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THEORETICAL CONSIDERATIONS REGARDING THE ULTRASONIC SHAPE OPIMISATION METHODS

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Abstract: *Ultrasonic horns are devices used in manufacturing for focusing the ultrasonic energy in the area of mechanical process. The model of the ultrasonic horns is based on Webster's wave propagation equation and one of the important problems is the optimization of their shape. In the present paper there are presented some theoretical consideration about the possibility of optimization of the ultrasonic horns. There are mentioned some work done by different authors and in the same time there are highlight the basic methods that can be used in ultrasonic horns.*
Keywords: *optimization methods theory, ultrasonic, horns*

1. INTRODUCTION

Ultrasonic horns are devices with an axisymmetric shape that provide acoustic energy in a specific tool. The acoustic energy is generating by a transducer, generally using a magnetostriction effect, considering an electrical input signal. In the same time, the acoustic horns can be directly tools or the tool can be attached at the small end of the horn. Main technological applications are cutting [1], [2], [3], drilling [4], [5], [6], turning [7], welding [8], [9], and also for machining different materials as carbon fiber reinforced composites [10], [11], [12].
The geometry of the ultrasonic horn involves a continuous varying cross section from an initial value S_0, in the origin of the reference system, to an end one value S_L (Figure 1).

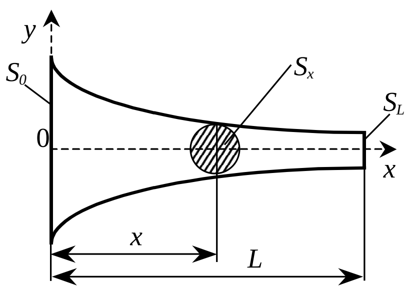

Figure 1: Solid horn geometry

The cross section variation generates a mechanical magnification of the input signal amplitude. The cross section variation is given by different mathematical functions (exponential [13], linear [14], tapered (or stepped) [15], catenoidal [14], Bézier [16], Gaussian, etc.). An important problem associated with the horns design refers to the shape optimization. The optimization can be associate with different design parameters as: working frequency [17], amplitude of the signal [18], [19], load transferred in the manufacturing area [20], the objective function [21], [22], etc. In the present paper, there is presented some theoretical considerations about the optimization procedure that can be used for optimization.

2. THEORETICAL CONSIDERATIONS

2.1. Introduction

The aim of any optimisation procedure is to find that values of some parameters that lead to a better functionality of a considered system, particularly a mechanical system. The success of any optimisation procedure depends on a mathematical representation of the system behaviour and the considered parameters [23], [24], [25], [26], [27], [28].

Generally there are found two types of optimisation functions:

a) ***Objective functions***, are that function which express a dependency between the quality characteristics (optimisation criteria) and variables that have to be optimised;
b) ***Restrictive functions***, that are represented as mathematical inequalities used to be found allowable domains for optimisation.

Any optimisation problem is correctly defined if the objective function meets the following conditions:

a) The function has to be a continuous one;
b) In a range, the function must have only one maximum or minimum or can have multiple maximum or minimum values (the case of the global optimum value);
c) It is an admitted domain delimited by restrictive functions in a manner that the extreme values are inside the domain or on its border.

In optimisation procedure of a system there are highlight two limit situations:

a) The system is well known and both dependences between variables and optimisation criteria and the restrictions of the objective function can be described;
b) The system can be evaluated by experiments.

The optimisation study involve the usage of specific methods and techniques that can be classified as follows:

a) Analytical methods – that use the differential equations to be found the maximum or the minimum value. Considering this method one can find general information about the objective function. It is necessary to be assured the continuity of the function and of its derivatives;
b) Numerical methods – that are based on an algorithm that drive the numerical evaluations of the objective function to its maximum or minimum values. In the frame of the algorithm it is included a criteria of finishing of the search that refers to the consequent optimisation variables or to the consequent values of the optimisation criteria;
c) Mixt methods – where the objective function is treated as black-box system and the numerical values of the objective function are found combined with experiments;
d) Experimental statistical methods – the optimisation variables are controlled modified.

The determination of the maximum or of the minimum of some mathematical expressions that contain unknown functions can be easy done using the variational calculus.

This is the case of the optimisation problem in the case of ultrasonic horns. The basic equation is the wave propagation equation through beams with variable cross section:

$$\frac{\partial^2 \Phi}{\partial x^2}+\frac{\partial \Phi}{\partial x}\frac{\partial}{\partial x}\left(\ln S_x\right)=\frac{1}{c^2}\frac{\partial^2 \Phi}{\partial t^2} \tag{1}$$

Based on the Weierstrass theorem one can say that a continuous functions $f(x)$ that is defined in a point in a closed range $[a,b]$ can has an absolute maximum ore minimum value. In case that the maximum or minimum value of the function is inside of the range than that value is a local extreme.

In case of the extreme problems the leading principle is to obtain a derivative of the function in the considered range in a point excepting a finite number of points where can be reached the absolute maximum:

a) A point where $\frac{df(x)}{dx}=0$, point defined as stationary point or critical point, al resulted points being stationary values;
b) A point at the end of the range;
c) A point where de function $f(x)$ has no derivative.

2.2. Problems with extreme values with connections. The method of Lagrange multipliers

Considering a function of two variables $z=f(x,y)$ defined in a domain Ω where the variables *x* and *y* are tied by the supplementary condition $g(x,y)=0$. According with [23] the function $z=f(x,y)$ has an extreme in a point defined by coordinates (a, b) it necessary to be fulfil in that point the condition:

$$\frac{f_x}{g_x}=\frac{f_y}{g_y}=-\lambda \tag{2}$$

where from it is obtained the system:

$$\begin{cases} f_x+\lambda\cdot g_x=0; \\ f_y+\lambda\cdot g_y=0. \end{cases} \tag{3}$$

The relations from (3) together with the condition $g(x,y)=0$ are enough to find the coordinates of the extreme point defined by the coordinates (a, b) and the multiplier λ.
The above mentioned conditions is defined as the method of Lagrange's multipliers and by λ was defined the Lagrange's multiplier.
This method can be resumed [n the following algorithm:

a) It is generate the function $F(x,y,\lambda)=f(x,y)+\lambda\cdot g(x,y)$ and the unknown values, (x,y,λ) are treated as independent variables;
b) It is equated the partial derivatives of the function *F* with zero:

$$\begin{cases} \dfrac{\partial F}{\partial x}=f_x+\lambda g_x=0; \\ \dfrac{\partial F}{\partial y}=f_y+\lambda g_y=0; \\ \dfrac{\partial F}{\partial}=g_x=0. \end{cases} \tag{4}$$

In the case of multiple variables, the system (4) becomes:

$$\begin{cases} \dfrac{\partial F}{\partial x_i}=\dfrac{\partial f}{\partial x_i}+\sum_{j=1}^{k}\lambda_j\dfrac{\partial g_i}{\partial x_i}=0, i=1,2,\ldots\ldots,n \\ \dfrac{\partial F}{\partial_j}=g_j(x_1,x_2,x_3,\ldots,x_n)=0, j=1,2,\ldots,k \end{cases} \tag{5}$$

where:

$$F(x_1,x_2,\ldots,x_n,\lambda_1,\lambda_2,\ldots,\lambda_k)=f(x_1,x_2,\ldots,x_n)+\sum_{j=1}^{k}\lambda_j\cdot g_j(x_1,x_2,\ldots,x_n) \tag{6}$$

2.3. Variational principles

In the case of a classical problem of variational calculus these are called isoparametric problems. These types of problems involve finding an extreme value of an integral if it is known the value of another integral.
It is defined the term of „*functional*” that has the meaning of avalue, or a whole function that depens on the variation of the graph of one or more functions and do not have a discrete number of variables [23].
It has to be mentioned that the domain of the valability of a functional is represented by a domain of alloweble functions that belong of a space or of a class of alawable functions.
The most interesting problems in the functional functions study are given by the stationary values of these. Considering an explicit function $F(x,y,y',y'')$ and x_0 and x_1 two given values, the shape of the functional is given by:

$$\mathbf{I}(y)=\int_{x_0}^{x_1}F(x,y,y',y'')\,dx\,. \tag{7}$$

The problem resume to the finding an external function $y=y(x)$ that realise an extreme of the functional (7) while y and y' are known in points $x=x_0$, and $x=x_1$.
In the theory of the variational calculus it is considered a variation of the function y with a quantity dy with the conditions:

$$\begin{cases} dy|_{(x=x_0)}=0; \\ dy'|_{(x=x_0)}=0. \end{cases} \tag{8}$$

In the case that the functional $\mathbf{I}$ has an extreme value, its variation is:

$$\delta\mathbf{I}(y)=\int_{x_0}^{x_1}\left(\frac{\partial F}{\partial y}\delta y+\frac{\partial F}{\partial y'}\delta y'+\frac{\partial F}{\partial y''}\delta y''\right)dx=0\,. \tag{9}$$

The relation (9) is defined as *the first variation of the functional* $\mathbf{I}$. Eliminating the variations dy' and dy'' and integrating and taking into consideration the boundary conditions it is obtained:

$$\frac{\partial F}{\partial y}-\frac{d}{dx}\left(\frac{\partial F}{\partial y'}\right)+\frac{d^2}{dx^2}\left(\frac{\partial F}{\partial y''}\right)=0. \tag{10}$$

Differential equation (10) it is known as *Euler's differential equation* that corresponds to the functional (7). Based on (10) and taking into consideration the boundary conditions one can find the function $y=y(x)$. Generally, the variation of the functional is zero $(\delta\mathbf{I}=0)$ for the points $x=x_0$, and $x=x_1$ if there are fulfil simultaneous the following conditions:

a) For a specified $y=y(x)$:

$$\frac{\partial F}{\partial y'}-\frac{d}{dx}\left(\frac{\partial F}{\partial y''}\right)=0 \tag{11}$$

b) For a given y':

$$\frac{\partial F}{\partial y''}=0 \tag{12}$$

and the equation (10) is satisfied.

In many problems from mechanical engineering domain it is used the theorem of the minimum potential energy. According with this theorem, from all displacements of the stable structure that satisfy the given boundary conditions of the displacements that which satisfy the equilibrium equations minimise the potential energy.

Reciprocal from all displacements that satisfy the boundary conditions, those which minimise the potential energy satisfy the equilibrium equations. This reciprocity is used in the frame of the variational principle. Thus, it is obtained:

$$\delta\mathbf{I}=0 \tag{13}$$

2.4. The variational principle of Hamilton

As is shown in [126] and [127] the Lagrange's equations:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_k}\right)-\frac{\partial L}{\partial q_k}=0,\ (k=1,2,\ldots,n) \tag{14}$$

can be seen as Euler's equations in a functional shape as:

$$\mathbf{I}=\int_{P_1}^{P_2} L(q_1,q_2,\ldots,q_n,\dot{q}_1,\dot{q}_2,\ldots,\dot{q}_n,t)\,dt\ , \tag{15}$$

where P_1 and P_2 are two points that are situated in the generalized coordinate plane $q_1,q_2,q_3,...,q_n$ known as the configuration space. Thus, the functional $\mathbf{I}$ is stationary if it is satisfied the condition (13) and results:

$$\delta\int_{P_1}^{P_2} L\,dt=0\ . \tag{16}$$

The integral (16) is expressed by the product between the kinematic potential L and time. The quantity

$$A=\int_{P_1}^{P_2} L\,dt\ , \tag{17}$$

is defined as action.

3. OPTIMAL DESIGN OF ULTRASONIC HORNS

In case of the ultrasonic horns, taking into consideration their shape, one can say that the problem of optimisation is focused on finding a function of the cross section variation which leads to a minimum volume, at a high level of the signal energy, at a minimum stress in a given point [27].

Thus, it is necessary to extreme an objective functional in conditions that verify the equilibrium equations of the structure. More, one can use the boundary conditions considering the effect of the fatigue as a result of the ultrasound waves.

The equations Euler-Lagrange obtained from the functional together with the conditions that has to reach the functional leads to an optimised mathematical model.

Generally, the problem of the ultrasonic horns optimisation is an isoparametric problem. The problem of obtaining of a minimum cross section can be reported to a problem of a minimum weight of the structure which leads to a problem of potential energy optimisation for a given volume [28].

3. CONCLUSION

The problem of ultrasonic horns optimization is a complex one. The optimization procedure involves finding a functional and the main parameters that can be used to obtain a proper shape for a maximum magnification of the signal. The shape of the wave propagation equation (1) involves to be used the Euler-Bernoulli method for separating the two components, the space one and the time one. The space component equation is used to define the optimum cross-section.
Any optimization method that is used combine the analytical method with the imposed designed parameters.

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