THE DAMPED DYNAMIC VIBRATION ABSORBER – A NUMERICAL OPTIMIZATION METHOD

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Abstract: *The aim of this contribution is to provide a numerical optimization method solution for passive control of structures by damped vibration absorber (DVA). We will use a generalized Den Hartog's model that includes damping in both, the main structure and the absorber. To develop a numerical solution, Den Hartog's method will be adapted and implemented in MATLAB Optimization Toolbox.*
Keywords: *vibration absorber, numerical optimization*

1. INTRODUCTION

The dynamic vibration absorber (DVA) or tuned-mass damper (TMD) is a widely used passive vibration control device. Watts [1] in 1883 and Frahm [2] in 1909 reported on the first use of a dynamic vibration absorber. The tuned mass damper (TMD) system, is a passive energy absorbing device to reduce undesirable vibrations. Den Hartog's model consisting of a translational mass, translational spring and translational viscous damper attached to a vibrating structure Figure 1.

Most of the researchers agreed that the performance of TMDs is sensitive to the accuracy of tuning the natural frequency of the TMD to the natural frequency of the structure. Den Hartog [4] has derived the formula for the optimum values of the TMD parameters for an undamped SDOF structure

subjected to a harmonic excitation. The key design parameters of a damped DVA are its tuning parameter and damping ratio. The first mathematical theory on the damped DVA was presented by Ormondroyd and Den Hartog [3]. In [4], Den Hartog first tackled the optimum solution of a damped DVA that is attached to a classical primary system, i.e., a system free of damping. His study utilized the feature of ''fixed-point'' frequencies, i.e., frequencies at which the response amplitude of the primary mass is independent of the absorber damping. Based on the ''fixed-points'' theory, Den Hartog found the optimum tuning parameter and defined the optimality for the optimum absorber damping. In the following treatment such a model will be referred to as the *classic* system. Because of its elegance and historical Importance, the design procedure proposed by Den Hartog is reported by the vast majority of textbooks on mechanical vibrations.

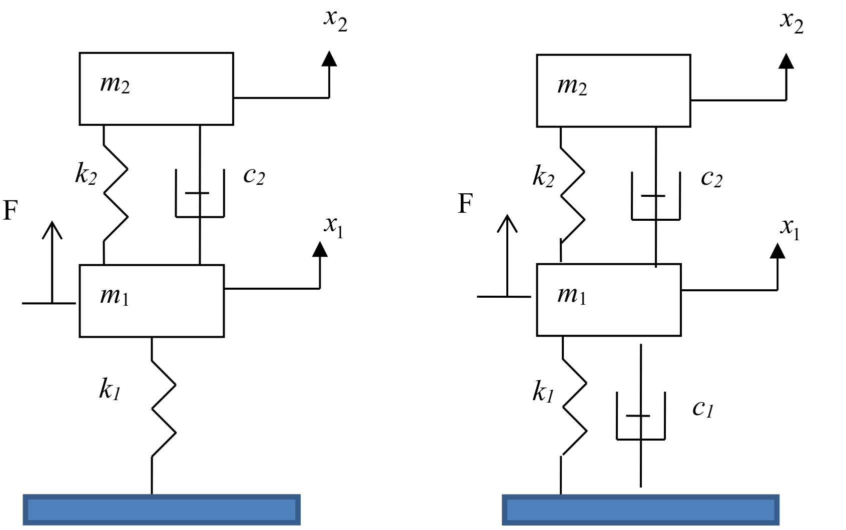
A real system possesses a certain degree of damping. Figure 1 b, shows a damped primary system attached by a damped DVA. When a primary system is damped, the useful ''fixed-points'' feature is lost. Randall *et al*. [6] considered the more realistic situation of viscous damping between the two masses. They have shown that the optimal parameters for the damped linear system differ significantly from those obtained for the classic system.

A number of studies have focused on the numerical solutions. These include a numerical optimization scheme proposed by Randall et al. [5], an optimal design of linear and non-linear vibration absorbers using nonlinear programming techniques by Soom [6], and an optimum design using a frequency locus method by Thompson [7]. Pennestri [8] used the min-max Chebyshev's criterion to seek the optimum solutions.

In the present work, the Den Hartog optimization procedure for the DVA parameters with harmonic loading applied to an undamped SDOF structure is extended to consider the damping of the main structure. The minimization of the maximum displacement of the primary mass is usually set as an objective functions. Thus, when there is viscous damping on both masses, the design problem can be formulated as follows. Given a primary mass m_1, connected to the ground with a spring, dashpot element and subjected to the force $F_0 \sin \omega t$, compute the values of secondary mass m_2 stiffness k_2 and viscous damping c_2 such that the frequency response curve of the main mass has two equal maximum amplitudes. Considering the requirements for the shape of such a response curve, it seems appropriate to solve the present design problem by use of the MATLAB Optimization Toolbox . The resulting systems of non-linear equations will be solved numerically.

2. MATHEMATICAL MODELS

Figure 1 shows the models used in this paper. The structure consist in a SDOF with properties m_1, k_1 and c_1. A tuned mass damper (TMD) systems with properties m_2, k_2 and c_2 is attached to structure.
The resulting system is two degrees of freedom system. This model is similar to the Den Hartog Model except that Den Hartog neglected the structure damping (i.e. c_1=0) in his study.
The motion of the two-degree-of-freedom system is described by a frequency analysis. The structure is subjeted to harmonic force $F(t) = F_0 e^{j\omega t}$ with angular frequency ω.



Model Den Hartog Model Model Generalized Den Hartog Model

Figure 1: Dynamic dampers models for SDOF systems

Displacement amplitude vector is $x = \left[x_1, x_2\right]^T$. The equations of motion are

$$\left(K + j\omega C - \omega^2 M\right)\mathbf{x} = \boldsymbol{F} \tag{1}$$

where

$$M = \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}, C = \begin{bmatrix} c_1 + c_2 & -c_2 \\ -c_2 & c_2 \end{bmatrix}, K = \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix} \tag{2}$$

If we introduce the following definitions

$$\begin{aligned} &\omega_{ni} = \frac{k_i}{m_i},\ m_r = \frac{m_2}{m_1},\ \tau = \omega_{n1} t\ i = 1,2 \\ &\omega_r = \frac{\omega_{n2}}{\omega_{n1}} = \frac{1}{\sqrt{m_r}}\sqrt{\frac{k_2}{k_1}},\ 2\varsigma_i = \frac{c_i}{m_i \omega_{ni}} \quad i = 1,2 \end{aligned} \tag{3}$$

and frequency ratio $\Omega = \dfrac{\omega}{\omega_{n1}}$ and normalized force $\boldsymbol{f} = \begin{bmatrix} f = \\ 0 \end{bmatrix} = \begin{bmatrix} F/k_1 \\ 0 \end{bmatrix}$ the equation (1) takes the form

$$\begin{bmatrix} 1 + m_r\omega_r^2 - \Omega^2 + 2j\left(\varsigma_1 + m_r\omega_r\varsigma_2\right)\Omega & -m_r\omega_r^2 - 2jm_r\omega_r\varsigma_2\Omega \\ -m_r\omega_r^2 - 2jm_r\omega_r\varsigma_2\Omega & m_r\omega_r^2 - m_r\Omega^2 + 2jm_r\omega_r\varsigma_2\Omega \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} F/k_1 \\ 0 \end{bmatrix} \tag{4}$$

The response x_1 of the structural mass is obtained from (4)

$$x_1 = H_f\left(\omega\right)\cdot f = G\left(\Omega\right)\cdot\left(F_0/k_1\right) \tag{5}$$

where

$$G(j\Omega)=\frac{p(j\Omega)}{q(j\Omega)} \tag{6}$$

$$\begin{aligned} p(j\Omega) &= \omega_r^2-\Omega^2+2j\omega_r\varsigma_2\Omega \\ q(j\Omega) &= \Omega^4-2j\left(\varsigma_1+\varsigma_2\omega_r\left(1+m_r\right)\right)\Omega^3-\left(1+\omega_r^2\left(1+m_r\right)+4\varsigma_1\varsigma_2\omega_r\right)\Omega^2+ \\ &+2j\left(\varsigma_2\omega_r+\varsigma_1\omega_r^2\right)\Omega+\omega_r^2 \end{aligned} \tag{7}$$

The magnitude of the frequency response function is given by

$$H(\Omega)=k_1\left|G(j\Omega)\right| \tag{8}$$

The natural frequencies of the undamped two-degree-of-freedom system is obtained by setting $\varsigma_1=\varsigma_2=0$ and finding the positive solutions of $q(j\Omega)=0$.
So, from (7) is obtain characteristic equations

$$\Omega^4-\left(1+\omega_r^2\left(1+m_r\right)\right)\Omega^2+\omega_r^2=0 \tag{9}$$

and solutions, natural frequencies,

$$\Omega_{1,2}=\sqrt{\frac{1}{2}\left(1+\omega_r^2\left(1+m_r\right)\mp\sqrt{\left(1+\omega_r^2\left(1+m_r\right)\right)^2-4\omega_r^2}\right)} \tag{10}$$

3. OPTIMIZATION METHOD

Many methods of optimization have been developed to opportunely design this vibration control technique. In the classical textbook on mechanical vibrations, Den Hertzog (1940) pointed out a remarkable feature: for any fixed values of ω_r and m_r, curves $\left|X_1\right|/\left(F/k_1\right)$ intersect in two points P and Q (named "invariant points"), as shown in Fig. 2, independently of the value of ς. These points are situated close enough to the peaks of the frequency-amplitude curve. Den Hartog suggested to choose the parameter ω_r to equalize ordinates of P and Q. Secondly, ς was taken to satisfy the condition of "almost horizontal" tangents in the invariant points. Thus the values

$$\omega_{r\,opt}=\frac{1}{1+m_r}\ ,\quad \varsigma_{opt}=\sqrt{\frac{3m_r}{8\left(1+m_r\right)^2}} \tag{11}$$

where obtained.
This approach does not fit in the case when the main body is damped itself [10]. So, in such cases, one have to rely upon numerical methods for optimization. However, when a damped DVA with a small mass ratio is attached to lightly or moderately damped primary systems, the normalized amplitude curves roughly join at two points. When the primary system damping ratio approaches zero, these two points converge to the ''fixed-points P and Q, respectively. Therefore it is justified to assume that the ''fixed-point'' theory also approximately holds even for the case when a damped DVA is attached to a lightly or moderately damped primary system. Based on this assumption, we derive an approximate solution for the optimum tuning parameter for the generalized damped model, Fig.1.
The objective is to determine the value of parameters for absorber such that the amplitude of the primary system with respect to the frequency is minimal. In this paper the minimax optimization is used. The parameters that are chosen to optimize the response are ς_2 and ω_r.
The minimax optimization

$$\min_{\omega_r,\varsigma_2}\max_{\Omega}\left\{H(\Omega)\right\} \tag{12}$$

finds the values of design parameters which minimize the maximum main mass displacement over a range of frequency [9], [10].
A representative graph of the functions H(Ω) given by Eq.(5) is shown in Figure 3. With respect to Figure 3, the goal is to find these absorber parameters for which the peak amplitude $H(\Omega_A)$ and $H(\Omega_B)$ are equal and are as small as possible while the minimum between this peaks $H(\Omega_C)$ is as close to $H(\Omega_A)$ and $H(\Omega_B)$

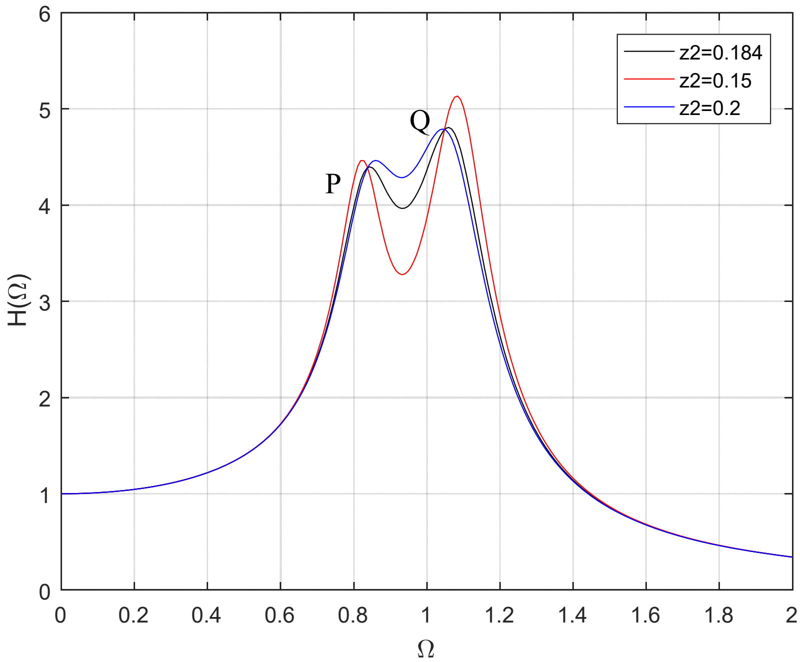


Figure 2: invariant points P and Q

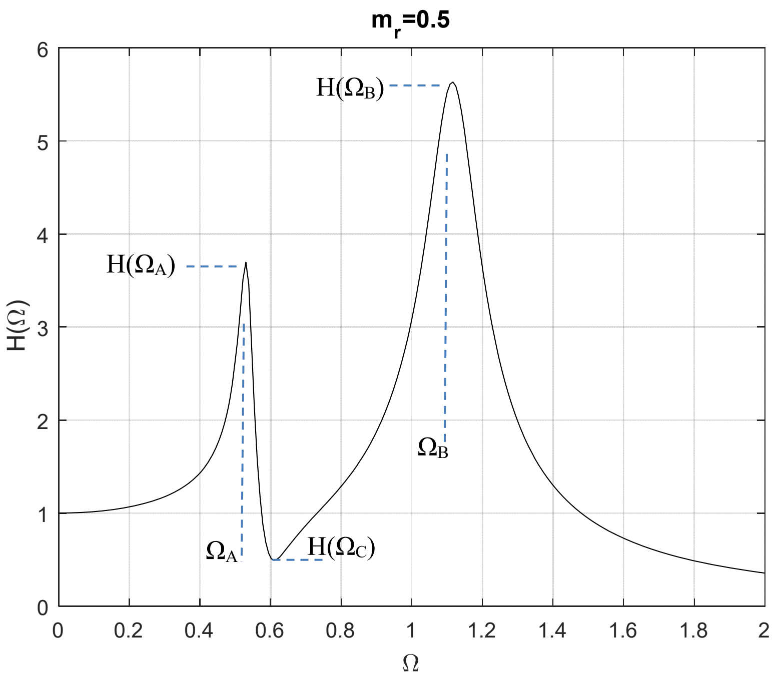


Figure 3: graph of the functions H(Ω)

In other words, we would like to find the system parameters that minimize each of the following three maximum values simultaneously:

$$\min_{\omega_r, \varsigma_2} \left\{ H\left(\Omega_A\right) \right\}, \ \min_{\omega_r, \varsigma_2} \left\{ H\left(\Omega_B\right) \right\}, \ \min_{\omega_r, \varsigma_2} \left\{ H\left(\Omega_C\right) \right\}, \tag{13}$$

subject to: $\begin{matrix} \omega_r > 0 \\ \varsigma_2 > 0 \end{matrix}$

The peaks occur approximately at $\Omega_A = \Omega_1$ and $\Omega_B = \Omega_2$, given by (10). The minimum between the two peaks is specified to occur at $\left(\Omega_A + \Omega_B\right)/2$ [9].

The solutions of optimization problems are obtained using two functions from the Optimization Toolbox: `fminsearch` and `fminimax [11].`

4. NUNERICAL RESULTS

The numerical example is taken from reference [6] and [9]. Consider a linear damped system with the following characteristics m_1 = 100 kg, ς_1 = 0.10, ω_1 = 100 rad/s. The design constraints are such that the m_r = 0.10. The optimal solution for Den Hartog, classic systems (i.e., ς_1 = 0) is $\omega_{r\,opt\,Hartog} = 0.909$ and $\varsigma_{2\,opt\,Hartog} = 0.185$.

A compareson of result obtained with the proposed numerical method and those from the other authors is reported in Table 1. In Figure 4, the values of optimal parameters and the magnitude of the amplitude–response function corresponding to these optimal values are presented, for classical and for this investigation model.

Table 1: A comparison of result given by different methods

Parameters	ς_1	m_r	$\omega_{r\,opt}$	$\varsigma_{2\,opt}$	H_{max}
Den Hartog [4]	0.00	0.10	0.909	0.185	4.59
This investigations	0.00	0.10	0.909	0.185	4.53
Randall [5]	0.10	0.10	0.861	0.204	2.63
Thompsonă [7]	0.10	0.10	0.862	0.192	2.62
Pennestri [8]	0.10	0.10	0.861	0.202	2.62
This investigations	0.10	0.10	0.862	0.199	2.54

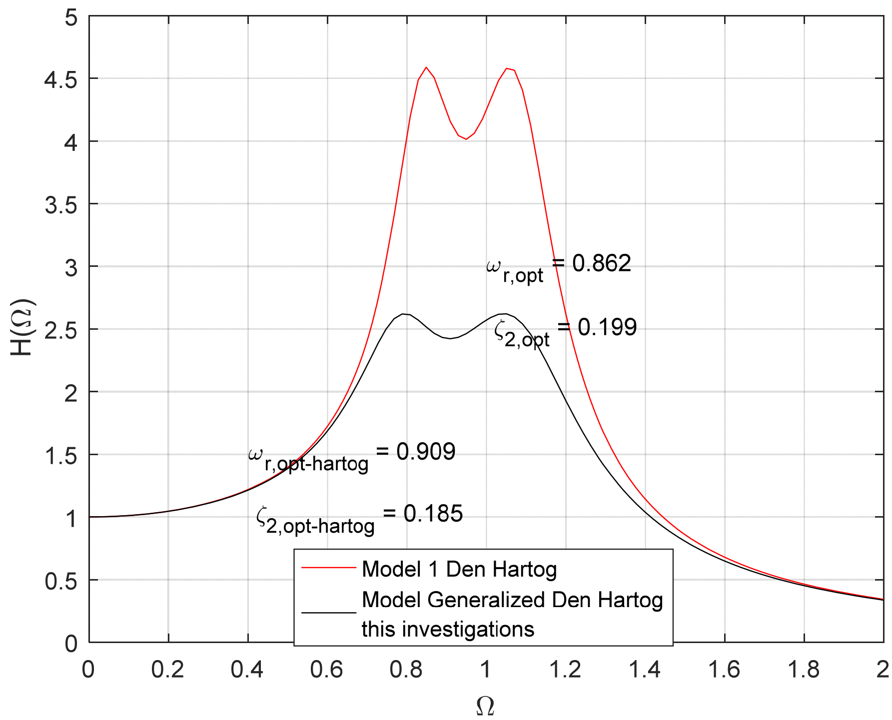


Figure 4: The amplitude response and optimum parameters

5. CONCLUSION

The design method presented allows the optimal choice of parameters of the damped dynamic vibration absorber. Observing Figure 4 one concludes that the case $\varsigma_1 = 0$ gives an upper bound for the primary mass vibration amplitude. For the case $\varsigma_1 = 0$ the results given by this method and those obtained with formula (11) are the same. The comparison with others methods show good correlation between the results obtained by different authors. Such results are almost coincident for those authors who preferred the algebraic approach to the use of non-linear programming codes. The graphs presented in this paper show the influence of the design aerometers on the performances of the damped dynamic vibration absorber.

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