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THE DETERMINATION OF THE STRESS INTENSITY FACTORS OF A STEEL PLATE

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Abstract: *This paper presents the FEM and numerical approaches of determining the stress intensity factors (K_I, K_{II}, K_{III}) of a steel plate having a central elliptical cut-out subjected to different pressures. The failure of cracked components depends on the stresses in the crack tip vicinity. The singular stress contribution is characterized by the stress intensity factor K. For this analysis, a small initial crack was modelled. Each of the stress intensity factors correspond to a crack propagation method. For this analysis, a solid element type with 8 nodes was used, with quadrilateral and triangular elements. Around the crack tip a finer mesh was applied for better results. The first analysis that was run was a static analysis. After that a path and a new coordinate system corresponding to the crack tip were defined and used to determine the stress intensity factors. Then a percentage error can be calculated by using the results obtained in ANSYS and the numerical (calculated) ones.*
Keywords: *stress intensity factors, composite layered plate, Ansys APDL*

1. INTRODUCTION

The fatigue life of a material starts with the initiation of a crack and continues with its propagation. One important factor of fracture mechanics for the initiation phase of a crack is the stress intensity factor [1].
The stress intensity factors calculated for the tip of a crack can be used to predict the evolution of that crack [2]. There are three modes of crack propagations that each have a corresponding stress intensity factor: mode I (K_I) – opening, the most common facture case; mode II (K_{II}) – sliding and mode III (K_{III}) – tearing. These factors can be determined either by using fracture mechanics formulas, in which case the geometry of the crack and the loading conditions should be taken into account; or by using a finite element method (FEM) software. For the more complex crack geometries it's easier to use FEM [3]. In the vicinity of the crack a finer mesh is necessary for better results in this case. Multiple types of FEM have been developed over the years that can be used to determine the stress intensity factors and not all have the same accuracy for predicting the results [4].
For example, in 1975, Hellen T.K. [5] introduced the virtual crack extension method as the released strain energy per crack extension, a method that was later used and improved by other researchers, such as Lin or Hwang [6, 7, 8, 9, 10]. Another method, proposed in 1999 by Belytschko and Black [11] was the extended finite element method (XFEM), which was a "minimal re-meshing finite element method for crack growth".
By using both an analytical method and FEM an error percentage can be calculated between the two methods.

2. METHODOLOGY

2.1. Analysis description

In this paper a steel plate with a central elliptical cut-out is analyzed. The model of this plate (figure 1) is the same as the one used in another paper [12], except that, in this case, it's a steel plate and also a crack with a length of 100 mm was added to it for the determination of the stress intensity factors. A solid element type (PLANE183) was used for this analysis with a triangular shape with a length of 125 mm, except in the vicinity of the crack, where a finer mesh (62.5 mm) was applied.
The following boundary conditions were applied to the plate:

– on the left side the plate is fixed, all degrees of freedom are blocked;
– near the crack, initially, symmetry conditions were applied. This was done to be easier to analytically calculate the first stress intensity factor and the error percentage between the results. A second analysis was run without these conditions;
– on the right side different uniformly distributed pressure cases from 100 to 1000 MPa (tension/compression) were applied (table 1).

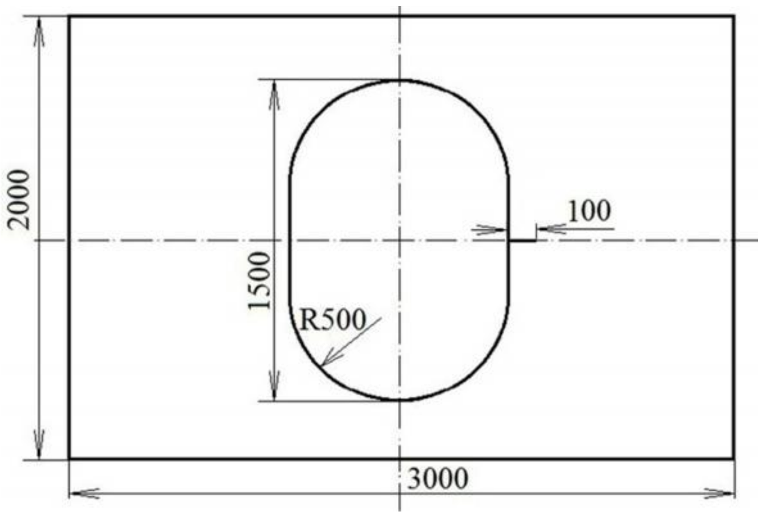


Figure 1: Plate geometry with a crack (dimensions in mm) [12]

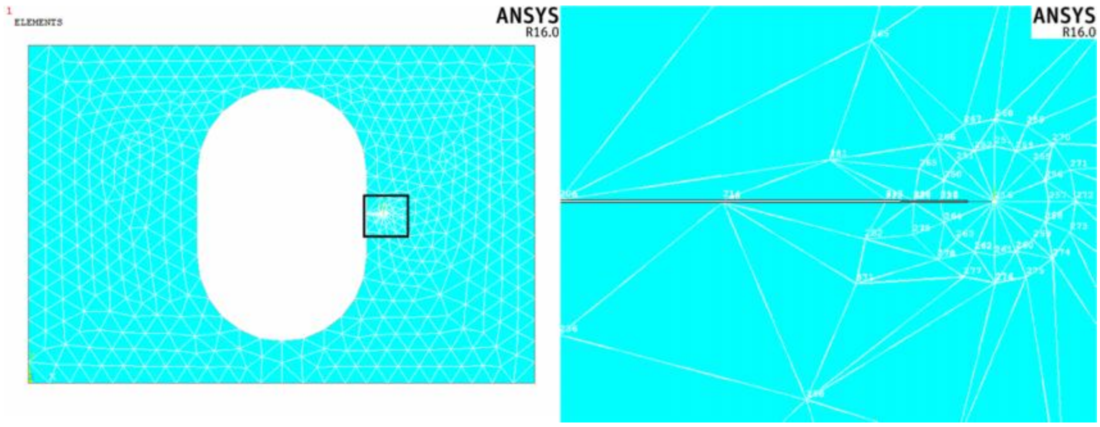


Figure 2: The meshed plate and a close-up on the crack

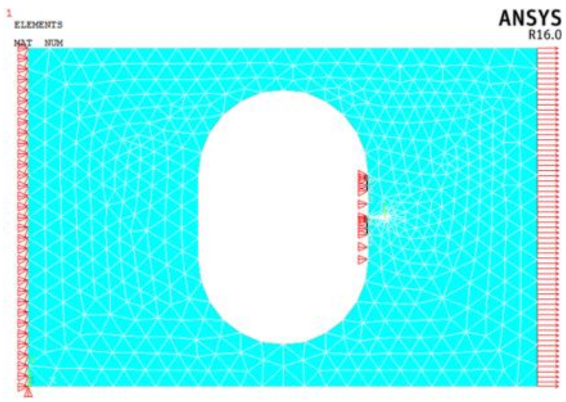


Figure 3: The initial boundary conditions applied to the plate

2.2. Analytical method

Before using the FEM software Ansys, for each loading case, an analytical calculus was made, by using the equation [13, 14]:

$$K_I = C\sigma\sqrt{\pi a} \qquad (1)$$

in which:

C – a constant based on the dimensions of the specimen and those of the crack; in the case of a plate with a crack is:

$$C = \left(1 - 0.1\eta^2 + 0.96\eta^4\right)\sqrt{1/\cos(\pi\eta)} \qquad (2)$$

σ – the pressure applied to the plate;
η – a notation for a/W;
a – the size of the crack, usually the length;
W – the width of the plate (or its length).
For the first pressure, 100 MPa, by using the first formula, the first stress intensity factor is

$$K_I = 1.006 \cdot 100 \cdot \sqrt{\pi \cdot 0.1} = 56.39\,\text{MPa} \cdot \sqrt{\text{m}} \qquad (3)$$

All the calculated results are presented in table 1.

2.3. FEM method

First, a static analysis was used to determine the maximum stress. After that, for determining the stress intensity factors, a path with 5 points was defined – the tip of the crack and 2 points along each of the two sides of the crack – and a new local coordinate system was assigned at the crack's tip.
Initially, symmetry conditions were applied to be able to calculate as if it were a plate with a crack (without a cut-out) and determine the error percentage:

$$e = \frac{K_I^{ANSYS} - K_I^{analythical}}{K_I^{analythical}} \cdot 100 \qquad (4)$$

After that, the symmetry conditions were removed and the analyses were run again.
In the figures bellow there are two of the von Mises stress diagrams (one for a tension case and one for a compression case) for the plate with a central cut-out and a crack and the result window for the stress intensity factors from Ansys. The mode I stress intensity factors calculated by Ansys (and the error percentages between results) are presented in the tables 1 and 2.

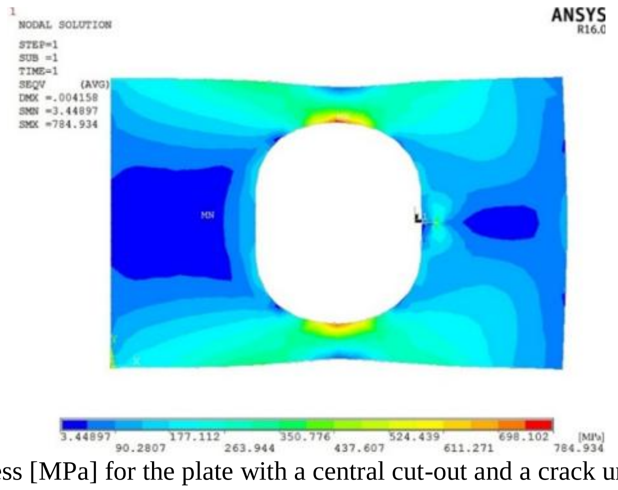


Figure 4: Von Mises stress [MPa] for the plate with a central cut-out and a crack under a tension pressure of 100MPa

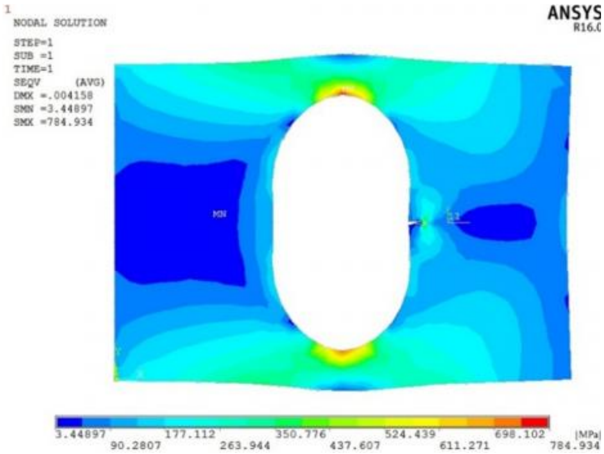
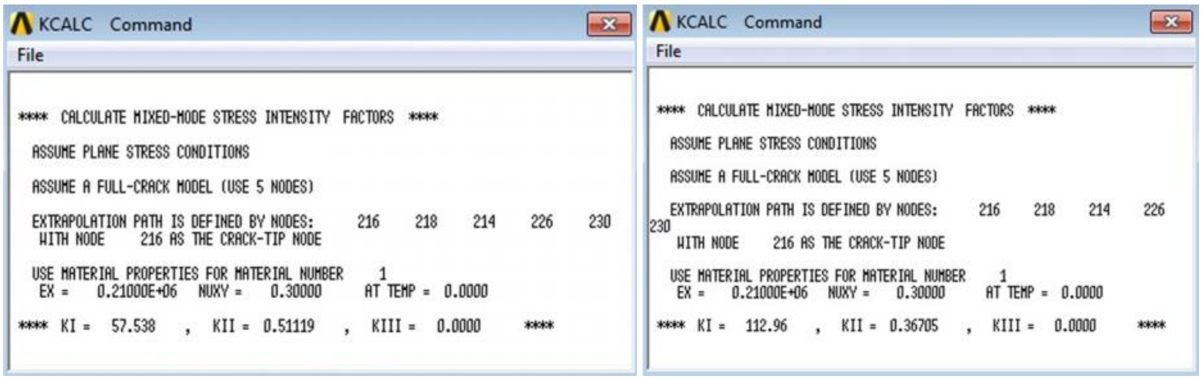


Figure 5: Von Mises stress [MPa] for the plate with a central cut-out and a crack under a compression pressure of 100MPa



a. plate with a crack b. plate with a central cut-out and a crack

Figure 6: The stress intensity factors [MPa·√m] for the plate with a pressure of -100MPa

3. RESULTS AND DISCUSSIONS

The first stress intensity factor (K_I) had much higher values than the other stress intensity factors (K_{II}, K_{III}) for all the loading cases, meaning that the crack propagates mainly by opening.

Table 1 contains the von Mises stresses and a comparison between the results calculated with the formula (1) and those obtained from Ansys for the plate with a crack. The error percentages were between 2.042 and 2.048, meaning that the difference between the results is pretty small, so for the second analysis the error percentages should be similar. For that reason the stress intensity factors were calculated only with the software Ansys for the plate with a central cut-out and a crack (table 2). A comparison between the Ansys results for the two cases is presented in the figure 7.

Table 1: Maximum von Mises stress and the stress intensity factors calculated analytically and by using the Ansys software for the plate with a crack

Pressure [MPa]	Maximum von Mises stress [MPa]	K_I (analytical) [MPa·√m]	K_I (Ansys) [MPa·√m]	Error [%]
100/-100	299.151	56.386	57.538	2.043
200/-200	598.303	112.77	115.08	2.048
300/-300	897.454	169.16	172.62	2.045
400/-400	1196.61	225.54	230.15	2.044
500/-500	1495.76	281.93	287.69	2.043
600/-600	1794.91	338.32	345.23	2.042
700/-700	2094.06	394.70	402.77	2.045
800/-800	2393.21	451.09	460.31	2.044
900/-900	2692.36	507.48	517.85	2.043
1000/-1000	2991.51	563.86	575.38	2.043

Table 2: Maximum von Mises stress and the stress intensity factors calculated by using the Ansys software for the plate with a central cut-out and a crack

Pressure [MPa]	Maximum von Mises stress [MPa]	K_I (Ansys) [MPa·√m]
100/-100	784.934	112.96
200/-200	1569.87	225.92
300/-300	2354.80	338.88
400/-400	3139.74	451.84
500/-500	3924.67	564.80
600/-600	4709.60	677.76
700/-700	5494.54	790.72
800/-800	6279.47	903.68
900/-900	7064.41	1016.6
1000/-1000	7849.34	1129.6

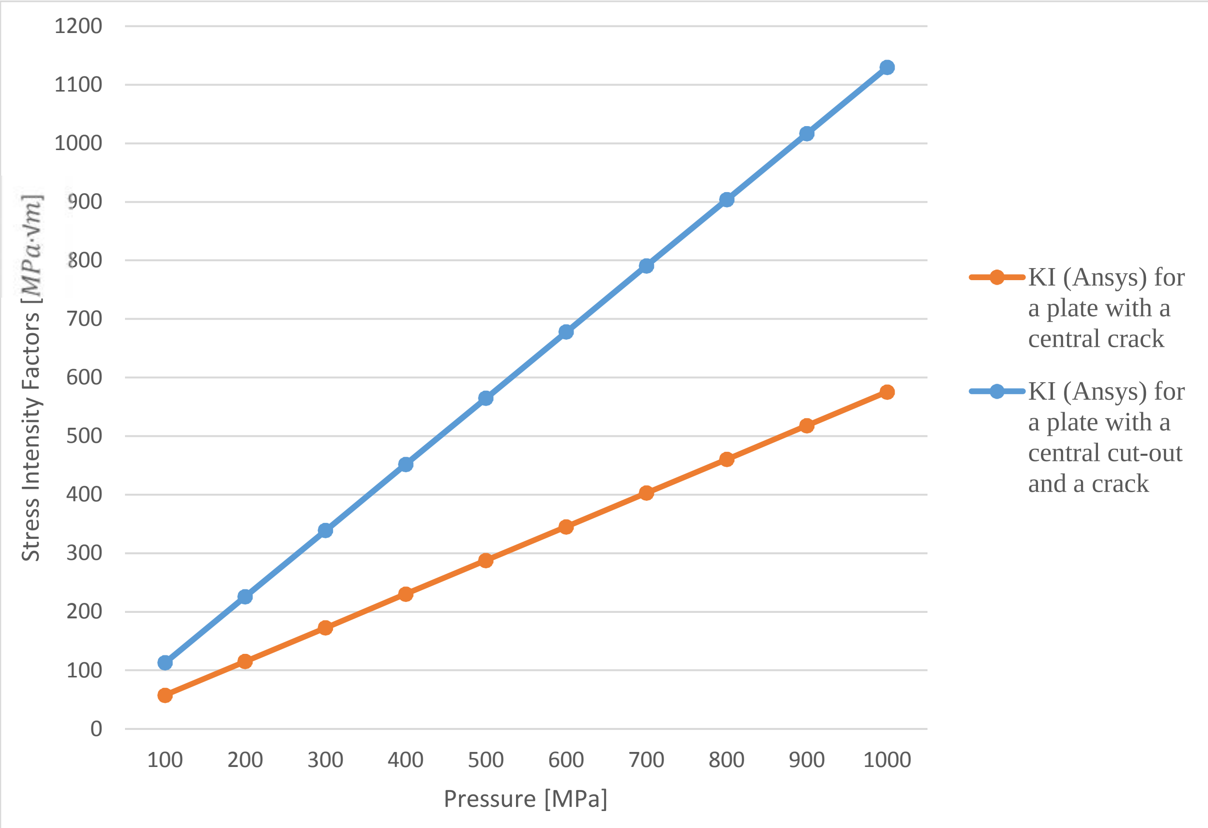


Figure 7: Comparison between the K_I stress intensity factors for the two analyzed cases

4. CONCLUSION

The stress intensity factors for a plate with a crack and for a central cut-out and a crack were calculated in this paper. For the first case the results were obtained analytically and with the software Ansys. By calculating the error percentage it can be observed that the results were not very far apart between the two methods, so for a plate with a more complicated geometry, the Ansys results should be accurate enough.

After analyzing the model with a central cut-out and a crack, the stress intensity factor K_I results obtained (Table 2) are almost double than those of the case of the plate with only a crack (Table 1). As it can be seen in the figure 7 the stress intensity factors increased linearly for both cases as the pressure increased.

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