The 40th International Conference on
Mechanics of Solids, Acoustics and Vibrations &
The 6th International Conference on
“Advanced Composite Materials Engineering”
ICMSAV2016& COMAT2016
Brasov, ROMANIA, 24-25 November 2016

CONSIDERATIONS REGARDING THE OPTIONS FOR OPTIMAL CONTROL OF DYNAMIC MECHANIC SYSTEMS WITH FRICTION

Bogdan Ciorușa[1], Nicolae Pop[2], Mirela Coman[1]
[1] Technical University of Cluj-Napoca, North University Center Baia Mare, Faculty of Engineering, Baia Mare, ROMANIA, bciorutza@yahoo.com – comanmirela2000@yahoo.com
[2] Institute of Solid Mechanics – Romanian Academy, Bucharest, ROMANIA, nicpop@gmail.com

Abstract: *According to the dynamical systems theory, many dynamical systems (physical-mechanic, chemical, socio-economical, ecological) can be modeled by mathematical relations, such as deterministic and/or stochastic differential and/or difference equations. These systems change with time or any other independent variable or parameters (friction in this case) according to the dynamical relations which outlines the mathematical model. This paper analyzes, according to the scientific literature we consulted, a few options available for characterizing the friction models, such as identification and stability analysis of friction, characterization of the limit cycles caused by friction and, in the end, friction estimation and optimal control techniques.*

Keywords: *dynamic systems, friction, optimal control.*

1. INTRODUCTION

According to the dynamical systems theory [1], many dynamical systems (physical, chemical, economical, ecological) [2] can be modeled by mathematical relations, such as deterministic and/or stochastic differential and/or difference equations [3]. These systems change with time or any other independent variable or parameters according to the dynamical relations, which outlines the mathematical model. A mathematical model (**Fig. 1**) is, especially in this case, a precise representation of a system’s dynamics used to answer questions via analysis and simulation, describing the input/output behavior of systems and often in so-called “state space” form [13].

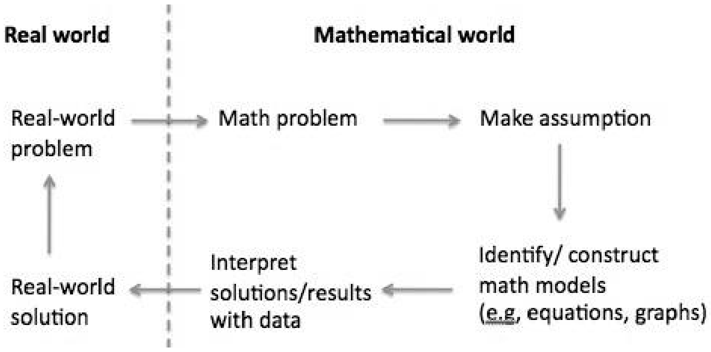


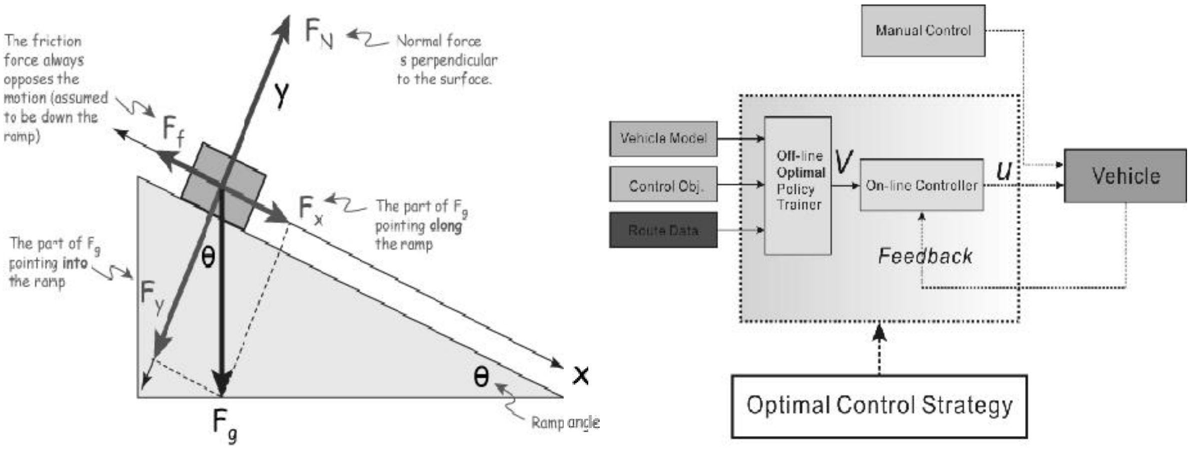
Figure 1: Mathematical models as a bridge between real and mathematical world
(source: math4teaching.com/what-is-mathematical-modeling?)

It is possible to steer these systems from one state to another state by the application of some type of external inputs or controls. If this can be done at all, there may be different ways of doing the same task. If there are different ways of doing the same task, then there may be one way of doing it in the "best" way. This best way can be minimum time to go from one state to another state, or maximum thrust developed by a rocket engine. The input given to the system corresponding to this best situation is called "optimal" control [4]. For high-performance engineering systems, model-based controllers are typically required to accommodate for the system nonlinearities [14].

Unfortunately, developing accurate models for friction has been historically challenging. Typical models are either discontinuous and many other models are only piecewise continuous [12].

Friction is generally described as the resistance to motion [9], when two surfaces slide against each other. In most cases friction is a useful phenomena making many ordinary things like walking and the brake in a car possible. On the other hand friction can also cause undesirable effects. For high precision mechanical motion systems for example, friction can deteriorate the performance of the system.

Possible unwanted consequences caused by friction are steady-state errors, limit cycling and hunting. In motion control a possible way to minimize the influences of friction is to compensate for it. In order to be able to compensate the effect of friction, it is necessary to describe the frictional behavior (**Fig. 2a**).



a) b)

Figure 2: a)The description of the frictional bahavior; b)The description of the friction's classic control system (source: www.drcruzan.com/friction.html / www3.imperial.ac.uk/.../energy)

Since no exact formula for the friction force is available, friction is normally described in an empiric model. By canceling the friction effect, the nonlinearity in the system, assuming no other nonlinear behavior is present, is removed. This is beneficial for classic control (**Fig. 2b**) which is based on linearity and where the feedback is therefore not able to completely compensate for frictional effects [9].

2. FRICTION MODELS: PROPERTIES AND CLASSIFICATION

Friction is the force resisting the relative motion of solid surfaces, fluid layers and material elements sliding against each other (**Fig. 3**). In the scientific literature, there are several types of friction:

- *dry friction* - resists relative lateral motion of two solid surfaces in contact. Dry friction is subdivided into static friction ("stiction") between non-moving surfaces, and kinetic friction between moving surfaces;
- *fluid friction* - describes the friction between layers of a viscous fluid moving relative to each other;
- *lubricated friction* - case of fluid friction, where a lubricant fluid separates two solid surfaces;
- *skin friction* - component of drag, the force resisting the motion of a fluid across the surface of a body;
- *internal friction* - is the force resisting motion between the elements making up a solid material while it undergoes deformation.

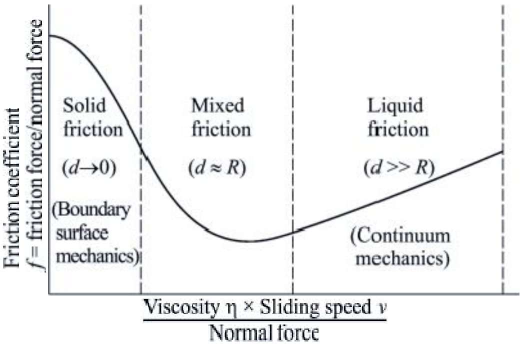


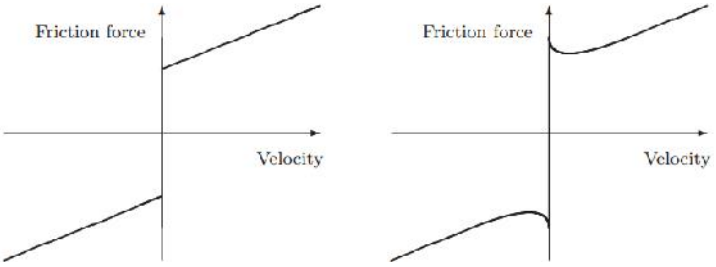
Figure 3: The description of friction as a force in different physical environments (source: http://virtual.cvut.cz/.../JXU+hgGQDwTGcHu7.html)

When surfaces in contact move relative to each other, the friction between the two surfaces converts kinetic energy into thermal energy. This property can have dramatic consequences, which may lead to performance degradation and/or damage to components.

The main purpose of this paper is to gain more insight into the available different friction models found in literature, such as static friction models (the Coulomb, the viscous and the Stribeck model), the Dahl model, the LuGre model, the Leuven (integrated friction) model and physics-motivated friction models, and their main differences, according to the expression of law used and the graphical reprezentation.

2.1. Static friction models

The most basic friction models contain Coulomb friction and linear viscous damping. For situations where the starting friction is higher than friction at a nonzero velocity "static" friction force (Fs) can be distinguished as can be seen in **Fig. 4a**. For the most common situations the friction decreases with increasing velocity for a certain velocity regime. This is called the Stribeck effect and is shown in **Fig. 4b**.



a) b)

Figure 4: The representation of the basic static friction force effects versus velocity [9]

These basic models describe a static relationship between the friction force and velocity. At rest where the velocity is zero, the friction force cannot be described as a function of velocity alone. For some applications this static model is adequate enough to describe the effects of friction. For practical high accurate positioning systems however other frictional properties have to be considered for a satisfying model.

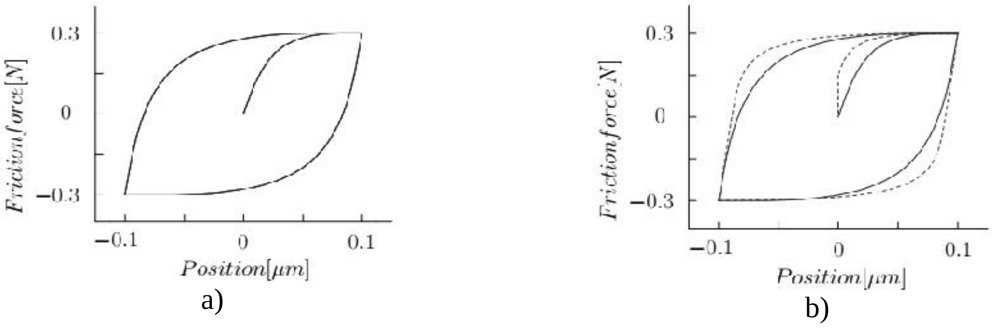
2.2. The Dahl and the LuGre models

Dahl [8] explained frictional behavior with an analogy for the stress-strain property for materials. For objects subjected to small displacements he observed that the objects returned to its original position.

Dahl compared this with the spring-like elastic material behavior, occurring in the bonding forces between the two surfaces. For larger displacements the bonding interface would undergo a plastic deformation resulting in a permanent displacement. In other words Dahl assumed that friction force is not only a function of the velocity, but of displacement as well (**Fig. 5a**).

The Dahl friction law is able to model predisplacement and hysteresis in a dynamic model, but it is unable to capture many other phenomena like the Stribeck effect and the ability to predict stick-slip motion.

The Dahl model forms the basis of the LuGre model (**Fig. 5b**) which replaces the constant Fc with a velocity-dependent function g(v) and adds two more terms. It has an additional damping σ1 associated with microdisplacement, and a memoryless velocity-dependent term f(v) [7].



a) b)

Figure 5: The representation of the main graphical differences between the Dahl and LuGre model [9]

The LuGre model represents a spring-like behavior for small displacements just like the Dahl model Another advantage of the LuGre is that both the presliding and the sliding regimes are described by the same model.

2.3. The Leuven and the physics-motivated friction models

The intergrated friction model structure, also known as the Leuven model [16], further improves the LuGre model by including presliding hysteresis with nonlocal memory. This type of hysteresis occurs for nonperiodic presliding and is an improvement for the model's accuracy with respect to reality.

The implementation for this hysteresis model requires two memory stacks for the non-local memory concept. One memory stack is needed for the minima of Fh and one for the maxima. Although the Leuven model is a great improvement with respect to the nonlocal memory hysteresis, it also introduces a number of implementation difficulties and a discontinuity in the friction force for some circumstances.

In order to describe the friction really fenomenon, experimental models were introduced. One of them was introduced by Shirakashi and Usui [15] who used anexponential law to relate frictional stress to normal stresses and maximum shear flow stresses (te). The model was initially conceived to non-ferrous metals and was derived from the concept of apparent/effective contact area. Later it was applied to metals in general.

Based on the experiment where a bar-shaped tool slids over the inner surface of a ring specimen, Iwata et al. [11] proposed an expression for frictional stresses dependent on Coulomb's friction coefficient, normal stress and Vickers hardness of the workpiece material (Hv). Eldridge et al. (1991) used an experimental curve which relates shear stress and yield stress in shear. The temperature dependency is accounted for by an exponential function so that the friction stresses and temperature are inversely proportional. Later, Wu et al. [18] assumed that the friction stress is directly proportional to the equivalent stress. Sekhon and Chenot (1993) adopted Norton's friction law which assumes that frictional stresses are proportional to the relative sliding velocity.

Table 1 presents the friction models used in the simulations of the really friction process.

Table 1. A few experimental friction models (source: www.scielo.br/.../scielo.php)

Method	Expression
Coulomb's law	$\tau_f = \mu \sigma_n$
Exponential law	$\tau_f = \tau_e \left[1 - \exp\left(-C \frac{\sigma_n}{\tau_e} \right) \right]$
Iwata et. al. (1984)	$\tau_f = \frac{H_v}{0.07} \tanh\left(\frac{0.07 \mu \sigma_n}{H_v} \right)$
Eldridge et al. (1991)	$\tau_f = \tau_f (T_0) \exp\left(\frac{A}{T} \right)$
Wu et al. (1996)	$\tau_f = - \Omega \sigma_{eq}$
Sekhon and Chenot (1993)	$\tau_f = - \alpha K \left\| \boldsymbol{v}_f \right\|^{p-1} \boldsymbol{v}_f$

The models described so far are all empirical-based models (**Fig. 6**). An other branch of friction models is the so called physics-motivated friction models. These describe friction on three different physical levels namely on atomic-molecular, asperity-scale and at tectonic-plate level.

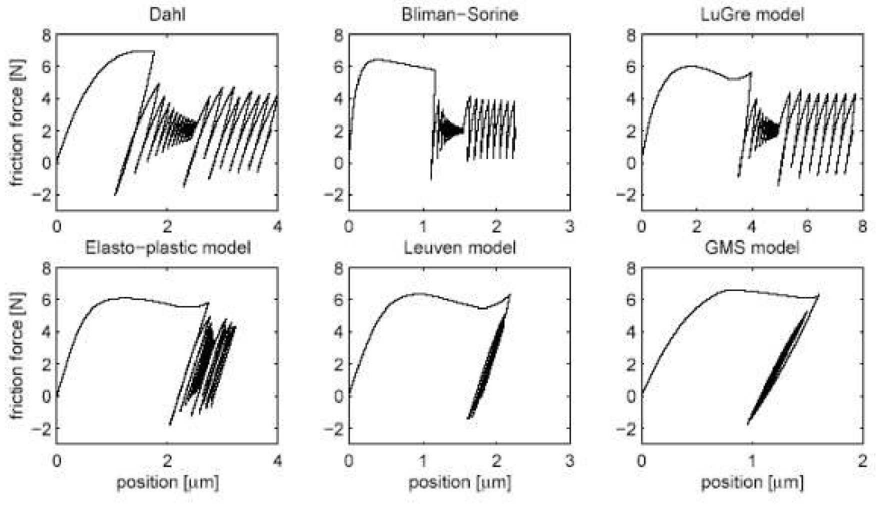


Figure 6: The main empirical-based friction models
(source: https://people.mech.kuleuven.be/~farid/tribology/friction/nondrifting.html)

In the literature many different physics-based friction models can be found which all show strong similarities until a certain extension. A generic friction model (GFM) was developed by Swevers et al. [16, 17]. The asperity has a lumped mass in the tip and is connected to the object with three springs. Due the nature of these models, more mechanisms can be taken into account compared to the empirical models such as normal creep, adhesion and impact of asperity masses for example. Haessig and Friedland [10] imagined asperities matching to bristles of a brush and called it the bristle model. Other closely related models are the Frenkel-Kontorova model, the Frenkel-Kontorova-Tomlinson model, the Burridge-Knopoff model and the Tomlinson-Prandtl atomic model [6]. Although the physics-based models are capable of capturing all friction-induced phenomena that are observed so far, they are much too complicated for online control purposes, or for the optimal control.

3. OPTIONS FOR OPTIMAL CONTROL OF DYNAMIC SYSTEMS WITH FRICTION

Friction related control problems have gained a lot of interest lately. In this section we only consider systems with a single friction interface. It has an associated relative velocity v, relative position x, friction force F and external force Fe, as seen in **Fig. 7**.

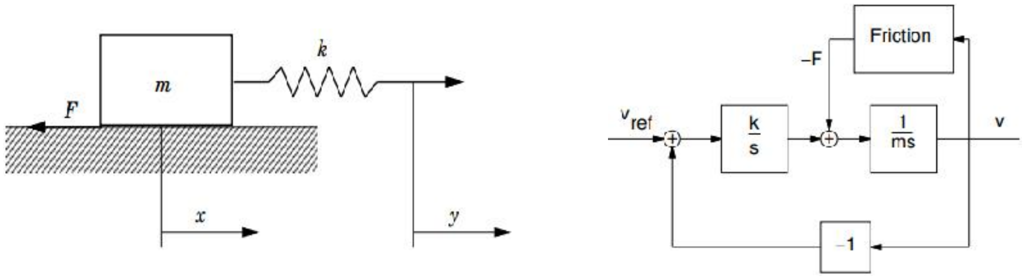


Figure 7: The friction model considered for optimal control and its associated system (Simulink block diagram)

Stick-slip motion (from Fig. 7) is a common problem occuring for low velocity motion. The resulting motion is jerky and switches between periods of sticking and slipping. Stick-slip motion is often highly undesirable in applications such as machine tools. Apart from poor control performance, the motion can also give vibrations and noise. It is, therefore, interesting to understand the nature of stick-slip motion and how to avoid it. Stick-slip motion is caused by the friction force at zero velocity, which is higher than at a small nonzero velocity.
According to the scientific literature we consulted there are a few options available for characterizing the friction models, such as:

- **identification and stability analysis of friction** using Lyapunov’s method;
- characterization of the limit cycles caused by friction (**stick-slip & hunting- "jump" motion**);
- **modeling and simulation of the dynamic behavior of systems with friction**, using continuosly differentiable models (e.g. f(q’) = 6 terms parameterizable form with tanh) – **Fig. 8**;

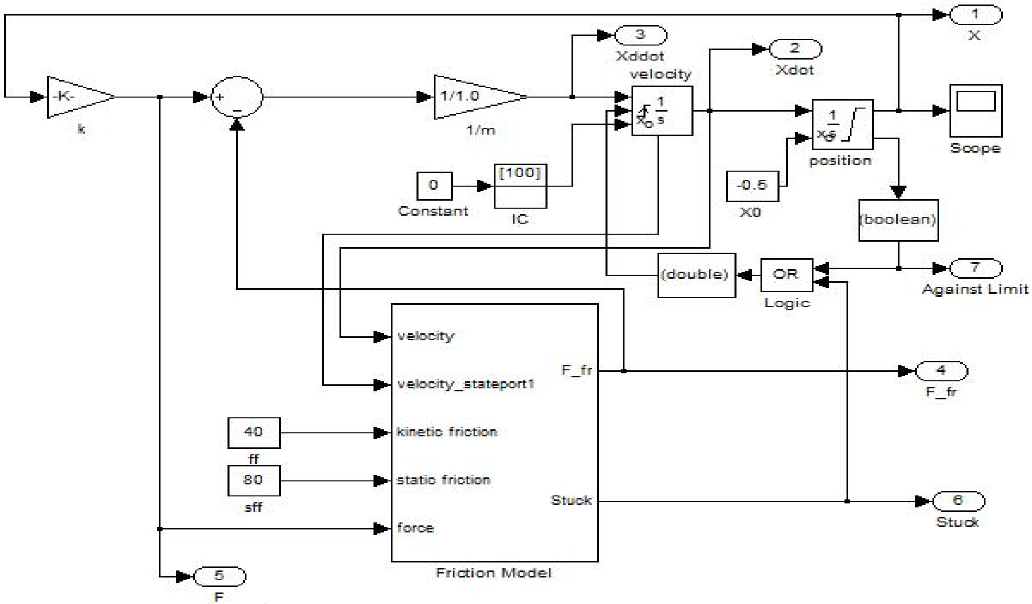


Figure 8: A Simulink$^{®}$ block diagram for optimal control of a dynamic system with friction

- **optimal control** & **friction estimation using LQR** (linear quadratic regulator), **PID** (proportional-integral-derivative), **SVM** (support vector machine) & **AAD** (adaptive anti-disturbance) – e.g. ABS (anti-lock braking technology system);
- **simple adaptive compensation**, in case of the dynamic LuGre friction model:
    - for tracking the given desired trajectory;
    - for control design of the unknown system parameters (identification or inverse problem), with nonlinear function α(x, x') known;
    - for control design for full set of unknown parameters;
- **adaptive compensation with diff. filter** & **fuzzy controller** (full-states, position measurements).

4. CONCLUSION

As we have seen, according to the scientifical literature, the desire to understand friction is old. Friction is found in almost every mechanism with moving parts. Engineering examples where friction is present abound, e.g. bearings, transmissions, hydraulic and pneumatic cylinders, valves and brakes.

Friction is an important aspect of many control systems, both highprecision servo mechanisms and simple pneumatic and hydraulic systems. It may cause large control errors, unwanted oscillations and excessive wear. Friction-related problems are very often encountered by the practicing control engineer. Other nonlinearities that are common in practice are back-lashes, dead zones, and saturations. They all have very special nonlinear structures and taking these into account can enable design of improved control laws. If friction is dealt with properly it may result in better quality, economy and safety.

It is important to take the control problem into account already at the hardware design stage. For a given control system it is the control engineer's task to understand how the system is affected by friction, in what way friction limits the performance and what the difficulties are, and then to find control strategies that makes the best of the situation. This means that it is important to deal with friction in a systematic way in the design of control laws.

REFERENCES

[1] ***, https://en.wikipedia.org/wiki/Dynamical_systems_theory
[2] ***, https://en.wikipedia.org/wiki/Dynamical_system
[3] ***, https://en.wikipedia.org/wiki/Differential_equation
[4] ***, https://en.wikipedia.org/wiki/Optimal_control
[5] ***, https://en.wikipedia.org/wiki/Friction
[6] Braun O.M., Kivshar Y.S., *Nonlinear dynamics of the frenkel-kontorova model*, *Physics Report*, 306(1), 1998
[7] Canudas C., Olsson H., Astrom K., Lischinsky P., *New model for control of systems with friction*, IEEE Transactions on Automatic Control, 40(3): 419–425, 1995
[8] Dahl P., *A solid friction model*, The Aerospace Corporation, Technical report, 1968
[9] Geffen V., *A study of friction models and friction compensation*, Technische Universiteit Eindhoven, Eindhoven, 2009 www.mate.tue.nl/11194.pdf
[10] Haessig D., Friedland B., *On the modeling and simulation of friction*, Journal of Dynamic Systems, Measurement and Control, Transactions of the ASME, 113(3): 354–362, 1991
[11] Iwata K., Osakada K., Terasaka Y., *Process Modelling of Orthogonal Cutting by the Rigid Plastic Finite Element Method*, J. Engng. Ind., Trans. ASME, Vol. 106, pp. 132-138, 1984
[12] Makkar, C., Dixon, W. E., Sawyer, W. G., Hu G., *A New Continuously Differentiable Friction Model for Control Systems Design*, Proceedings of the International Conference on Advanced Intelligent Mechatronics, 24-28 July 2005, Monterey, California nersp.osg.ufl.edu/~wsawyer/AIM2005.PDF
[13] Murray, R., *Analysis and Design of Feedback Systems*, Computing and Mathematical Sciences (CMS) Department, California Institute of Technology, pg. 31-60, 2012 www.cds.caltech.edu/~murray.pdf
[14] Naidu, D.S., *Optimal control systems*, Electrical engineering textbook series, CRC PRESS LLC, 2003 dl.offdownload.ir/optimal control systems.pdf
[15] Shirakashi T., Usui E., *Simulation Analysis of Orthogonal Metal Cutting Mechanism*, Proceedings of the Int. Conf. Prod. Engng. (part I) , Tokyo, Japan, pp. 535-540, 1974
[16] Swevers J., Al-Bender, *An integrated friction model structure with improved presliding behavior for accurate friction compensation*, IEEE Transactions on Automatic Control, 45(4): 675–686, 2000
[17] Swevers J., Al-Bender F., Lampaert V., *A novel generic model at asperity level for dry friction force dynamics*, Tribology Letters, 16(1-2): 81–94, 2004
[18] Wu J.S., Dillon Jr., Lu W.Y., *Thermo-Viscoplastic Modeling of Machining Process using a Mixed Finite Element Method*, J. Manuf. Sci. Engng., Trans. ASME, Vol. 118, pp. 470-428, 1996