STARTING PERFORMANCES CALCULATION PARTICULARITIES OF CVT-EQUIPPED VEHICLES

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Abstract: *The paper presents some aspects concerning the launching performances calculation in the case of the motor vehicles equipped with friction continuously variable transmissions (CVTs). Having the possibility to realize an endless number of transmission ratios, the CVTs may confer the advantage to select the engine's optimum running speed independently of the vehicle driving conditions. This will confer some particularities to the vehicle starting process, as the finding of the best instantaneous ratio or as the continuous change of the vehicle's apparent mass. These differences with respect to gear-transmission vehicles and some driveability aspects will be presented. Also, some computation results obtained for the launching of a hypothetical vehicle will be shown.*

Key words: *CVT, starting performances, calculation, equation of motion*

1. INTRODUCTION

Continuously variable transmissions (abbreviated CVTs) are transmissions having as core element a variator, device able to change its transmission ratio in a continuous, un-stepped manner. Due to the variator, the CVTs are capable to realize an infinite number of transmission ratios, therefore offering the theoretic advantage of the possibility to decouple the engine's running regime from the vehicle's driving (or working) conditions [5].

The variators used today on mass-production vehicles are modifying the parameters (speed and torque) of the rotational output power with respect to the rotational input power using intermediate transformations in electric or hydraulic power or by generation of mechanical friction forces applied at variable radii.

Even normally the term CVTs includes transmissions of different types, in the common understanding the term "CVTs" designates mainly "friction continuously variable transmissions", i.e. continuously variable transmissions working on the principle of mechanical friction. Further in this article, the term "CVT" will be used with this last meaning, excluding the transmissions that contain electric or hydraulic devices for power adjustment.

In the case of the vehicles equipped with gear transmissions (conventional, stepped transmissions), the engine and the driving wheels are normally connected through mechanisms ensuring constant ratio for each engaged gear, which means the engine speed and the wheels peripheral speed have a fixed ratio. In the case of the CVT equipped vehicles, the engine and wheels speed ratio can be modified as preferred between two extreme (limiting) values. For this reason, the calculation of the dynamic performances in general (and the acceleration performances in particular) presents some differences and complications for the CVT vehicles if compared with stepped transmission vehicles.

The aim of this paper is to present some calculation particularities of CVT-vehicle launching process.

2. DYNAMIC AND MATHEMATIC MODEL OF THE VEHICLE

The starting (launching) ability of a vehicle is given by its quality to accelerate as fast as possible. The starting performances are indicated by the evolutions of acceleration, speed and travelled distance as functions of time. The starting process can be considered in different road or vehicle-loading conditions, beginning from a certain vehicle speed value, normally zero (start from rest).

To calculate the starting performances it is necessary first to write the vehicle's equation of motion (acceleration versus time) and then, integrating once this equation, to obtain the function of speed and, integrating again, the function of distance versus time.

In many studies the vehicle starting performances are indicated versus speed (acceleration, launching time and launching distance as functions of speed), which imply supplementary transformations of the equations.

To reach the vehicle's equation of motion it is necessary firstly to imagine a simplified model of the vehicle (the so called "dynamical model") and then to apply the laws of dynamics (Newton's laws) in order to obtain a set of equations (the so called "mathematical model") which will be solved. Finally, the results will be plotted and interpreted.

For the study, some simplifying hypotheses will be made. These will be presented gradually.

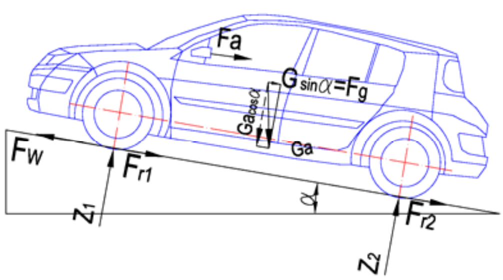

Fig. 1 – The external forces acting on a rolling vehicle

During moving, the vehicle interacts with the environment: the Earth gravity, the road and the atmosphere. In the **figure 1** it is presented a vehicle (planar model) isolated from the environment (all the interactions are replaced by forces). Assuming a straight line movement and no lateral interactions, the external forces acting against the vehicle are:

- the vehicle's gravitational force (the weight) G, acting in the centre of gravity;
- the air resistance force (aerodynamic drag) F_a (acting parallel to the road) and the aerodynamic lift force Z_a (acting perpendicular to the road and unrepresented on the figure); these components of the total aerodynamic force act in a point called "centre of pressure";
- the sum force of the vehicle's weight G and of the aerodynamic lift force Z_a induces the road's reaction forces Z_1 and Z_2, normal to the road surface, which, in turn, determines the rolling resistance forces F_{r1} and F_{r2} on the wheels of the front and rear axles, respectively;
- the driving force at the wheel F_w that propel the vehicle; this cumulative force represents the summation of all the traction forces generated by the driving wheels through the physical process of grip.

F_w is the **driving force** of the vehicle, while $F_r = F_{r1} + F_{r2}$, $F_g = G \sin\alpha$ and F_a represent the three resistance forces of any rolling vehicle (rolling, grade and aerodynamic resistances, respectively). The rolling and aerodynamic resistances depend on the vehicle speed, while the grade resistance is independent of the vehicle speed.

The **grade resistance** F_g is in fact the gravity's component parallel to the ground:

$$F_g = G \sin\alpha = m_t\, g \sin\alpha \qquad (1)$$

where: m_t is the vehicle mass, $G = m_t\, g$ – the vehicle weight and α – the longitudinal slope angle of the road.

The **rolling resistance** is mainly a consequence of the hysteresis (loss of energy, transformed partly in heat) manifested under deflection by the pair tire-ground and by the suspension dampers and bushing. Its value can be computed with the equation:

$$F_r = f\,(G \cos\alpha - Z_a) = f\,(m_t\, g \cos\alpha - Z_a) \tag{2}$$

where: $G \cos\alpha$ is the vehicle's weight component perpendicular to the ground surface (pressing the tires) and f is the dimensionless coefficient of the rolling resistance. Experiments and finite element method simulations of the tires dynamic behavior indicate an increase of the f value if the road is bumpier or the vehicle speed is higher [2], [3].

Assuming no wind, the **air resistance** (aerodynamic drag) can be computed with the formula:

$$F_a = (1/2)\,\rho\, c_d\, A\, v^2 \tag{3}$$

where ρ is the air density; c_d – the aerodynamic drag coefficient of the vehicle; A – the frontal area of the vehicle.

Applying the Newton's second law for the vehicle's planar model from **figure 1** results the instantaneous acceleration of the vehicle:

$$a = (F_w - F_r - F_g - F_a)/m_t \tag{4}$$

In that equation, the acceleration a is a function of time, vehicle speed v and traveled distance x. Because the speed and the distance can be obtained by integration,

$$v = \int_{t_0}^{t} a\, \mathrm{d}t \qquad\qquad x = \int_{t_0}^{t} v\, \mathrm{d}t \tag{5}$$

the vehicle movement (and the starting performance) is described by the system formed by the three equations (4) and (5).

In some particular cases, the forces F_w, F_r and F_a in the equation (4) can be considered as functions of only one parameter, the vehicle speed v, while the force F_g can be considered constant. In this situation it is preferred to express the vehicle's acceleration a and distance x as functions of the speed v. For that, the equations (5) will be written as differentials:

$$a = \mathrm{d}v/\mathrm{d}t \qquad\qquad v = \mathrm{d}x/\mathrm{d}t$$

$$\mathrm{d}t = \frac{1}{a}\,\mathrm{d}v \qquad\qquad \mathrm{d}x = v\,\mathrm{d}t = \frac{v}{a}\,\mathrm{d}v \tag{6}$$

that leads to the equations:

$$t = \int_{v_0}^{v} \frac{1}{a}\,\mathrm{d}v \qquad\qquad x = \int_{v_0}^{v} \frac{v}{a}\,\mathrm{d}v \tag{7}$$

Using the equation of motion (4) seems to be a very easy way to calculate the vehicle's starting performances. Unfortunately, because the (conventional) driving force at the wheel F_w is obtained from the contribution of the grip forces of all the vehicle's driving wheels, this force can be limited under two aspects:

- by the grip properties at the tire-road interfaces (at the driving wheels);
- by the possibilities of the engine, transmission and driveline.

In other words, the driving force at the wheels F_w is the minimum of the force F_{wgr} limited by the tire-road grip and the force F_{wdt} limited by the drivetrain characteristics:

$$F_w = \min(F_{wgr}, F_{wdt}) \tag{8}$$

If $F_{wdt} > F_{wgr}$ the driving wheels will spin-out evidently, the tires leaving black marks on tarmac or concrete roads.

The driving force limited by the grip F_{wgrj} generated by one particular wheel j depends on the instantaneous wheel load Z_j exerted by the road, on the grip coefficient μ_j and on the instantaneous wheel slip-coefficient s_j (through a function ξ_j). Thus, the driving force limited by the grip of the wheel j calculates as:

$$F_{wgrj} = Z_j \, \mu_j \, \xi_j(s_j) \tag{9}$$

In the case of the vehicle launching, the function $\xi(s) = F_{wgr} / F_{wgrmax}$ depends on the tire and road stiffness, takes values in the interval 0…1 and has a shape as the one presented in **figure 2**, while the slip-coefficient s_j of the wheel j is given by the equation:

$$s_j = 1 - \frac{v_j}{\omega_j \, r_j} \tag{10}$$

where v_j, ω_j and r_j are, respectively, the translational speed, the rotational speed and the dynamic radius of the wheel j. If the vehicle accelerates in straight line movement, the speeds v_j of the wheels centers and the vehicle speed v can be considered equal.

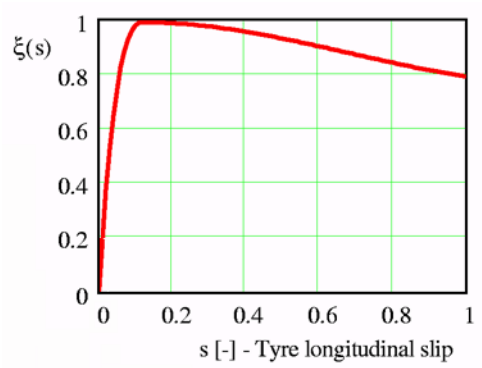

Fig. 2 – The relative grip coefficient versus slip

As can be figured-out, computing the force F_{wgr} as the sum of the particular forces F_{wgrj} isn't an easy task, mainly because the normal loads Z_j and the dynamic radii r_j of the tires vary due to the driving force. In terms of computation, this means also that, for each integration step, an algebraic system of equation must be solved in order to found the instantaneous values Z_j. Moreover, the dynamic load transfer between axles will generate pitch movements of the vehicle body, some energy being accumulated in the suspension springs or dissipated in the dampers and affecting so the precision of the calculation.

3. APPARENT MASS OF THE VEHICLE

During the vehicle launching, the mechanical energy produced by the engine is used not only to overcome the resistances and to accelerate in translation the vehicle mass m_t, but also to accelerate all its rotating parts. As a result, a smaller driving force F_w as expected will be measured at the wheels, i.e. the force F_{wdt} limited by the drivetrain characteristics. Due to that phenomenon, to calculate the force F_{wdt} it is needed an energetic approach [4]: it will be applied the principle of energy conservation, which states the derivative (the instantaneous change rate) of the vehicle's total mechanical energy is equal with the engine power P_e minus the total power losses:

$$\frac{dE_{tot}}{dt} = P_e - \sum P_{loss} \tag{11}$$

Assuming only negligible amounts of potential energy can be accumulated by the drivetrain shafts and suspension springs, the vehicle's mechanical energy consists on the kinetic energy E_k of the parts in translation and rotation and on the potential energy E_p, determined by the vehicle altitude:

$$E_{tot} = E_k + E_p = E_{kt} + E_{kr} + E_p \tag{12}$$

The kinetic energy E_k includes the one of the translational mass m_t and those of any rotating part j kinematically connected with the wheels:

$$E_k = \frac{m_t v^2}{2} + \sum_j \frac{I_j \omega_j^2}{2} = \frac{m_{ap} v^2}{2} \tag{13}$$

where ω_j and I_j are the speed and the moment of inertia of the rotating part j.

The vehicle behaves as having a bigger translational mass, the apparent mass m_{ap}:

$$m_{ap} = m_t + \sum_j I_j \left(\frac{\omega_j}{v}\right)^2 = m_t \left[1 + \sum_j \frac{I_j}{m_t}\left(\frac{\omega_j}{v}\right)^2\right] = m_t\, \delta \tag{14}$$

which accumulates in translation the same kinetic energy as the entire vehicle. In the last equation, the term

$$\delta = 1 + \sum_j \frac{I_j}{m_t}\left(\frac{\omega_j}{v}\right)^2 = 1 + \lambda_{wn} + \lambda_{wd} + \lambda_e + \lambda_v \tag{15}$$

is called coefficient of the rotating parts influence, is always bigger as 1 and will decrease if the vehicle mass m_t increases. The terms λ_{wn}, λ_{wd}, λ_e and λ_v represents the contributions of the main rotating parts to the overall influence coefficient, respectively the non-driving wheels, the driving wheels, the engine and the intermediate parts (belt, chain, rollers) of the variator.

The influence of the wheels can be computed with the next equations:

$$\lambda_{wn} = \frac{I_{wn}}{m_t}\left(\frac{\omega_{wn}}{v}\right)^2 \qquad \lambda_{wd} = \frac{I_{wd}}{m_t}\left(\frac{\omega_{wd}}{v}\right)^2 \tag{16}$$

where I_{wn} and I_{wd} are the equivalent moments of inertia of the non-driving and driving wheels and ω_{wn} and ω_{wd} are the corresponding rotational speeds.

From the equation (10) results:

$$\frac{\omega_w}{v} = \frac{r}{1-s} \tag{17}$$

which shows that the apparent mass of the vehicle will depend on the wheels slip s.

During vehicle launching, for the non-driving wheels the slip is always very small ($s_{wn} \approx 0$), and so the equations (16) become:

$$\lambda_{wn} = \frac{I_{wn} r^2}{m_t} \qquad \lambda_{wd} = \frac{I_{wd} r^2}{m_t}\left(\frac{1}{1-s_{wd}}\right)^2 \tag{18}$$

In the conditions of small slip, the influence of the wheels inertia is generally reduced over the apparent mass of the cars and small trucks: λ_{wn} = 0.01…0.02 and λ_{wd} = 0.02…0.03 (that means the “added” mass by the wheels rotation represents 3…5% of the true vehicle mass.

The influence of the engine inertia versus the apparent mass may be important due to the engine’s high rotation speed and can be computed with the next equation:

$$\lambda_e = \frac{I_e}{m_t}\left(\frac{\omega_e}{v}\right)^2 = \frac{I_e}{m_t}\left(\frac{i_t\,\omega_{wd}}{v}\right)^2 = \frac{I_e\,r^2}{m_t}i_t^2\left(\frac{1}{1-s_{wd}}\right)^2 \quad (19)$$

where i_t is the total (overall) instantaneous transmission ratio (between engine and driving wheels) and I_e is the equivalent moment of inertia of the parts rotating solidary with the engine. Due to the proportionality with the square of the overall transmission ratio, the term λ_e can be many times bigger as the term λ_w that corresponds to all the wheels.

Most of the CVT types found today on the vehicles market use variators with intermediate parts connecting by friction the input and output shafts: push or pull belts disposed between pulleys or rollers disposed between toroidal disks. The instantaneous speed ω_v of these elements (belt, chain or rollers) depends on the engine speed and on the current variator ratio i_v:

$$\omega_v = C\frac{\omega_e}{1+i_v} \quad (20)$$

where C is a constant depending on the variator design. Using equations (19) and (20) and remembering that the variator ratio i_v is included in the overall transmission ratio i_t ($i_t = i_v\, i_a\, i_0$, with the adaptation ratio i_a and the final drive ratio i_0 having constant values), the inertia influence term λ_v of the variator intermediate parts can be calculated easily.

The left side of the **figure 3** presents, as an example, a possible law of the transmission ratio ($i_{tr} = i_v\, i_a$) variation versus vehicle speed (in km/h), alongside with the plot of the rotating-parts influence-coefficient δ. In the considered case, δ maximum reach the value 1.34 (at low speeds), while the minimum value is 1.03.

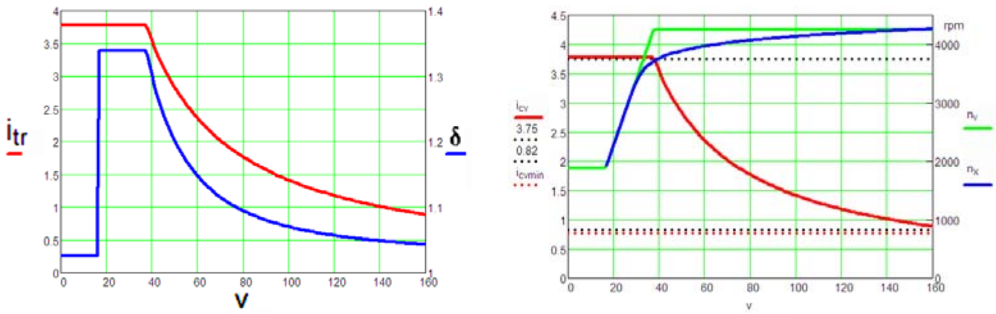

Fig. 3 – Example of the variation versus vehicle speed of the transmission ratio (red plots), of the rotating-parts influence-coefficient for loaded vehicle (left side, blue plot) and of the engine speed (right side, green and blue plots)

Comparing gear-transmission vehicles with CVT vehicles it observes some differences:

- considering small wheel slip, the term λ_e remains constant when a gear is engaged (and also the δ coefficient) on a conventional vehicle, while on the CVT vehicles this term can continuously change its value;
- for the same transmission ratio, the moment of inertia of the engine connected parts I_e is normally bigger at the CVT vehicles (due to the variator rotors);
- the term λ_v will not exist for gear-transmission vehicles.

4. EQUATION OF VEHICLE MOTION

In order to deduce the equation of vehicle's motion it is necessary to come back to the energy conservation principle applied to the vehicle, i.e. to the equation (11).

The potential energy of the vehicle is:

$$E_p = m_t\, g\, h \quad (21)$$

where $g = 9.81$ m/s^2 represent the gravitational acceleration and h the altitude.

Introducing equations (12)…(14) and (21) in the equation (11) obtains:

$$\frac{dE_{tot}}{dt} = m_{ap}\, v\, \frac{dv}{dt} + m_t\, g\, \frac{dh}{dt} =$$
$$= m_{ap}\, v\, a + m_t\, g\, v \sin\alpha = P_e - \sum P_{loss} \qquad (22)$$

where $\frac{dh}{dt} \approx \frac{dx}{dt} \sin\alpha = v \sin\alpha$ and a is the vehicle's instantaneous acceleration.

From the equation (22) results:

$$m_{ap}\, a = P_e/v - (\sum P_{loss})/v - m_t\, g \sin\alpha =$$
$$= P_e/v - (P_f + P_r + P_a)/v - m_t\, g \sin\alpha =$$
$$= (P_e - P_f)/v - P_r/v - P_a/v - m_t\, g \sin\alpha =$$
$$= P_{ef}/v - P_r/v - P_a/v - m_t\, g \sin\alpha =$$
$$= F_{ef} - F_r - F_a - F_g = F_{ef} - (F_r + F_g + F_a) \qquad (23)$$

that permits to calculate the vehicle's acceleration, in the well known form indicated in the literature (for example [1], [2]).

A discussion it is necessary in this point of the presentation. Even the equations (4) and (23) looks similar, there are some differences:

- in the equation (4) the mass is the true mass m_t of the vehicle, while in the equation (23) appears the apparent mass $m_{ap} = m_t\, \delta$;
- in the equation (4) appears the true traction force F_w generated by the driving wheels, while in the equation (23) the force F_{ef} is a conventional force, bigger as F_w, obtained when divides to the vehicle speed the difference between the power of the engine and the power lost by friction in the driveline.

It is a common approach to consider the driveline friction through the help of the overall driveline efficiency η in steady-state condition: $P_e - P_f \approx P_{ess}\, \eta$. This can be permitted because the engine power in transient condition P_e is smaller as the engine power in steady-state condition P_{ess} (due to the delayed response of the engine), but also the driveline friction losses in transient condition P_f are smaller as the ones in steady-state condition. Thus, the conventional force at the wheel, in equation (23), is $F_{ef} \approx P_{ess}\, \eta/v$ (assuming small driving wheels slip).

Another important thing to remember is that the equation (23) is true and can be used as the unique equation of vehicle movement only if no or very small slippage appears in the driveline couplings. If this hypothesis isn't fulfilled, aberrant results may be obtained [1] (for example, the vehicle acceleration at zero speed is always zero!!!).

5. TRANSMISSION RATIO VARIATION LAWS

During vehicle launching, opposed to the gear-transmission vehicle, a CVT vehicle disposes of an infinite number of ratios. One problem was: haw to choose the best ratio at a certain traveling speed in order to obtain the biggest acceleration value? Mathematically, the answer consists in founding the value of the transmission ratio that corresponds to a null value of the acceleration derivative da/di_t – with the function $a(i_t)$ from the equation (23).

But, for light vehicles (car and small trucks) disposing of conventional CVTs (with only one power flow), at a certain engine power, the biggest ratio (in the domain permitted by the CVT) will generate the maximal acceleration.

A second question was: in which proportion must be used the engine power for the acceleration of the vehicle in translation and for the acceleration of the rotating parts? The answer is: the engine power must be used first to accelerate the engine itself as quick as possible, in order to dispose rapidly of its maximum power, to accelerate then the vehicle. This kind of actuation was implemented at the early CVT types and is presented in **figure 3** with green color.

Unfortunately, this control law was not agreed or accepted by the drivers because the vehicle seems to "hesitate" at a sudden kick-down of the accelerator pedal, producing discomfort or even panic. This CVT comportment, considered by many drivers as a lack in the driveability qualities, mainly at the overtaking maneuvers, was suggestive named „rubber band effect", „slingshot effect", „motorboat feel" or „slipping clutch syndrome" [5].

To realize an "acceptable" acceleration behavior, the engineers have changed the way the CVT vehicle accelerates, ensuring a continuous increase of the engine speed during vehicle launching, as can be observed the blue plot in **figure 3**, the right side. But, this new manner of CVT control generates poorer starting performances. New CVT types, having fuzzy-logic based actuation controls for example, are able to "learn" the driving style, "studying" the driver's commands and choosing the best control law.

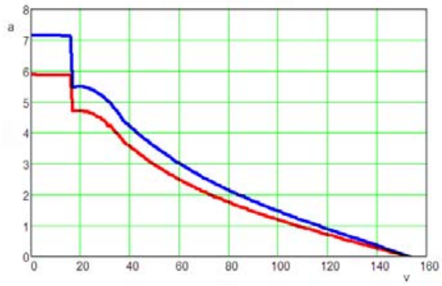

Fig. 4 – Starting characteristics for an example CVT vehicle (acceleration versus speed) red plot – loaded vehicle; blue plot – unloaded vehicle

In the **figure 4** are presented starting characteristics of a hypothetical vehicle equipped with a single power-flow CVT. The ratio changing strategy corresponds to the green plot in the **figure 3**, right side: the starting clutch it is engaged gradually up to 18 km/h, maintaining the engine speed at 1800 rpm; then the CVT's maximal ratio it is maintained until the engine reaches its rated speed (4300 rpm and 37 km/h); finally, the maximal engine power is maintained and the CVT's ratio is reduced continuously.

3. CONCLUSIONS

The paper presented specific aspects concerning the computation of the launching performances of the motor vehicles equipped with friction continuously variable transmissions (CVTs). General aspects of the vehicle starting process modeling and some particularities appearing in the case of CVT implementation were presented, as the continuous change of the vehicle's apparent mass and the finding of the best instantaneous ratio. These differences with respect to gear-transmission vehicles and some driveability aspects were also presented. Finally, few computation results obtained for the launching of a hypothetical vehicle were shown.

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