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J-INTEGRAL VARIATION FOR A LAYERED COMPOSITE PLATE

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Abstract: *In this paper the FEM approach of determining the J-integral for a cracked plate, using the Abaqus software, is presented. The plate was modelled as a shell, with a central cut-out and a small crack and meshed with quadrilateral and triangular elements. Only half of the plate was modeled with symmetry conditions for the other half, a fixed end and a loading at the other end.*
A static analysis was run with the added option to calculate the J-integral for the defined crack. Since J also can be considered as the energy flow into the crack tip the time variation of J-integral will be presented as a diagram with the coordinates: displacement energy release rate for the tip of the crack – time.
Keywords: *J-integral, composite layered plate, Abaqus*

1. INTRODUCTION

Because of their mechanical properties, composite materials have applications in various industries, such as aircraft, naval, automotive, military industries, the biomedical domain etc. One important aspect that needs to be taken into consideration for composites integrity and their fatigue life is the evolution of the cracks that appear during loading. All aspects and methods useful for studies are developed by fracture mechanics concepts [1].
In 1968, J. R. Rice introduced the concept of J-integral [2], which can be used to predict the evolution of a crack. Initially the J-integral could be used for 2D problems and, in time, researchers improved this method so it can also be used for 3D problems [3, 4, 5] and orthotropic composites [6, 7].
Many applications of numerical methods oriented to study the dynamic fracture problems have used the finite difference method developed by Chen and Wilkens [8].
Nishioka is the researcher who focused on evaluation of crack-tip parameters, by using J-integral and stress intensity factors in fracture problems, based on finite element analysis [9, 10, 11, 12].
FEA is the method that can evaluate fracture parameters by using various methods such as crack closure. On the other hand, this method offers difficulties in the area of coping with crack propagation, especially for the case of propagation in arbitrary directions [12]. The problem of FEA is difficult in the case of problems with large deformations or rotations. For these cases the finite element mesh could have big distortions and thus it is necessary a re-meshing, causing problems in calculation efficiency.

2. METHODOLOGY

2.1. Analysis description

A plate with two layers made from an epoxy-glass composite material is analyzed in this paper. Three orientation cases for the layers are considered: I. $0^o/0^o$; II. $0^o/90^o$; III. $45^o/-45^o$. The plate's dimensions are 3000 mm X 2000 mm, with a central circular cut-out with the radius of 500 mm and a crack with a length of 200 mm (figure 1), but only half of the plate with the crack (and with symmetry conditions) was modeled and analyzed. For comparing the results half of a plate with a central crack was also modeled and analyzed with the same conditions.

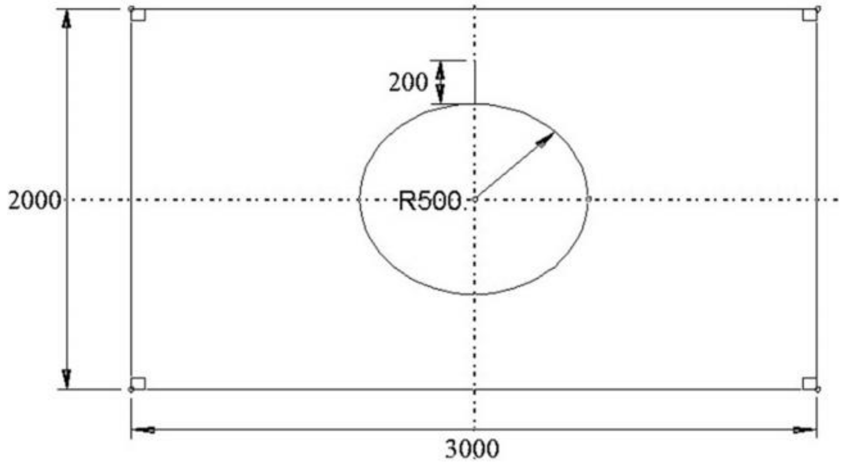

Figure 1: Plate geometry with a crack (dimensions in mm)

The plate was subjected to the boundary conditions: the left edge of the plate was fixed and symmetry conditions were applied on the bottom edge, because only the top part of the plate was modeled. A pressure was distributed uniformly on the right edge of the plate, with values between 100 and 500 MPa for five analysis cases (figure 2).

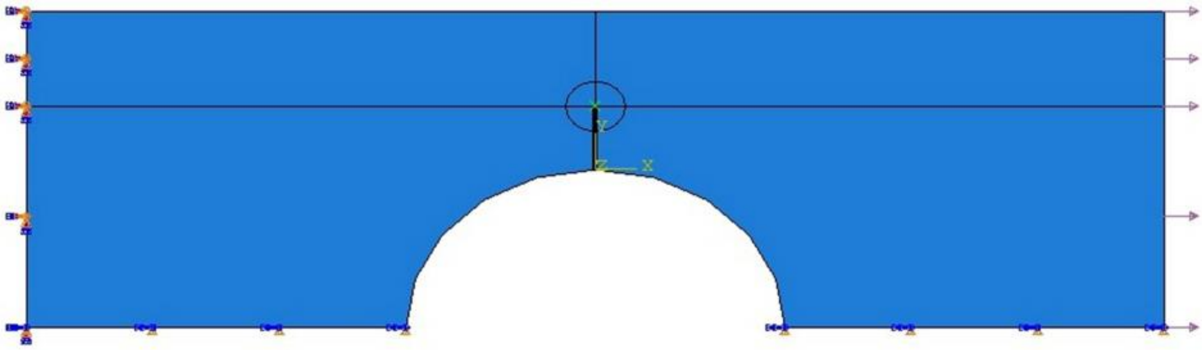

Figure 2: The boundary conditions for the plate with a central cut-out with the seam of the crack highlighted

The approximate global size of the applied seeded mesh was 50 mm with quadrilateral elements, except in the vicinity of the crack's tip where the applied mesh was finer with quadrilateral and triangular elements (figures 3 and 4).

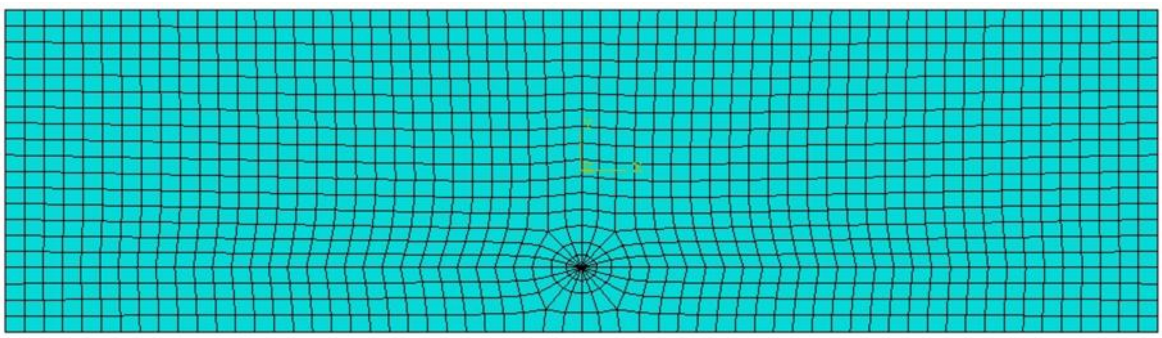

Figure 3: Meshed plate with a central crack

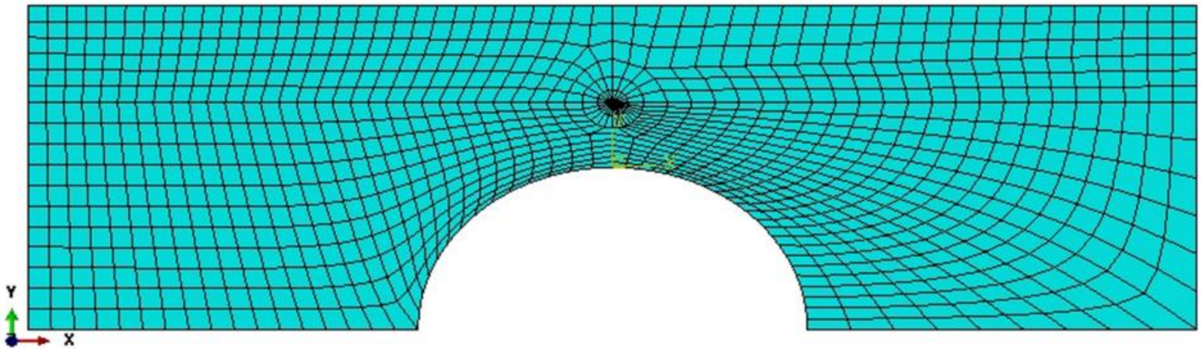

Figure 4: Meshed plate with a central cut-out and a crack

2.2. FEM method

A static analysis was run with an added option to calculate the first and second stress intensity factors and the J-integral. The von Mises stress for a tension pressure case for the plate with the layers oriented at $0^o/0^o$ and with a central cut-out is presented in the figure 5. The figures 6, 7 and 8 contain the variations of the stress intensity factors and of the J-integral in a time interval between 0 and 1 second. All FEM results can be found in table 1.

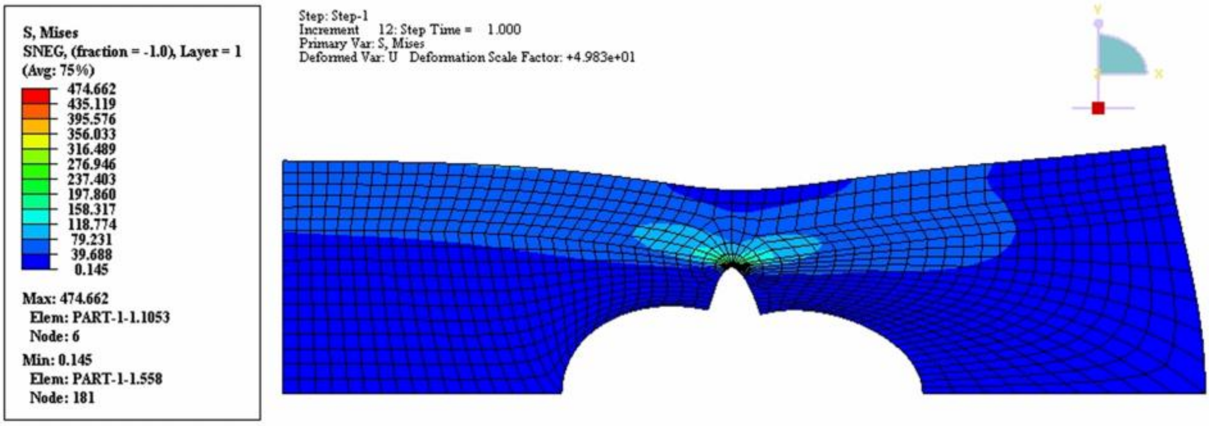

Figure 5: Von Mises stress [MPa] for the plate (layers oriented at $0^o/0^o$) with a tension pressure of 100MPa

	X	Y
1	0.05	101.641
2	0.1	203.282
3	0.175	355.743
4	0.275	559.025
5	0.375	762.307
6	0.475	965.589
7	0.575	1168.87
8	0.675	1372.15
9	0.775	1575.43
10	0.875	1778.72
11	0.975	1982
12	1	2032.82

Quantity Types
X: Time Y: Stress intensi

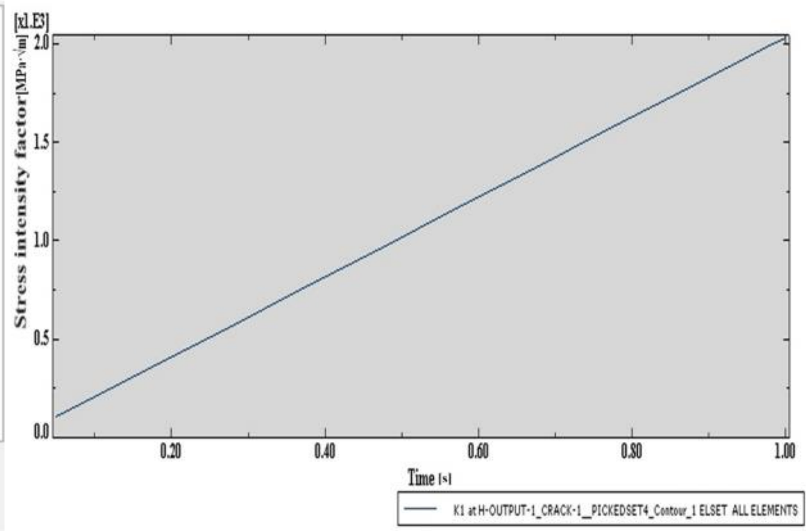

Figure 6: First stress intensity factor (K_I) [MPa·√m] variation for the plate (layers oriented at $0^o/0^o$) with a tension pressure of 100MPa

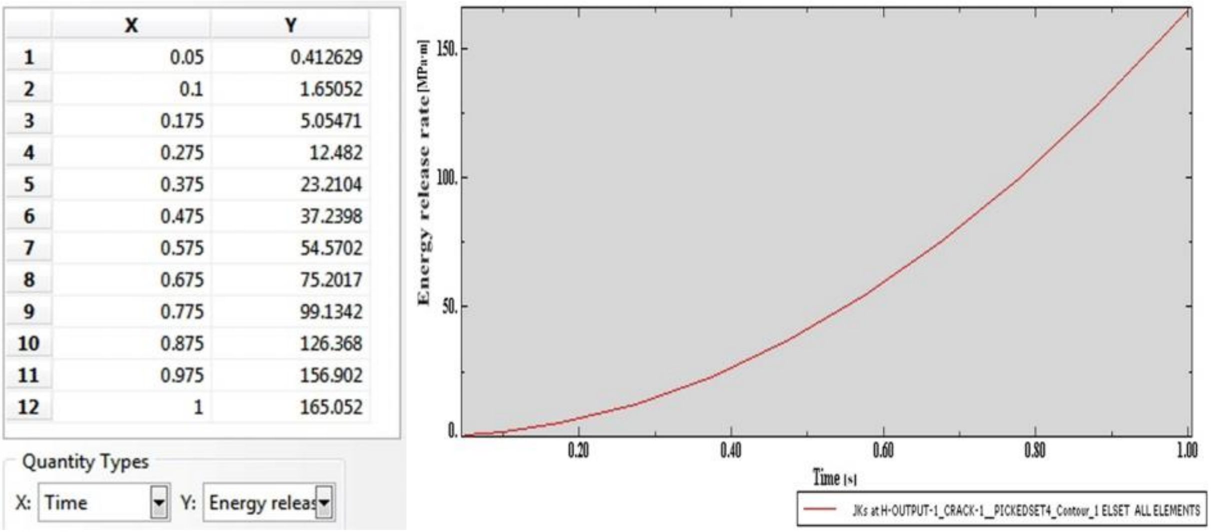

Figure 7: J-integral [MPa·m] variation for the plate (layers oriented at $0^o/0^o$) with a tension pressure of 100MPa

3. RESULTS AND DISCUSSIONS

Two plate geometry cases have been modeled: one with only a crack and one with a circular cut-out and a crack – with three cases of layer orientations ($0^o/0^o$, $0^o/90^o$, $45^o/-45^o$) and subjected to five pressure loading cases (100, 200, 300, 400 and 500 MPa).

The results obtained from Abaqus are presented in the tables bellow. To better compare the results for the maximum von Mises stress and J-integral two diagrams were also prepared (figures 9 and 10).

Table 1: Maximum von Mises Stress, the stress intensity factors and the J-integral calculated by using the Abaqus software for the plate with the layers oriented at $0^o/0^o$ (with/without a central cut-out)

Pressure [MPa]	Maximum von Mises Stress [MPa]		K_I [MPa·√m]		K_{II} [MPa·√m]		J-integral [MPa·m]	
	Plate without a cut-out	Plate with a cut-out	Plate without a cut-out	Plate with a cut-out	Plate without a cut-out	Plate with a cut-out	Plate without a cut-out	Plate with a cut-out
100	151.576	474.662	635.22	2032.82	1.9415	146.313	15.9417	165.052
200	303.151	949.324	1270.44	4065.64	3.883	292.627	63.7667	660.207
300	454.727	1423.985	1905.66	6098.45	5.8245	438.94	143.475	1485.47
400	606.302	1898.647	2540.88	8131.27	7.766	585.254	255.067	2640.83
500	757.878	2373.309	3176.1	10164.1	9.7075	731.567	398.542	4126.29

Table 2: Maximum von Mises Stress, the stress intensity factors and the J-integral calculated by using the Abaqus software for the plate with the layers oriented at $0^o/90^o$ (with/without a central cut-out)

Pressure [MPa]	Maximum von Mises Stress [MPa]		K_I [MPa·√m]		K_{II} [MPa·√m]		J-integral [MPa·m]	
	Plate without a cut-out	Plate with a cut-out	Plate without a cut-out	Plate with a cut-out	Plate without a cut-out	Plate with a cut-out	Plate without a cut-out	Plate with a cut-out
100	83.425	359.283	702.128	2206.2	0.538548	44.1357	19.4765	192.457
200	166.85	718.565	1404.26	4412.39	1.0771	88.2713	77.9058	769.828
300	250.275	1077.848	2106.29	6618.59	1.61564	132.407	175.288	1732.11
400	333.7	1437.131	2808.51	8824.79	2.15419	176.543	311.623	3079.31
500	417.125	1796.413	3510.64	11031	2.69274	220.678	486.911	4811.42

Table 3: Maximum von Mises Stress, the stress intensity factors and the J-integral calculated by using the Abaqus software for the plate with the layers oriented at 45°/-45° (with/without a central cut-out)

Pressure [MPa]	Maximum Von Mises Stress [MPa]		K_I [MPa·√m]		K_{II} [MPa·√m]		J-integral [MPa·m]	
	Plate without a cut-out	Plate with a cut-out	Plate without a cut-out	Plate with a cut-out	Plate without a cut-out	Plate with a cut-out	Plate without a cut-out	Plate with a cut-out
100	187.223	663.789	655.669	2278.07	-92.3589	-304.2	24.3498	294.983
200	374.445	1327.577	1311.34	4556.14	-184.718	-608.4	97.3994	1179.93
300	561.668	1991.366	1967.01	6834.2	-277.077	-912.6	219.149	2654.84
400	748.891	2655.154	2622.67	9112.27	-369.436	-1216.8	389.598	4719.72
500	936.113	3318.943	3278.34	11390.3	-461.794	-1521	608.746	7374.56

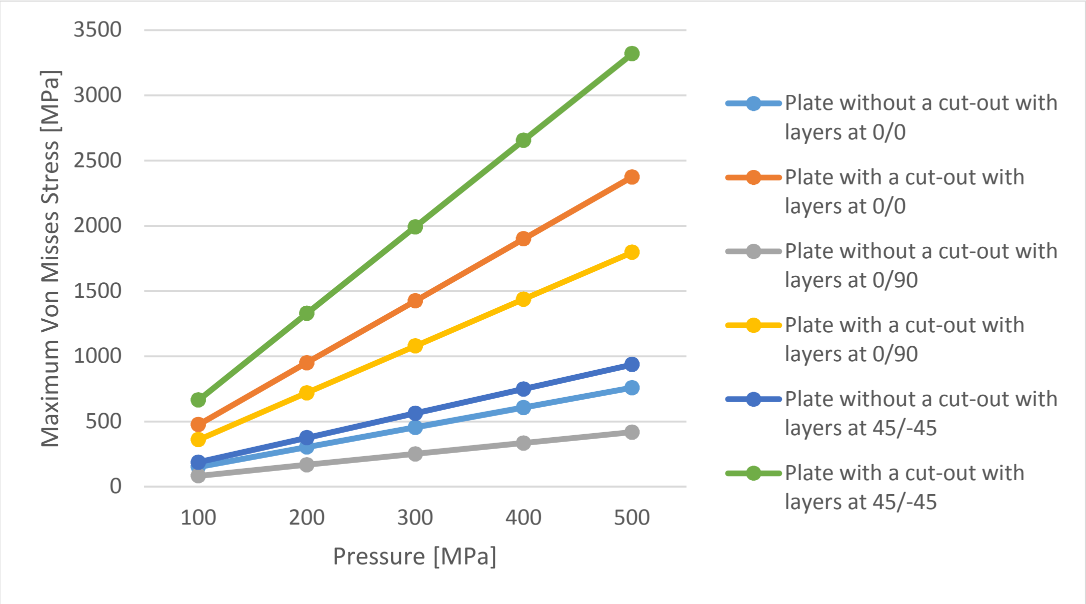

Figure 8: The variation of the maximum von Mises stresses for the analyzed cases

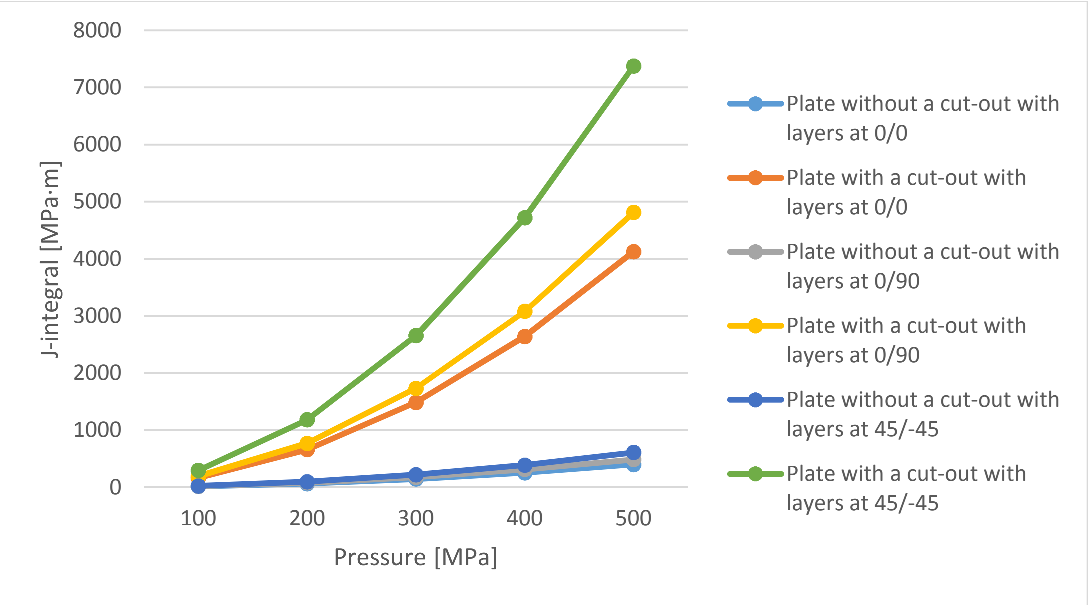

Figure 9: The variation of the J-integral values for the analyzed cases

4. CONCLUSIONS

In this paper an epoxy-glass composite plate with two layers (with and without a central cut-out) and a central crack has been analyzed to estimate the variation of the J-integral during the loading with a pressure distributed uniformly on the right edge of the plate.

By increasing the applied pressure, it can be observed that the von Mises stresses have a linear growth, while the J-integral values have an exponential growth. As it is observed, the plate with a cut-out and the fibers oriented at $45^o/-45^o$ has the weakest properties in this case, the plate without a cut-out and with the fibers oriented at $0^o/90^o$ has the best stress resistance (figure 8) and the plate without a cut-out and with the fibers oriented at $0^o/0^o$ has the lowest J-integral value (figure 9).

The reason for the discrepancy existing between the results may be the modeling approach of the plates that can introduce errors during interlaminar studies [13, 14]. Future work should be aimed at the fracture problems including crack propagation.

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