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DETERMINATION OF CONTACT TENSIONS BETWEEN ORTHOTROPIC (COMPOSITE) MATERIALS BY USING THE BOUNDARY ELEMENT METHOD

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Abstract: *The boundary element formulation and the computer implementation of the 2-D contacvt problem with small displacements and strains between elastic anisotropic materials are presented in this paper.The contact program include isoparametric linear, quadratic,and quater-point-traction-singular elements. several contact zones with different friction coefficients berween the solids. Several exemples have been included, specially the computation od contact tractions in composite material plates with bolted joints or the influence on the stress intensity factor of the crack closure effects.*
Keywords:contact, tension,composite,boundary,element*

1. INTRODUCTION

The increasing requirements in the design of machanical elements imply the necessity to include in the analysis differernt aspects that traditionally have only: appear inside them due to this effect.
It is true that, in most case, these stress are reduced to a very small region in the neighbourhood of the contact zon, and they do not effect the behavior of this structure. However, in order cases, the contact stresses are either the most important ort else they modify substantialy the reponse oThis is the case of joining elements, tribology or crack closure effects, among many others.
Over the last few years important advancements have been made in the inclusion of contact formulations into standard finite element, or boundary element programs. This last method seems to have proved advantageous in treating the linear contact problem, taht is the contact between lineart elastic solids with small displacements and strains, as occurs for instance along the crack lips of elastic bodies.
The formulation of the BEM is primarily incuded for completeness, so are the formulation and algoritma used to solve the contact problems between two solids. Finally several exemples are explained in detail, specially the study of contact traction in bolted joints in composite laminates.

2.THE BEM IN 2-D ELASTICITY

The boundary element method (BEM) consist basically of the transformation of the system of partial differen'ial equation that rules the elastic problem into a set of singular integral equation which allows the representation of the displacements, strains and stress in any internal point as a function of the boundary displacements and tractions. By a limiting process to the boundary and by approximating these boundary distributions, it is possible to obtain an algebraic linear system with the displacementsand tractions in some boundary points as unknows.
The first equation of the BEM in its direct formulation is the well-known Somiagliani', which express the displacement components $u_i(Q)$ of a point Q of a domain Ω as a function of a displacement $u_i(P)$ of the boundary points P and the body forces X_i:

$$C_{ik}u_i(Q) = \int_{\partial\Omega} U_{ik}(Q,P)t_i(P)d\partial\Omega - \int_{\partial\Omega} T_{ik}(Q,P)u_i(P)d\partial\Omega + \int_{\partial\Omega} U_{ik}(Q,P)X_i(P)d\Omega \tag{1}$$

where U_{ik} is the Kelvin fundamental solution of the Navier'equations, T_{ik} are tractions corresponding to those fundamental solutionof the Navier's equations, $T_{ik}$are the tractions corresponding to those fundamental solutions (the expresisions for the orthopic case are included in the Appendix), and C_{ik} can be expressed as:

$$C_{ik} = \begin{cases} \delta_{ik} \to Q \in \Omega \\ C_{ik}^{'} \to Q \in \delta\Omega \\ 0 \to Q \notin QU\delta\Omega \end{cases} \tag{2}$$

with

$$C_{ik}^{'} = \frac{1}{4\pi(1-\nu} \times \begin{bmatrix} 2(1-\nu)(\pi+\alpha_1-\alpha_2) + \frac{1}{2(sen2\alpha_1 - sen2\alpha_2)} & sen^2\alpha_1 - sen^2\alpha_2 \\ sen^2\alpha_1 - sen^2\alpha_2 & 2(1-\nu)(\pi+\alpha_1-\alpha_2) - \frac{1}{2}(sen2\alpha_1 - sen\alpha_2) \end{bmatrix}$$

for isotropic materials, for exemple, where α_1 and α_2 have the geometrical mecaning shown in Fig.1

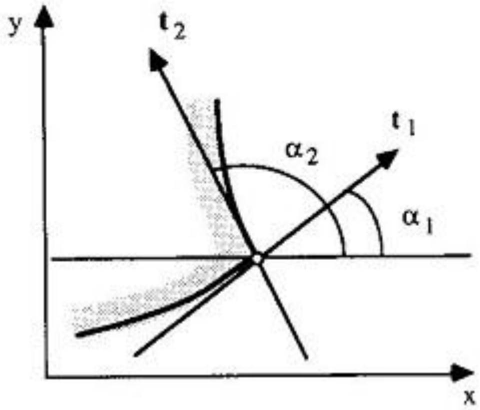

Figure.1 Geometrical mean of α_1 and α_2

Under certain conditioins, the domain integral in (1) can be rewritten as the sum of two boundary integrals, in such a way that it is possible to express the displacements of any point of the domain Ω in terms of only boundary integrals. In this work no body forces are consideredso that only bouindary integrals appear in (1).

If a boundary discretization is used with Ne elements (Fig.2), and the displacements and tractions are approximated inside each element in terms of nodal values in the classical way of the BEM

$$u_i^j = \sum_{k=1}^{Nnj} (u_i^j)_k \varphi_k \qquad t_i^j = \sum_{k=1}^{Nnj} (t_i^j)_k \varphi_k \tag{3}$$

where Nnj is the number of nodes of the element j, and φ_k the shape function for 2-D continuous elements, then the eq(1) can be approximated by

$$C_{ik}u_i(Q) = \sum_{j=1}^{Ne} \int_{\delta Q_j} U_{ik}(Q,P)\left[\sum_{m=1}^{Nnj}(t_i^j)_m \varphi_m\right] d\delta\Omega_j - \sum_{j=1}^{Ne} \int_{\delta\Omega_j} T_{ik}(Q.P)\left[\sum_{m=1}^{Nnj}(u_{ij})_m \varphi_m\right] d\delta\Omega_j \qquad (4)$$

In the case, for exemple, of linear elements (Nnj=2)

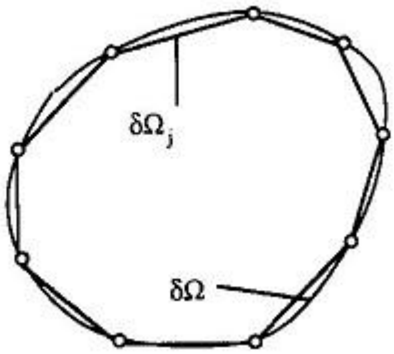

Figure.2 Discretization of the boundary

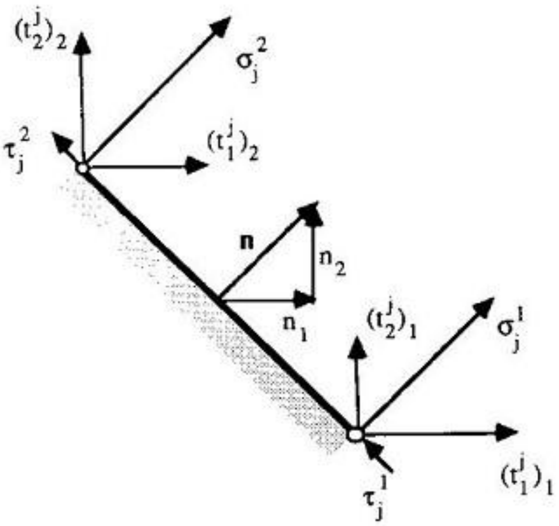

Figure.3 Local For coordinates

the above equation can be written as

$$\begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}\begin{bmatrix} u_1(k) \\ u_2(k) \end{bmatrix} + \sum_{j=1}^{Ne}\begin{bmatrix} A_{111}^{kj} & A_{211}^{kj} & A_{112}^{kj} & A_{212}^{kj} \\ A_{121}^{kj} & A_{221}^{kj} & A_{122}^{kj} & A_{222}^{kj} \end{bmatrix} \times \begin{bmatrix} (u_i^j)_1 \\ (u_i^j)_2 \\ (u_i^j)_3 \\ (u_i^j)_4 \end{bmatrix} =$$

$$= \sum_{j=1}^{Ne}\begin{bmatrix} B_{111}^{kj} & B_{211}^{kj} & B_{112}^{kj} & B_{212}^{kj} \\ B_{121}^{kj} & B_{221}^{kj} & B_{122}^{kj} & B_{222}^{kj} \end{bmatrix}\begin{bmatrix} (t_i^j)_1 \\ (t_i^j)_2 \\ (t_i^j)_3 \\ (t_i^j)_4 \end{bmatrix} \qquad (5)$$

$$A_{imn}^{kj} = \int_{\delta Q_j} \varphi_i T_{mn}^{kj} d\delta\Omega_j \qquad\qquad B_{imn}^{kj} = \int_{\delta Q_j} \varphi_i U_{mn}^{kj} d\delta\Omega_j$$

If this eq. (5) is applied to each of nodes and the known boundary conditions are also included, it is posible to obtain, an algebric linear system with $\lfloor 2\sum_j (Nn_j - 1) \rfloor$ equations and unknowns, corresponding to the displacements and tractions of the boundary nodes.[7] If the collocation point (nodes for which the eq. (5) is applied, (CP) is not one of the nodes of elements along which the integrals in (5) are computed, a stanmdard Gauss-Legendre, quadrature is used (with some care when the node is very close to element). However, when the CP constants B are computed by means of a quadruature with logarithmic weight function and the singular constanta A,together with the free term C_{jk} , by imposing a rigid body condition on the system(5). For ewach nodes it is possible then to write two equations, six being the num,ber of initial unknowns (two displacement and two left and right tractions) corresponding to each node.In most cases the tractions are expressed in local (tangent-normal) coordinates, so that the tractions vector is transformed as (fig.3);

$$\begin{bmatrix} (t_i^j)_1 \\ (t_i^j)_2 \\ (t_i^j)_3 \\ (t_i^j)_4 \end{bmatrix} = \begin{bmatrix} n_1 & 0 & n_2 & 0 \\ -n_2 & 0 & n_1 & 0 \\ 0 & n_1 & 0 & n_2 \\ 0 & -n_2 & 0 & n_1 \end{bmatrix} \begin{bmatrix} \sigma_j^1 \\ \sigma_j^2 \\ \tau_j^3 \\ \tau_j^4 \end{bmatrix} \qquad (6)$$

Once the equations and the boundary conditions have all been considered an algebric system obtained

$$Kx = f \qquad (7)$$

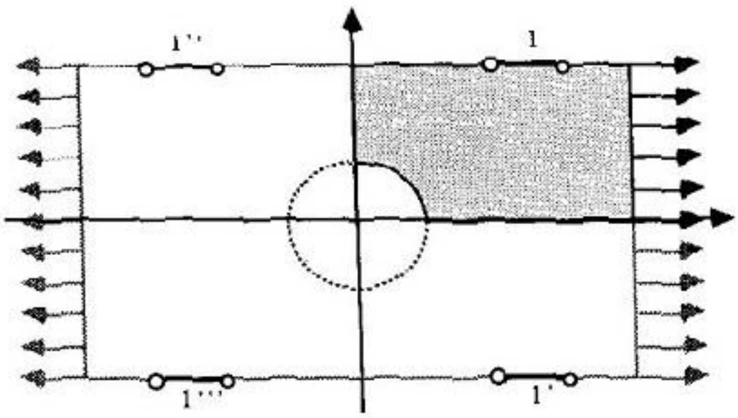

Figure.4 Symmetries

The solution oof this system can be obtained by any of the of the well-known linear algebric solves depending on its size.Once the boundary displacements and tractions are known,the displacements at any interior point can be computed by means of eq.(1), andr the stress by the applicationof the stress operator to the same equation.
Finally, an important aspect is the use the symmetry conditions in order to reduce the number of resulting equations. For exemple, in (fig.4) only thr characteristics of element 1 have to be given, being the ones correspondingto the elements1', 1'', and 1''' automatically obtained by the program on applying symmetry conditions.

3. FORMULATION OF THE CONTACT PROBLEM BETWEEN ELASTIC SOLIDS WITH SAMALL STRAINS AND DISPLACEMENTS

The unilateral contact problem with small displacements and strains is just a linear elastic problem for each solid under non-linear and initialy unknown boundary conditions, along an unknown contact surface. These conditions depend on load level and the deometry of solids in contact.
In this case, only the contact problem between two elastic solids will be considered. The extensionto multibody problems or the particularization to rigid base problems are straightforward once the above formulation has been obtained, regardless of the implementation and modelling difficulties impled. Let $\delta\Omega$ be the boundary neighbourhood of a point P on the contact surface, and $n=f_A(t)$, $n=f_B(t)$ the equations that represent the undeformed surfaces for both solids A, B along the contact zone, in a local coordinates system tangent-notmal to

the surface $\delta\Omega_A^c$, $\delta\Omega_B^c$,(if a small displacement problem is considered those equations must be essentially the same, and also similar to the equation of the final contact zonbe after the deformation $\delta\Omega^c$) (Fig.5).
The non-penetration condition at point P is established as

$$d_N + u_N \leq 0 \tag{8}$$

with d_N the projection of the initial vector joining the equivalent points P, P' (same position after the contact) along the normal to $\delta\Omega^c$, and u_N the projection of the relative displacement beween the two points along the same normal.

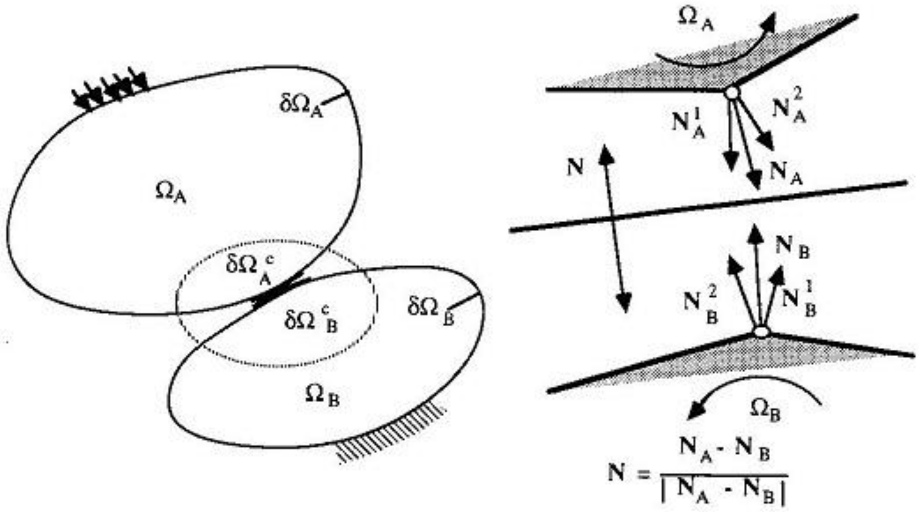

Figure.5 Intermediate normal between the two solids

It can shown that conditions is no more than the linerized non-penetration condition for large displacements along the coordinate t in the neighbourhood of the point P. This kinematic expression is then the approximated real condition and it is exact only when both solids have the same normal at the points P and P' which remain fixed along the contact process, and the relative displacement has the direction of that normal. When the normals to both solids are orthogonal, the error is maximum, even marking is possible to violate the non-penetration condition. However, cases close to this are unreal in practical in practical small displacement contact problems.
The static boundary conditions, in the unilateral case, wich a Coulomb friction law, as the one considered in this paper, can be expressed as

$$\sigma_N \leq 0 \qquad \tau \leq \mu \cdot \sigma_N \quad \text{with } \mu \text{ the friction coefficient} \tag{9}$$

The direction N that has been used to project the displacements and tractions in order to impose the boundary conditions , is the average between the two normals to both solids in correspondiong boundary nodes (Fig.5).
Besides those kinematic and static conditions for each domain, the equilibrium and compatibility conditions between both solids in both solids must be fulfilled along the contact zone. In order to do this, different zones along the boundaries are defined (Fig.6):

- Out of contact zone (zone 1). It is the one that is never in contact;
- Candidate to contact zone (zone 2). It is the one which is not in contact yet, but can be cantact for a certain load level;
- Sliding zone (zone 3), $|\tau| = \mu \cdot \sigma_N$;
- Adhesive zone (zone 4), $|\tau| \prec \mu \cdot \sigma_N$;
- Welded zone (zone 5). It is really a contact zone, but an interface between two domains, but has been included as a contact zone in order to generalize the program including the possibility of bilateral contact problems with zones under tension.

The contact problembetweem two solids consists then in establishment of the BE equations for each of the solids under contact, including implicity or explicity thr boundary conditions (equilibrium and compatibility) along the contact region for each load level, and the standard boundary regions.

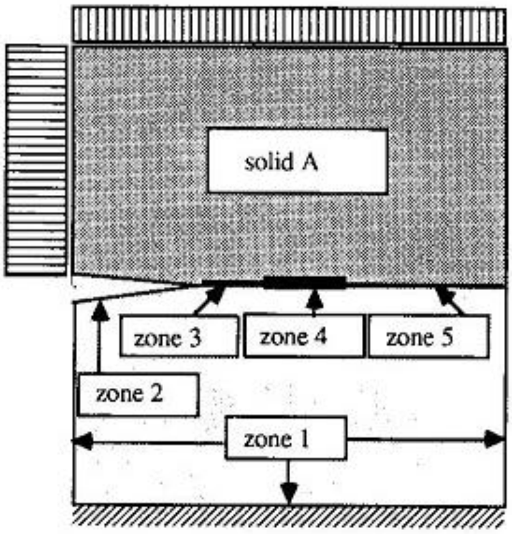

Figure. 6 Contact zone

4. CONCLUSIONS

It has been sshown that the B.E.M. may be used yo study the problem of propagating craks in orthotropic bodies in a similar form yo the previouis works on isotropic materials. Also, the singular boundary elements give very good results in the computationof stress intensity factors every with very coars meshes, specially using a direct traction being only necessary the modification of the fundamental solution of a standard isotropic boundary element method.

In the most of cases, the method which gives rise to the best results in the SIF is the one the singular traction approximation,using the nodal value of the singular node as ther parameter wkich allows the obtantion of the SIF, although it is very impostant the choice of the length of siongular element.

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