BENDING TORQUES ON SOLID BEAM AXLE DURING CORNERING

Prof.Ph.D.Eng. Ion PREDA, Transilvania University of Braşov, Ph.D.Eng. Alexandru FLUERAŞ, INAR Braşov, Assoc.Prof.Ph.D.Eng. Ovidiu CÂMPIAN, Transilvania University of Braşov

ABSTRACT

The paper presents considerations regarding the bending torque produced on vehicle's rigid axle. The torque values are obtained only for the forces acting on the vertical-transversal plane when the vehicle is cornering or is skidding. The idea of this work is that the axle's most stressed section has no fixed position, because this is changing with respect to the value of lateral acceleration.

INTRODUCTION

Many titles of the automotive literature suggest that the most stressed section of the vehicle rigid axle is in the region where the suspension spring is fixed and recommend making verifications especially for this section. This paper intends to demonstrate that the assumption is not entirely true for the vehicle cornering or skidding.

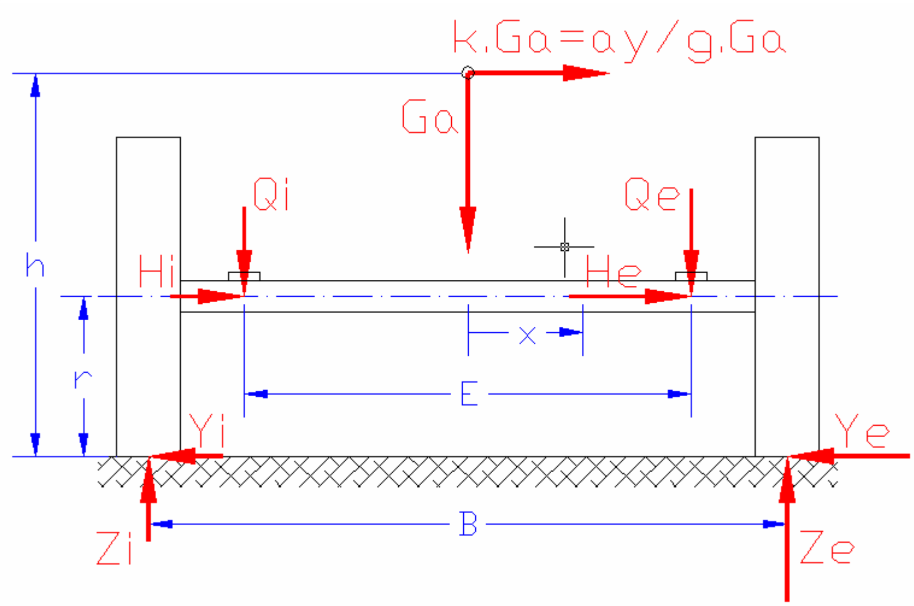

Fig. 1 Scheme of the forces acting on the axle

Figure 1 presents schematically the forces acting on the axle beam when the vehicle moves in a turn. The significance of the symbols used in figure is: G_a – axle load; *B* – axle tread (track width); *E* – distance between axle springs; *h* – height of the centre of gravity; r – wheel radius; Z – dynamic vertical force on the wheel; Y – lateral force on the wheel; Q – vertical load on the spring; x – the distance from a current section to the middle of the axle; a_y – transversal acceleration during cornering or side-skid; g – acceleration of gravity; $k = a_y/g$ – relative transversal acceleration. The indices i and e refer respectively to the interior and exterior side of the cornering curve.

For easy calculation, some simplifying assumptions are made:

- the vertical and lateral tyres' deformations can be neglected;
- the roll movement is reduced and is not considered;
- the axle's own weight is small in comparison with the axle load.

THE FORCES ACTING ON THE AXLE IN LATERAL-VERTICAL PLANE

The balance of the vertical forces and torques acting on the sprung mass permits to write the equations for the forces Q compressing the springs:

$$Q_i = G_a\left(\frac{1}{2} - \frac{h-r}{E}k\right) \qquad Q_e = G_a\left(\frac{1}{2} + \frac{h-r}{E}k\right). \tag{1}$$

The vertical reactions on the axle wheels by the road are obtained for the balance of the vertical forces and torques acting on the whole system (containing sprung mass and axle):

$$Z_i = G_a\left(\frac{1}{2} - \frac{h}{B}k\right) \qquad Z_e = G_a\left(\frac{1}{2} + \frac{h}{B}k\right). \tag{2}$$

The lateral reactions on the interior and exterior wheels are:

$$Y_i = \xi_i Z_i \qquad Y_e = \xi_e Z_e, \tag{3}$$

where ξ represents the "used friction". From the equilibrium of the lateral forces obtains

$$k = \frac{1}{2}\frac{\xi_i + \xi_e}{1 + (\xi_i - \xi_e)\dfrac{h}{B}} \tag{4}$$

and if considers lateral forces proportional to the vertical load for both wheels ($\xi_1 = \xi_1 = \xi$), results

$$k = \xi. \tag{5}$$

Considering $k = \xi$ and dividing equations (1), (2) and (3) by the axle load G_a obtain six coefficients

$$q_i = \frac{1}{2} - \frac{h-r}{E}k \qquad q_e = \frac{1}{2} + \frac{h-r}{E}k, \tag{6}$$

$$z_i = \frac{1}{2} - \frac{h}{B}k \qquad z_e = \frac{1}{2} + \frac{h}{B}k, \tag{7}$$

$$y_i = k\left(\frac{1}{2} - \frac{h}{B}k\right) \qquad y_e = k\left(\frac{1}{2} + \frac{h}{B}k\right), \tag{8}$$

representing respectively the load forces of springs, the vertical and lateral forces to the wheels, all produced by an axle load force equal with one unit (1 N).

Figure 2 presents the coefficients' dependence on the lateral relative acceleration k for an example truck (B = 2 m, E = 1.5 m, h = 1.2 m, r = 0.5 m).

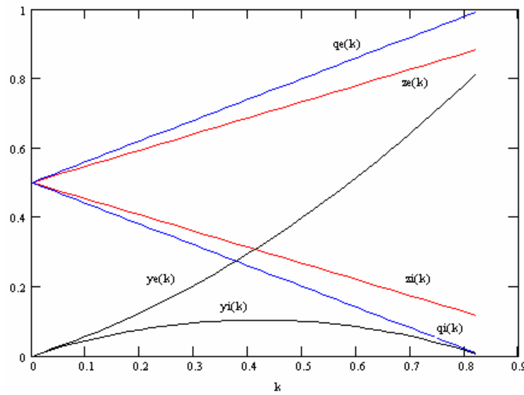

Fig. 2 Dependence on lateral relative acceleration *k* for spring forces and for wheels' vertical and lateral forces, all produced by a unit axle load

The maximal obtainable relative acceleration k is the minimum between the value that produces the vehicle rollover (z_i = 0) and the maximum friction coefficient μ_{max} on the wheel-road interface:

$$k_{\max} = \min\left(\frac{B}{2h}, \mu_{\max}\right). \qquad (9)$$

For many trucks and road trains, $B/(2h)$ is smaller than μ_{max} = 0.8…1 and in these circumstances the rollover happens before lateral skid.

THE AXLE'S BENDING TORQUE ACTING IN LATERAL-VERTICAL PLANE

The bending torque on the axle beam considers negative when no lateral acceleration exists. Similarly to the forces' coefficients, it is defined a bending coefficient, $m = M/G_a$, that expresses the bending torque produced by an axle load force equal to one unit. The bending coefficient has tree mathematical expressions depending on the zone where considers the calculation section (fig. 1):

$$m = \left[\begin{array}{ll} m_e = y_e r - z_e\left(\frac{B}{2} - x\right) = r\frac{h}{B}k^2 - \left(\frac{h-r}{2} - \frac{h}{B}x\right)k - \frac{B}{4} + \frac{x}{2} & if \quad \frac{E}{2} < x < \frac{B}{2} \\ m_a = m_e + q_e\left(\frac{E}{2} - x\right) = r\frac{h}{B}k^2 - \left(\frac{h-r}{E} - \frac{h}{B}\right)xk - \frac{B-E}{4} & if \; -\frac{E}{2} \le x \le \frac{E}{2} \\ m_i = m_a + q_i\left(-\frac{E}{2} - x\right) = r\frac{h}{B}k^2 + \left(\frac{h-r}{2} + \frac{h}{B}x\right)k - \frac{B}{4} - \frac{x}{2} & if \; -\frac{B}{2} < x < -\frac{E}{2} \end{array}\right. , \qquad (10)$$

In the figure 3, the thin curves on left side represent the bending coefficient m for different values of lateral relative acceleration and on the right side is an axonometric view for the function $m = f(k,x)$. For each calculation section ($-B/2 < x < B/2$) is a different k value that determine the maximum absolute bending torque |m| (figure 4 and thick curve in figure 3). To find these values $k_t(x)$ first writes the partial derivative $\partial m/\partial k$

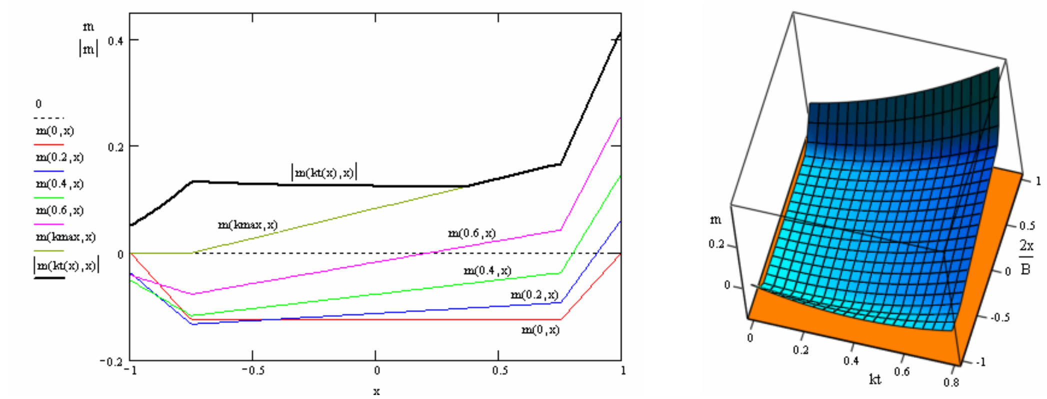

Fig. 3 Dependence on lateral relative acceleration *k* for the bending torque *m* produced by a unit axle load

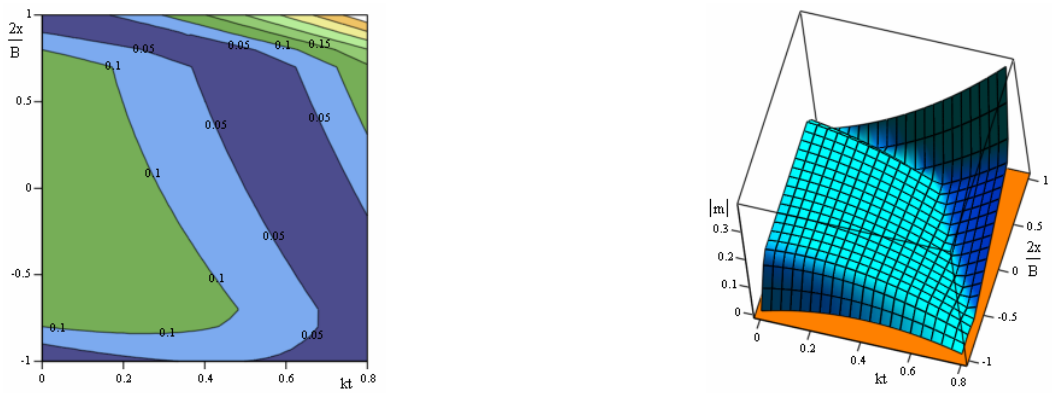

Fig. 4 Two representations of the absolute bending torque |*m*| produced by an unit axle load

$$\frac{\partial m}{\partial k}=\begin{cases}\dfrac{\partial m_e}{\partial k}=2r\dfrac{h}{B}k-\left(\dfrac{h-r}{2}-\dfrac{h}{B}x\right) & if \quad \dfrac{E}{2}<x<\dfrac{B}{2}\\[2ex] \dfrac{\partial m_a}{\partial k}=2r\dfrac{h}{B}k-\left(\dfrac{h-r}{E}-\dfrac{h}{B}\right)x & if \ -\dfrac{E}{2}\le x\le\dfrac{E}{2}\\[2ex] \dfrac{\partial m_i}{\partial k}=2r\dfrac{h}{B}k+\left(\dfrac{h-r}{2}+\dfrac{h}{B}x\right) & if \ -\dfrac{B}{2}<x<-\dfrac{E}{2}\end{cases}, \tag{11}$$

and then, equalling with zero, obtains (fig. 5)

$$k_{t1}=\begin{cases}k_e=\dfrac{(h-r)B-2hx}{4rh} & if \quad \dfrac{E}{2}<x<\dfrac{B}{2}\\[2ex] k_a=\dfrac{h(B-E)+rB}{2Erh}x & if \ -\dfrac{E}{2}\le x\le\dfrac{E}{2}\\[2ex] k_i=\dfrac{-(h-r)B-2hx}{4rh} & if \ -\dfrac{B}{2}<x<-\dfrac{E}{2}\end{cases}, \tag{12}$$

The values of k_t must be positive or zero, to maintain the initial assumptions (otherwise the exterior wheel becomes interior wheel).

$$k_{t2}=\max(k_{t1},0), \tag{13}$$

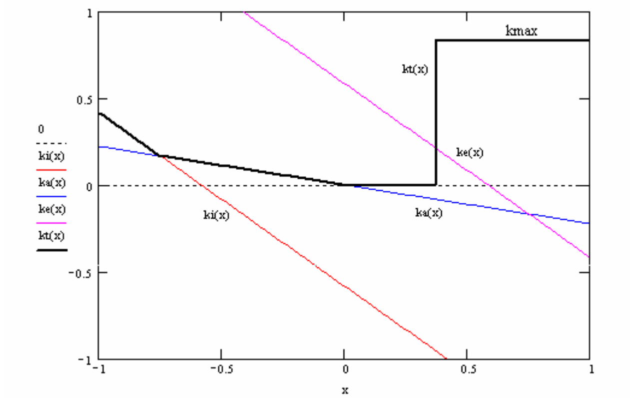

Fig. 5 The values of relative lateral acceleration that determine the maximum absolute bending torque on the axle beam

Also, because on the exterior side of the axle ($x > x_c$) the equation 12 produces no value k_t in the interval [0, k_{max}] (figure 5) and because the maximal absolute value of the bending coefficient is obtained for the maximal relative lateral acceleration (figure 3, left), results that $k_t = k_{max}$ for the interval $x_c < x < B/2$. So, the values of relative lateral acceleration which determine maximal bending torque (on a section placed at the distance x) are given by

$$k_t = \begin{bmatrix} k_{t2} & if \quad -\frac{B}{2} < x < x_c \\ k_{\max} & if \quad x_c < x < \frac{B}{2} \end{bmatrix} . \qquad (14)$$

and are represented with thick line in the figure 5.

The value of the distance x_c can be found if solves the equation

$$\left| m(k = k_{\max}) \right| = \left| m(k = k_{t2}) \right| . \qquad (15)$$

Now, introducing equations 14 in equation 10, the maximal absolute bending coefficient for each axle section ($-B/2 < x < B/2$) can be calculated and is obtained the thick curve on the left side of figure 3.

THE INFLUENCE OF THE CONSTRUCTIVE PARAMETERS

The presented algorithm permits to study the influence of the dimensional parameters over the axle's maximal bending torque. Figure 6 shows curves plotted considering different distances between springs.

As can be seen, at the axle's end situated to the exterior side of the vehicle trajectory (the right side of the figure), an identical value for the bending torques is obtained for any distance between springs. If the springs distance E diminishes from B to 0, the maximal torque decreases on the exterior side and increase on interior. The most reasonable value for E/B is 0.72 (in the case of the example vehicle) and determines constant torque for about 70% of axle length.

If the springs are mounted so E/B is under this optimal value, the bending torque increases rapidly to a maximal value, obtainable near the centre of the axle. The distance $E = 0$ is specific to the tractors with pivoting front axle (in this case, the height h represents the distance between the pivot axis and the ground).

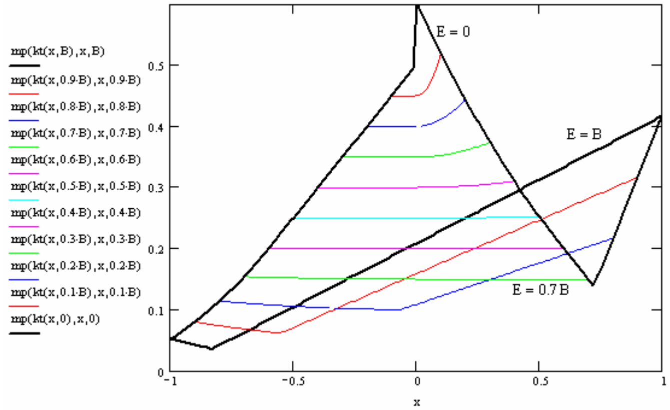

Fig. 6 Influence of springs distance over the axle's maximal bending torque

Figure 7 presents the effect of load height *h* over the axle's bending torque: low height increases the torque, mainly in the exterior part.

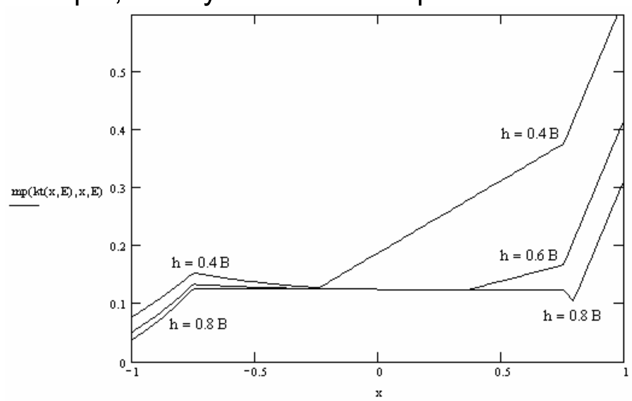

Fig. 7 Influence of the center of gravity height over the axle's maximal bending torque

Other influences could be taken into account: wheel radius, shape of the axle ($r = f(x)$), roll effect of the lumped mass, torque produced by longitudinal forces, supplementary stress generated by traction-compression or shear forces, different friction coefficient for interior and exterior wheels ($\xi_i \neq \xi_e$), etc.

CONCLUSIONS

The paper demonstrates that the maximal bending torque acting on vehicle rigid axles is a complex function with many parameters and the most stressed section change its position with respect to the lateral acceleration. A good design for axle and suspension needs a careful analysis of these factors, but a dimensional optimisation is possible.

REFERENCES

[1] Untaru,M. Poțincu,Gh. Stoicescu,A. Pereş,Gh. Tabacu,I. Dinamica autovehiculelor pe roți. Editura Didactică şi Pedagogică, Bucureşti, 1981.

[2] Untaru,M. Seitz,N. Frățilă N. Poțincu,Gh. Pereş,Gh. Tabacu,I. Macarie,T. Calculul şi construcția automobilelor. Editura Didactică şi Pedagogică, Bucureşti, 1982.