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## **THE DYNAMIC MODEL OF THE LOWER LIMB MOVEMENT**

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**Abstract:** *The aim of this paper is to present the dynamical model of the lower limb movement, of the subjects suffering from osteoarthritis who were surgically treated with total knee prosthesis. There are presented some theoretical considerations about the dynamic model and kinematic chains. For the human motion analysis is used a motion system capture using markers to detect and track points. There are presented some conclusions.*

**Keywords:** *dynamic model, multibody systems, medical recovery, kinematic chain*

### **1. INTRODUCTION**

Numerical simulation of human movement requires the application of mathematical models that accurately describe the behavior of the human body as a result of its interaction with the environment. The use of the dynamic methods of the multibody systems and of the mathematical optimization techniques have proved to be excellent to describe this type of mathematical models, with quality results and low cost. The information obtained are the result of applying the laws of classic mechanics to living structures and have the advantage of obtain data about human mechanical behavior without using invasive techniques.

The human body can be shaped like a multibody system that include several body segments connected together by joints, [1]. Kinematic pair are two solid bodies or two body segments connected by a joint with one linear or angular degree of freedom, [2], [3].

The leg bone and the femurs are the segments that we are concerned in this paper, in order to perform human motion analysis during postoperative recovery. These are connected to the knee joint, forming a kinematic coupling, which forms a series kinematic chain, [4].

### **2. THEORETICAL CONSIDERATIONS**

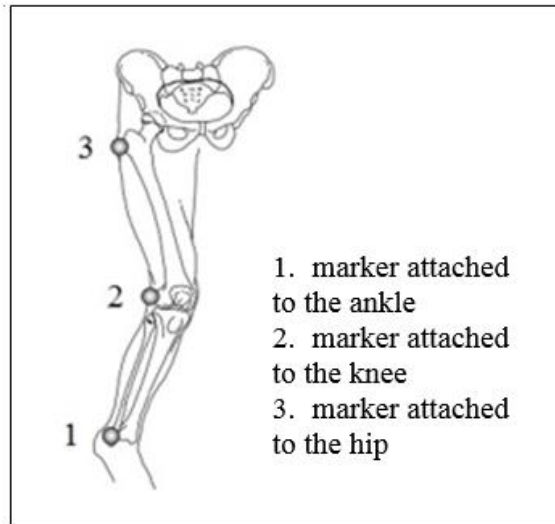
Human motion analysis using artificial vision is an interesting topic in biomechanical analysis of the human body and recovery techniques.

The recovery techniques should objectively evaluate and analyze the patient's performance of the recovery sessions prescribed by a physiotherapist. For this reason, the patient motion capture is a priority. Currently, there are several monitoring systems of human motion that generates real-time data which represents measurements of human motion based on different technologies.

In this paper framework is used a motion system capture and analysis through processing digital techniques of images, using markers to detect and track points.

The technique is based on tracking specific markers by optical sensors, such as cameras, capturing markers that are placed in specified locations on the human body, see Figure 1, [5].

It is developed a mathematical model in order to analyze the motion of the segments involved the movement of the subjects after the osteoarthritis surgery.



**Figure 11:** The positioning systems of the markers, [5].

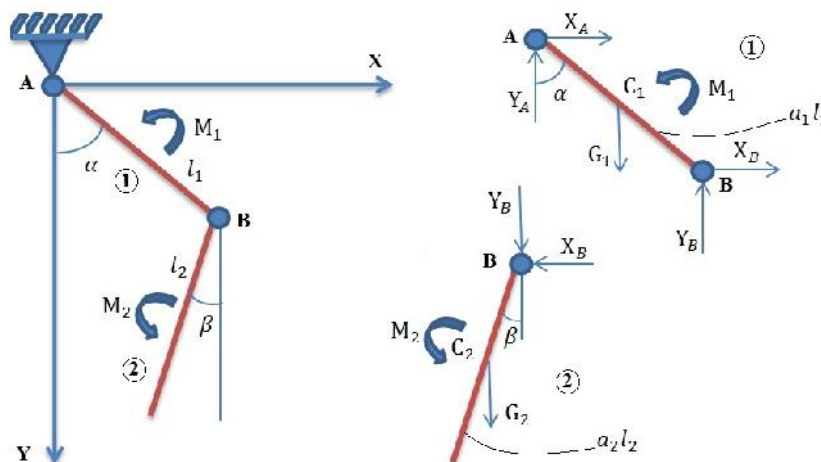
## 2.1. Dynamic model

The markers positions represents the motion parameters. The simplified model of the kinematic chain is shown in Figure 2.

Is proposed a model with two elements, with plane parallel motion. The model has two degrees of freedom and presents lower limb movement.

The two elements are connected by knee joint and represent the femurs - leg bone pair. This chain represent the human leg without motion in the ankle joint.

There are known  $l_1$  the length of the segment 1 – femurs,  $l_2$  the length of the segment 2 - leg bone, B the knee joint and A the hip joint, where is consider fixed the leg bone extremity. The angle  $\alpha$  is the angle of hip and the angle  $\beta$  is the knee joint angle.



**Figure 2.** The dynamic model used for the motion parameters, [5]

## Forward dynamics

The forward dynamics equation based of the presented dynamic model are, [5]:

$$\begin{cases} m_1 \ddot{x}_1 = x_A + x_B \\ m_1 \ddot{y}_1 = y_A + y_B - G_1 \\ J_1 \nu_1 = M_1 - x_A a_1 l_1 \cdot \cos \alpha - y_A a_1 l_1 \cdot \sin \alpha + x_B (1 - a_1) l_1 \cdot \cos \alpha + y_B (1 - a_1) l_1 \cdot \sin \alpha \end{cases} \quad (1)$$

$$\begin{cases} m_2 \ddot{x}_2 = -x_B \\ m_2 \ddot{y}_2 = -y_B - G_2 \\ J_2 v_2 = M_2 - x_B a_2 l_2 \cdot \cos S - y_B a_2 l_2 \cdot \sin S \end{cases} \quad (2)$$

By derivation of the  $r$  and  $s$  angles is obtain, [5]:

$$\begin{aligned} \dot{r} &= \check{S}_1 \\ \dot{s} &= \check{S}_2 \end{aligned} \quad (3)$$

$$\Rightarrow \begin{aligned} \check{S}_1 &= v_1 \\ \check{S}_2 &= v_2 \end{aligned} \quad (4)$$

$$\begin{aligned} &\begin{bmatrix} m_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & m_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & J_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & m_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & m_2 & 0 \\ 0 & 0 & 0 & 0 & 0 & J_2 \end{bmatrix} \cdot \begin{Bmatrix} \ddot{x}_1 \\ \ddot{y}_1 \\ v_1 \\ \ddot{x}_2 \\ \ddot{y}_2 \\ v_2 \end{Bmatrix} = \\ &= \begin{Bmatrix} x_A + x_B \\ y_A + y_B - G_1 \\ M_1 - x_A a_1 l_1 \cdot \cos \Gamma - y_A a_1 l_1 \cdot \sin \Gamma + x_B (1 - a_1) l_1 \cdot \cos \Gamma + y_B (1 - a_1) l_1 \cdot \sin \Gamma \\ -x_B \\ -y_B - G_2 \\ M_2 - x_B a_2 l_2 \cdot \cos S - y_B a_2 l_2 \cdot \sin S \end{Bmatrix} \end{aligned} \quad (5)$$

In the end are obtained, [4]:

$$\begin{cases} Q_1 = -G_1 a_1 l_1 \cdot \sin \Gamma + M_1 + G_2 l_1 \cdot \sin \Gamma \\ Q_2 = G_2 a_2 l_2 \cdot \sin S + M_2 \end{cases} \quad (6)$$

$$[J] = \begin{bmatrix} m_1 a_1^2 l_1^2 + J_1 + m_2 l_1^2 & m_2 a_2 l_1 l_2 \cdot \cos(\Gamma - S) \\ m_2 a_2 l_1 l_2 \cdot \cos(\Gamma - S) & J_2 + m_2 a_2^2 l_2^2 \end{bmatrix} \quad (7)$$

$$[J'] = \begin{bmatrix} 0 & m_2 a_2 l_1 l_2 \cdot \sin(S - \Gamma) \\ m_2 a_2 l_1 l_2 \cdot \sin(S - \Gamma) & 0 \end{bmatrix} \quad (8)$$

### Inverse dynamics

$$[J] \begin{Bmatrix} \ddot{r} \\ \ddot{s} \end{Bmatrix} + [J'] \begin{Bmatrix} \dot{r}^2 \\ \dot{s}^2 \end{Bmatrix} = \begin{Bmatrix} -G_1 a_1 l_1 \cdot \sin \Gamma + G_2 l_1 \cdot \sin \Gamma \\ G_2 a_2 l_2 \cdot \sin S \end{Bmatrix} + \begin{Bmatrix} M_1 \\ M_2 \end{Bmatrix} \quad (9)$$

$$\begin{Bmatrix} M_1 \\ M_2 \end{Bmatrix} = [J] \begin{Bmatrix} \ddot{r} \\ \ddot{s} \end{Bmatrix} + [J'] \begin{Bmatrix} \dot{r}^2 \\ \dot{s}^2 \end{Bmatrix} - \begin{Bmatrix} -G_1 a_1 l_1 \cdot \sin \Gamma + G_2 l_1 \cdot \sin \Gamma \\ G_2 a_2 l_2 \cdot \sin S \end{Bmatrix} \quad (10)$$

### 3. CONCLUSION

It shows that the dynamics of multibody systems is a great base for analyzing the multibody systems.

There is no need for a complex model, with many degrees of freedom, because the movement of the subjects, who are in the medical recovery period, is very limited. However, the model adopted will be very sensitive to the change of some parameters. Taking into account the low mobility of subjects (unlike the sports movement, where we have ample movements in large spaces) we were looking for simplified models, but effective, to respond to the purpose of the present study.

It was proposed and analyzed a mathematical model with two elements, with plane parallel motion, which has two degrees of freedom because the movement of the subjects is very limited.

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