model, the elaboration of a method and of devices for experimental research provided the opportunity to study the behavior of the tribosystem in unstable operating.

Keywords: tribosystem, friction, sliding, wearout, Lagrange equation

1 Introduction

As functional components of mechanical systems, tribosystems collaterally affect the former's dynamic behavior and have a predominant role in energy dissipation. The evolution character of the dissipative process is influenced and correlatively connected with the friction characteristic (law) occurring at the relative motion of contact surfaces. At present, a series of different laws have been formulated only for dry friction: simple-static; complicated-dynamic. Considering also lubrication (with Stribeck effect), the series of the friction laws diversify [2], [5], [8]. In fact, the laws of friction are complex and include the influence of a series of factors of a different nature related to working, geometric and micro geometric, to tribosystem structure, to the source, properties, and characteristics of the materials for triboelements, and to the working environment.

When nonlinearities of different orders occur into the friction characteristic (with fluctuations in the frictional force), in the mechanical system elements are generated noises under the form of self-oscillations with a wide spectrum of frequencies [2], [5], [8]. The structure of the spectrum, the amplitude and shape of the oscillations are influenced by charging parameters, the friction regime and energy dissipation factors, the properties and state of the materials for triboelements and the lubricant, the origin and intensity of processes arising in the contact area. For problems of such complexity a reliable research method remains the experimental one. However, experimental modeling should be formalized and executed within the framework of the fundamental equations of nonlinear dynamics.

2 Dynamic modeling of mechanical system-tribosystem interaction

A model commonly used to describe and study the oscillatory processes in different systems is the harmonic oscillator. The oscillator is the basis of both mathematical models and necessary technical devices for tests and the experimental research of the studied systems.

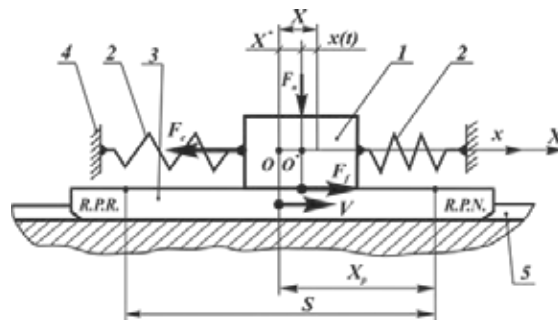


Fig. 1. Mechanical oscillator scheme

The oscillator has been accepted as a model for studying the interaction between the tribosystem characteristics and the mechanical system (figure 1). It consists of block 1 with mass m linkconnected to housing 4, fixed on both sides by means of two similar elastic elements 2, of low rigidity c . The angular frequency of the oscillator is $\omega = \sqrt{c/m}$. The tribological connection between the oscillator and the triboelements is realized through the contact between block 1 and platform 3. Platform 3, driven by a crank-type mechanism, performs a translational reciprocating movement on guide 5 within distances S , with speed:

$$V = r\Omega \left(\sin\varphi_m + \frac{\lambda}{2} (1 - \sin 2\varphi_m) \right) \tag{1}$$

where: Ω - the angular speed of the crank, r - the radius of the crank; l - the length of the rod; $\lambda = \frac{r}{l}$; φ_m - the rotation angle of the crank.

Initially, the X coordinate's origin of the gravity center of block 1 is in the stable equilibrium point O . When the platform begins to move with speed V on distance S distance, block 1, influenced by the friction force F_f , will move in direction X with speed \dot{X} . The relative speed between the contact surfaces of the friction bond becomes $v_r = \dot{X} - V$.

Connecting the oscillator to the tribosystem results in a system composed of two subsystems of different nature (mechanical and dissipative) with own dynamic behaviour, influencing each other during working. The evolution of the dissipative process (of energetic essence) can be studied only from the perspective of Lagrangean formalism, according to which the generalized dissipative force Q_d derives from a force function called Rayleigh dissipative function [6, 7], defined by the relationship:

$$\Phi_d = \sum_{j=1}^N k_j \int_0^{v_j} f_j(u) du \tag{2}$$

where: k_j and $f_j(u)$ – the positive functions defined on spaces j of the contact real elementary areas that are dependent on the $q = X$ coordinate and on the generalized speed $\dot{q} = \dot{X}$ of the oscillator, on speed V of the platform, and on the internal and external parameters of the tribosystem; v_j – the relative local speed of the surfaces on the contact real elementary areas of the spaces; N – the number of real elementary areas within the boundaries of the contact nominal area.

In the sliding tribosystem the role of dissipative generalized force Q_D is played by the total force of friction F_f , defined by the gradient of the dissipative force in the direction of the relative motion of the contact surfaces [2], [8]

$$Q_d = F_f = -\frac{\partial}{\partial (\dot{X} - V)} \sum_{j=1}^N k_j \int_0^{v_j} f_j(u) du = -\frac{\partial \Phi_d}{\partial v_r} \tag{3}$$

where: $\dot{X} - V = v_r$ – the generalized speed is represented by the relative speed of the surfaces when the platform and the block of the oscillator move.

The movement of the oscillator under the action of the dissipative forces is described by the Lagrange equation

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{X}} \right) - \frac{\partial L}{\partial X} = -\frac{\partial \Phi_d}{\partial v_r} \tag{4}$$

where: $L = (T - \Pi)$ – the Lagrange function (kinetic potential); $(T = m \frac{\dot{X}^2}{2})$ - the kinetic energy of the oscillator; $(\Pi = c \frac{X^2}{2})$ – the potential energy accumulated into the elastic elements of the oscillator.

If the working parameters are maintained at constant level, the oscillator asymptotically stabilizes his position temporarily “freezing” in the vicinity of the unstable equilibrium point O^* (figure 1) with coordinate X^* . When the position of the block is stabilized, speed $\dot{X} \rightarrow 0$, and the contact relative speed becomes $v_r = -V$. In this state the oscillator passes in a steady and stable working mode relative to the point X^* , where block 1 will be in the balance of forces in the movement direction of the platform ($F_f + F_e = 0$), where: $F_f = F_c$ - the constant component of the Coulomb type friction force; $F_e = -cX^*$ - the force of elasticity. On the oscillator’s passing in steady state, the dissipative friction force between the block and the platform remains linearly dependent on the generalized coordinate X^* and the platform speed V .

Lagrange equation for the steady state conditions of the oscillator takes the form.

$$-\frac{\partial L}{\partial X^*} = \frac{1}{2}c \frac{\partial}{\partial X^*} (X^*)^2 = cX^* = -\frac{\partial \Phi_d}{\partial (-V)} = F_c = const \quad (5)$$

According to (5), the friction force F_c and the energy dissipation power $P_d(V)$ for motion with relative speed $(-V)$.

$$F_c = cX^* \quad P_d(V) = -cX^*V \quad (6)$$

The loss of stability violates the balance of forces, the movement of the oscillator being determined by the variation of the dissipative forces. In the event of some instability in the operation of the tribosystem, with disruptive fluctuations of the friction force, the oscillator passes into a self-oscillation regime maintained from the external source of energy. The nature and evolution of the dissipative process can be efficiently set in the analysis result of the oscillator’s motion in the phase space (figure 2) built in the phase coordinates $Y = X$ and $Z = \frac{\dot{X}}{\omega}$. To this aim, the ratio between the motions of the representative point M on the phase trajectories for each two cycles in a row (i) and ($i + 1$) are examined step by step.

In examining the movement on the phase trajectory of cycle i in self-oscillation state (figures 1, 2) the coordinate and speed of the oscillator are

$$X_i = X_i^* + x_i, \dot{X}_i = \dot{X}_i^* + \dot{x}_i,$$

where: $X_i^* = (X_i^{max} + X_i^{min})/2$ - average component; x_i - variable nent; $\dot{X}_i^* = \Delta X_i^* \omega$ - speed of passage from the previous cycle ($i - 1$) to cycle i . On the i cycle path, two types of movement can be identified: 1 - with low speed \dot{X}_i^* , determined by the variation of the mean component coordinate X_i^* of the cycle; 2 - with high speed \dot{x}_i , determined by the variation of the x_i coordinate and angular own frequency ω of the oscillator.

$$\begin{aligned}
\frac{dE_i}{dt} &= c \left(X_i \dot{X}_i + \frac{\dot{X}_i \ddot{X}_i}{\omega^2} \right) \\
&= c \left[X_i^* \dot{X}_i^* + \left(X_i^* \dot{x}_i + \frac{\dot{X}_i^* \ddot{x}_i}{\omega^2} \right) + \left(x_i \dot{x}_i + \frac{\dot{x}_i \ddot{x}_i}{\omega^2} \right) \right] \quad (8) \\
&= c R_i V_n^i
\end{aligned}$$

where: V_n^i – the projection of the phasic speed V_Φ^i on the direction of the normal \bar{n}_i , drawn through the M_i representative point moving on the phasic trajectory of the cycle when intersecting the representative orbit h_i . From a dynamic point of view, V_n^i is the rate of change (dissipation) of the oscillator's energy at movement of the M_i point on the phasic path.

According to relation (8), the variable component of the oscillator's energy includes three groups of factors:

1. $X_i^* \dot{X}_i^*$ – for variation of the low speed and low frequency component;
2. $x_i \dot{x}_i + \frac{\dot{x}_i \ddot{x}_i}{\omega^2}$ – for variation of the high speed and high frequency component;
3. $X_i^* \dot{x}_i + \frac{\dot{X}_i^* \ddot{x}_i}{\omega^2}$ – for mutual influence of the low and high speed factors.

Setting the mechanical status of the oscillator by experimental methods, based on expressions (7) and (8), the normal component V_n^i of the phasic speed is determined, which comprises the three groups of influence factors.

$$\begin{aligned}
V_n^i &= \frac{X_i \dot{X}_i + \frac{\dot{X}_i \ddot{X}_i}{\omega^2}}{R_i} = \frac{X_i^* \dot{X}_i^* + \left(X_i^* \dot{x}_i + \frac{\dot{X}_i^* \ddot{x}_i}{\omega^2} \right) + \left(x_i \dot{x}_i + \frac{\dot{x}_i \ddot{x}_i}{\omega^2} \right)}{\sqrt{X_i^2 + \left(\frac{\dot{X}_i}{\omega} \right)^2}} \quad (9) \\
&= (V_n^i)^{**} + (V_n^i)^* + (V_n^i)^v
\end{aligned}$$

At temporary “freeze” of the oscillator's mechanical status in the point of coordinate X_i^* , the instant steady state functioning conditions are obtained fixed on the average level of the oscillation cycle generated by the stationary friction force F_c^i at the platform movement with speed V^i . Based on relations (5) and (6), the solution for friction force and energy dissipation power on the average cycle component is obtained.

$$F_c^i = c X_i^* \quad (10)$$

$$P_d^i(V^i) = -c X_i^* V^i \quad (11)$$

The total energy dissipation power in the contact area for each movement cycle consists of two basic components: $P_d^i(V^i)$ – instantly constant, defined by speed V^i within cycle limits; variable $P_d^i(V_n^i)$, defined by speed V_n^i .

$$\begin{aligned}
 P_d^i &= P_d^i(V^i) + P_d^i(V_n^i) \\
 &= -c(X_i^*V^i + R_iV_n^i)
 \end{aligned}
 \tag{12}$$

Taking into account relation (9) the following is obtained:

$$\begin{aligned}
 P_d^i = F_f^i v_r^i &= -\frac{\partial \Phi_d^i}{\partial v_r^i} v_r^i \\
 &= -\left[cX_i^*V^i + cR_i(V_n^i)^{**} + cR_i(V_n^i)^* + cR_i(V_n^i)^v \right]
 \end{aligned}
 \tag{13}$$

The total instantaneous value of the friction force during cycle (i) conditioned by the achievement of the working unstable self-oscillation state:

$$F_f^i = \frac{P_d^i}{v_r^i} = -\left[cX_i^* \frac{V^i}{v_r^i} + cR_i \frac{(V_n^i)^{**}}{v_r^i} + cR_i \frac{(V_n^i)^*}{v_r^i} + cR_i \frac{(V_n^i)^v}{v_r^i} \right]
 \tag{14}$$

or

$$F_f^i = \left[F_c^i + (F_v^i)^{**} + (F_v^i)^* + (F_v^i)^v \right]
 \tag{15}$$

where: $F_c^i = -cX_i^* \frac{V^i}{v_r^i}$; $(F_v^i)^{**} = -cR_i \frac{(V_n^i)^{**}}{v_r^i}$; $(F_v^i)^* = -R_i \frac{(V_n^i)^*}{v_r^i}$; $(F_v^i)^v = -cR_i \frac{(V_n^i)^v}{v_r^i}$.

The component F_c^i (of Coulomb type) of the friction force is defined by the linear factors of the dissipative function Φ_d in the vicinity of the X_i^* coordinated point. The variable (fluctuating) components of the friction force $(F_v^i)^{**}$, $(F_v^i)^*$, $(F_v^i)^v$ occur as a result of various dynamic effects from the contact zone with higher order nonlinearities and may vary in a wide range of frequencies and amplitudes.

When condition $v_r = \dot{x} - V < 0$ is accomplished, the F_c component of the friction force has always the same sense as platform speed vector V and determines the energy dissipation level in the contact zone at relative motion of the surfaces. The variable components (depending on the speed signs $(V_n^i)^{**}$, $(V_n^i)^*$, $(V_n^i)^v$ sign) can change their sign according to direction within the limits of the same oscillation cycle. On the negative direction, the energy previously accumulated in the oscillator’s elements dissipates in the contact area, and, on the positive direction, through the tribosystem, a new portion of energy from the external source is introduced into the oscillator. This behaviour relates to the achievement of the dynamic effect of variable dissipation on

direction, also called “negative friction” effect, which is the main cause of engendering friction excited self-oscillations in the mechanical system [1], [2], [5], [7].

Accepting cycle (i) as a benchmark, at crossing to the next cycle ($i + 1$), the X_i^* (coordinated point (figure 2) acquires an additional movement $\Delta X_{(i+1)}^*$ with the speed $X_{(i+1)}^*$, where: $\Delta X_{(i+1)}^* = X_{(i+1)}^* - X_i^*$; $\dot{X}_{(i+1)}^* = \Delta X_{(i+1)}^* \omega$; $X_{(i+1)}^* = (X_{(i+1)}^{max} + X_{(i+1)}^{min})/2$. The coordinate, the absolute speed and the relative speed within the cycle limits ($i + 1$) with respect to cycle (i) will be: $X_{(i+1)} = (X_{(i+1)}^* + x_{(i+1)}) = (X_i^* + \Delta X_{(i+1)}^* + x_{(i+1)})$; $\dot{X}_{(i+1)} = (\dot{X}_{(i+1)}^* + \dot{x}_{(i+1)})$; $v_r^{(i+1)} = (\dot{X}_{(i+1)} - V^{(i+1)})$.

The prime integral of equation (4) for the cycle ($i + 1$)

$$E_{(i+1)} = \frac{1}{2} c \left[(X_{(i+1)}^*)^2 + \left(\frac{\dot{X}_{(i+1)}^*}{\omega} \right)^2 \right] + c \left[X_{(i+1)}^* x_{(i+1)} + \right. \tag{16}$$

Similarly, the instantaneous frictional force is determined at the motion of the oscillator during the ($i + 1$) cycle

$$F_f^{(i+1)} = - \frac{P_d^{(i+1)}}{v_r^{(i+1)}} = - \left[c X_{(i+1)}^* \frac{V^{(i+1)}}{v_r^{(i+1)}} + c R_{(i+1)} \frac{(v_n^{(i+1)})^{**}}{v_r^{(i+1)}} + \right. \tag{17}$$

$$F_f^{(i+1)} = \left[F_c^{(i+1)} + (F_v^{(i+1)})^{**} + (F_v^{(i+1)})^* + (F_v^{(i+1)})^v \right] \tag{18}$$

The loss of system stability can occur for both types of movements: of low frequency and low speed, and with high frequency and high speed. As an experimental criterion for assessing the movement regime, the deviations of the displacements $\Delta X_{(i+1)}^*$ and $\Delta x_{(i+1)}$ between each pair of consecutive cycles are used:

under steady state oscillatory motion

$$\begin{cases} \Delta X_{(i+1)}^* = X_{(i+1)}^* - X_i^* = 0 \\ \Delta x_{(i+1)} = (x_{(i+1)}^{max} - x_{(i+1)}^{min}) - (x_i^{max} - x_i^{min}) = 0 \end{cases} \tag{19}$$

under unstable state oscillatory motion

$$\begin{cases} \Delta X_{(i+1)}^* = X_{(i+1)}^* - X_i^* \neq 0 \\ \Delta x_{(i+1)} = (x_{(i+1)}^{max} - x_{(i+1)}^{min}) - (x_i^{max} - x_i^{min}) \neq 0 \end{cases} \tag{20}$$

3 Experimental results

Based on the above theoretical model, an original model of tribometer was made, with a cyclic translational motion and equipped with proper systems for measuring the dynamic characteristics (variables) of the system. As dynamic variables the following are used:

1. displacement X_i (mm) in the movement direction of the platform of the mass center of the oscillating part against point "O" of the oscillator's stable equilibrium;
2. linear speed $v_i = \dot{X}_i$ (mm/s) of the mass center of the oscillating part;
3. linear acceleration $a_i = \ddot{X}_i$ (mm/s^2) of the mass center of the oscillating part;
4. linear speed V^i (mm/s) of the platform in the movement direction within the limits of cycle i of the oscillator's oscillation;
5. cyclic frequency of the platform n_c (min^{-1});
6. experimental average temperature θ_k ($^{\circ}C$) in the contact area during cycle k of the platform movement.

The dynamic variables are recorded, step by step, in the form of time series for each cycle (i) of the oscillator's movement and over each cycle (k) of the platform movement.

Computerized technologies for recording the dynamic variables and for processing of the time series were used to study the behavior of the tribosystem in unstable operating conditions. The evolution of the friction process was estimated through the variation characteristic (law) of the friction force $F_f^k = F_f^k(v_r)$ for relative speed variation; the evolution of the dissipative process (energy dissipation) was estimated through the work of the friction force W_f^k cumulated for each cycle (k) of the platform movement.

In the case of the translational cyclical movement, the accuracy of local friction force determination depends on the resolution $s = \frac{T_k}{T}s$ of the system, established within the limits of the platform movement cycle, where $T = \frac{2\pi}{\omega}$ - the period of the oscillator's self-oscillation, $T_k = \frac{2\pi}{\Omega}$ - the period of cycle (k) of the platform movement. Resolution s represents the oscillator's number of cycles included in a platform movement cycle.

As a benchmark characteristic for establishing the experimental friction law, the integrated mean within the limits of cycle (i) period of the local friction force is used

$$F_f = \frac{\omega}{2\pi} \int_{-\frac{\pi}{\omega}}^{+\frac{\pi}{\omega}} F_f^i dt \quad (21)$$

where: F_f^i - the instantaneous value of the friction force within the cycle (i) limits.

The level of energy dissipation (represented by the work of friction forces integrated on period T of cycle (i))

$$W_f^i = \int_{-\frac{\pi}{\omega}}^{+\frac{\pi}{\omega}} F_f^i v_r^i dt \quad (22)$$

The work of friction forces (dissipated energy) W_f^k during the cycle (k) period of the platform movement summed

$$W_f^k = \sum_{i=1}^s W_f^i \quad (23)$$

The results of the experimental data analysis for a number of couples of materials and lubricants revealed a different and complex behavior of the frictional force (figure 3) on different portions and areas of the characteristics points, at relative movement of the contact on the platform cycle strokes (S). Within a cycle the following are identified: the DSM stroke of the platform motion on the direct sense of the movement and the OSM stroke for opposite movements; portions with acceleration movement (AM) per stroke and deceleration movement zones (DM); (ZRP) zones of return points of the contact per stroke and (PMS) zones of points of maximal speed of the cycle. If the platform is actioned by the crank mechanism, return points with different kinematic characteristics are obtained at the end of the strokes: (RPN) - return point near; (RPR) - return point removed. A pronounced dynamic behavior of the friction force occurs when changing speed direction at entry into and exit from the areas of return points.

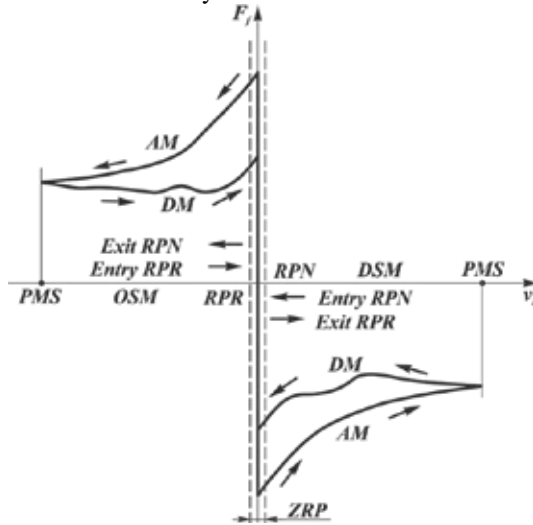


Fig. 3. Law of friction determined experimentally for cyclical translatory movement

During the experimental research, the following conditions were set: oscillator mass $m = 0,2kg$; rigidity of the elastic element $c = 100N/mm$; angular frequency of the oscillator $\omega = 6280s^{-1}$; angular frequency of the crank $\Omega = 31,141s^{-1}$; cyclic frequency of the platform $n_c = 300min^{-1}$; system resolution $s = 200$; contact load in two ways (1 – with constant normal loading during the testing period, 2 - loading in consecutive steps); drop lubrication. The experimental temperature θ in the

contact area was measured with a K type mini K thermocouple. Temperature evolution in the contact area occurs due to heat produced at the exhaust of mechanical energy. Contact form - flat with dimensions: width $b = 2mm$; length $l = 40mm$. Stroke length $S = 100mm$.

Under study was the tribological behavior of the materials of the pair: block triboelement - electrolytic chromium on a steel surface; platform triboelement - 38KH2MYUA steel (similar to 41CrAlMo7). The contact was lubricated with MT-16P GOST 6360-83 oil (similar to SAE 40 API CB).

Experimental results are shown in figures 4 and 5. For temperature increase in the contact area (figure 4) for a load $F_n = 2,0kN$, the energy dissipation level per cycle (k) of platform movement will vary non-linearly with a trend of asymptotic stabilization, between the work values of the friction forces of $W_f^k = (40 \dots 50)J$. If the operating parameters are maintained constant (F_n load and cyclic speed of the platform n_c), process stabilization is complete at temperature values $\theta = (240 \dots 250)^\circ C$.

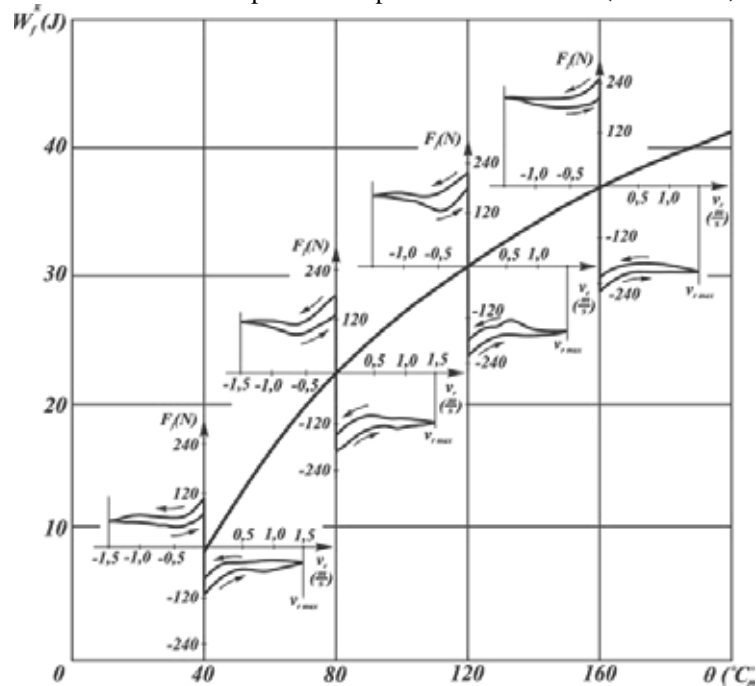


Fig. 4. Character of the dissipative process and evolution of the friction law with experimental temperature variation θ in the contact area for normal force loading $F_n = 2,0kN$.

Of special importance for determining the dynamic behaviour of mechanical systems is the evolution of the law of friction in concrete working conditions. Figure 4 shows a significant influence of temperature on friction force for relative speed v_r variation per cycle strokes of platform movement. When temperature increases, the friction force changes both values and the variation manner per stroke.

Different behaviour is identified if the variation of the contact load force F_n takes place at constant temperatures θ . To achieve a boundary lubrication regime, during the experiment the temperature was maintained at high-level in the contact zone, with a permissible variation within $\theta = (200 \dots 210)^\circ C$. When the contact was loaded (figure 5), a practically linear dependence of the level of energy dissipation W_f^k on force F_n was obtained. In this case, the evolution manner of the force in the experimental friction law changes.

4 Conclusion

In cases when the friction law (for the couple of materials used in the construction of the triboelements) higher order nonlinearities occur, the friction forces generate in the mechanical system elements noises as self-oscillations in a wide range of frequencies.

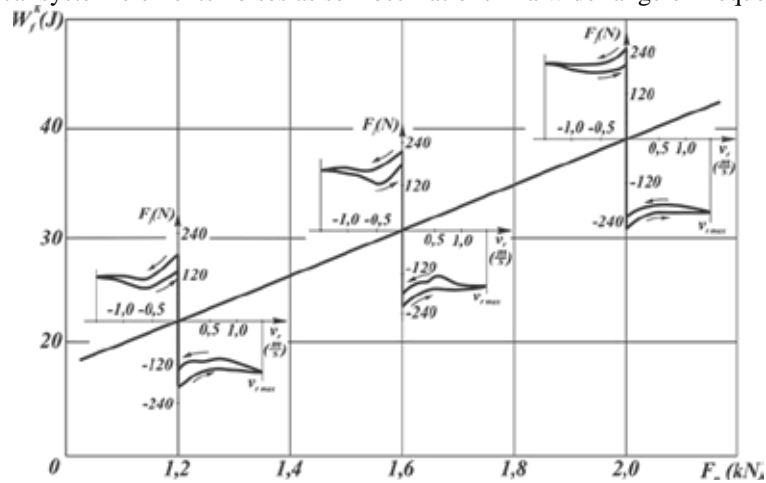


Fig. 5. Nature of the dissipative process and evolution of friction law for variation of the load force F_n at constant temperature θ into contact area

Based on Lagrange equation, the dynamic model of the interaction between the mechanical system and the tribosystem was developed. The harmonic oscillator with elastic elements was accepted as mechanical system for modeling.

The examination of the dynamic model identified the structure of the friction force in unstable operating conditions of the mechanical system. In the structure of the total friction force F_f four possible components appear: a component F_c^i (of Coulomb type), defined by the linear factors of the dissipative function Φ_d ; three variable (fluctuating) components within the limits of each oscillation cycle $(F_v^i)^{**}$, $(F_v^i)^*$, $(F_v^i)^v$ (occur as a result of various dynamic effects in the contact zone with higher order nonlinearities and can vary over a wide range of frequencies and amplitudes).

Based on the model with harmonic oscillator, an original model of tribometer has been made, with cyclical translational movement, equipped with proper measuring systems for the dynamic characteristics of the oscillator, and the method for experi-

mental determination [3] of the dynamic characteristics of the sliding tribosystem in unstable operating regime was developed.

References

1. Andronov A A, Vitt A A and Khaikin S E Teoriya kolebany. Moskva: Nauka (1981)
2. Armstrong-Helouvry B, Dupont P and Canudas De Wit C A Survey of Models, Analysis Tools and Compensation Methods for the Control of Machines with Friction Automatica, 30(7) 1083-138 (1994)
3. Crudu I Tribomodelarea. București: AGIR (2011)
4. Kragelskii I V Treniye i iznos. Moskva: Izdatelstvo Mashinostroyeniye (1968)
5. Kragelsky I V and Gitis N V Friksionnye avtokolebaniya. Moskva: Nauka (1987)
6. Landau L D and Lifshits Ye M Teoreticheskaya fizika: Mekhanika Tom1 Moskva: Nauka (1988)
7. Lurye A I Analiticheskaya mekhanika. Moskva: Izdatelstvo fiziko-matematicheskoy literatury (1961)
8. Wojewoda J, Stefanski A, Wiercigroch M and Kapitaniak T Hysteretic effects of dry friction: modeling and experimental studies Phil. Trans. R. Soc. A 366 747-65 (2008)

