INTERNATIONAL SCIENTIFIC CONFERENCE

CIBv 2010

12 – 13 November 2010, Braşov

ON SIMULATION OF CONTINUOUS WELDED RAIL BUCKLING

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Abstract: A 3D simplified model developed for the buckling analysis of continuous welded rail (CWR) subjected to thermal and vehicles loads in deterministic and probabilistic approach is presented in this paper. The model is based on a nonlinear analysis in total lagrangian formulation. The structure was modelled by beam elements and lateral, longitudinal and torsional spring elements with non-linear characteristics. The source of nonlinearity is due to the geometric nonlinearity of the rail high axial forces and also to the nonlinearity of material type for the resistances of the ballast and of the fasteners, respectively. A displacement control algorithm is used beyond the critical point and permits a more realistic computation of the structural safety. The great variability of the main parameters which govern the stability of the CWR track is introduced in the computational model by the statistical distributions. The algorithm is based on the evaluation of convolution integrals in a discrete approach.

Key words: continuous welded rail (CWR), track stability analysis, probabilistic computational model, non-linear stability analysis, structural safety

1. INTRODUCTION

It is impossible to talk about high-speed railway at the present day without taking into account the necessity of joints elimination. Impacts occur when a railway wheel encounters discontinuities generated by gaps of the rail joints. These large impact forces may cause damages to wheel, track and vehicle. A modern solution to solve this problem is to, i.e. to make CWR track, using the aluminothermic welding method or flash butt welding method [3, 6].

The welding of rail should be realized only into prescribed temperature range for decrease the risk of rail track buckling in a hot summer and the risk of rail breaking in cold winter [5, 8]. Nevertheless during hot summers several hundreds of track buckling occur world wide and they cause major damage. The number of CWR buckling and the costs of damages and repairs are increasing each year [5].

Because the physical model experiments in CWR track buckling are very expensive, the numerical simulations are more convenient [4, 5].

In 1992 ÷ 1999 period the International Union of Railways (UIC) commissioned a research program from European Rail Research Institute (ERRI) about improving the knowledge of continuous welded rail (CWR) track, including switches [8]. This research was necessary for revision and update of Leaflet UIC 720 which regulate the problems concerning the laying and maintenance of CWR track, which was from January 1986. In the new Leaflet UIC 720 [9], which was from March 2005, there were introduced concepts and criteria for the CWR buckling safety assessment and were shown case studies which appeal to the two CWR stability analysis softwares, one developed at TU Delft (Holland) for ERRI – software called initially CWERRI, and nowadays LONGSTAB – and the other developed at Foster&Miller company for Federal Rail Administration of United States of America (FRA) – software called CWR-BUCKLE [12]. In this context, at the Civil Engineering Faculty from the Transilvania University of Braşov, Romania, was developed a software for track stability simulation using a three dimensional simplified non-linear discrete model for CWR buckling analysis, in the presence of thermal and vehicle loads, model called **SCFJ** (**S**tabilitatea **C**ăii **F**ără **J**oante = Stability of CWR). An extended presentation of SCFJ model can be found in [4].

The validity of the SCFJ model is verified through a series of comparative analyses with other author's results [5].

The probabilistic computational algorithm of SCFJ program is based on the evaluation of convolution integrals [10] in a discrete approach [11], using the histograms of the main parameters which characterize the stability of the CWR track [13], in contrast with CWR-BUCKLE program, which use probability density functions.

2. THE LOADS AND PARAMETERS OF TRACK

The physical and geometrical parameters of the CWR track introduced in SCFJ model are [4]:

1. *The longitudinal resistance of the ballast* presents a linear or bi-linear displacement-force curve and it is introduced into the model by spring elements which are describing the material nonlinear behaviour of the ballast and of the fastenings for longitudinal displacements.
2. *The transversal resistance of the ballast* presents a linear, bi-linear or tri-linear displacement-force curve and it is modelled by spring elements which are describing the material nonlinear behaviour of the ballast and of the fastenings for transversal displacements. In all cases the elasto-plastic model includes softening. This kind of ballast behaviour has been measured for consolidated ballast.
3. *The torsional resistance of the fastenings* is introduced into the model by linear or tri-linear torsional springs.

In the case of loaded rail, the behaviour of the three above mentioned resistances can be corrected by taking into account the vertical force acting on each sleeper.

4. *The vertical stiffness of the track* is modelled by spring elements with linear behaviour.
5. *The rail* is modelled by beam elements having the following *geometrical and physical characteristics*:
    - area A of the cross section and second order moment about the vertical and horizontal axes I_z and I_y, respectively.
    - Young modulus E and thermal expansion coefficient α of the material.
6. *The misalignments of the rail* can be described by two types of curves: a complete or a half cosine wave having the total length λ and the amplitude δ (Fig. 1).
7. *The load* is introduced into the model by two pairs of vertical forces corresponding to a vehicle with two bogies.

The model allows parameterizations of the centre spacing between the bogies, of the spacing between the axles in a bogie and of the axle loads.

8. *The length of the model* is an input of the program. At the end of the model special infinite boundary elements are introduced - equivalent with the theoretical infinite rail [5]. These elements are reducing the length of the model and hence the computational effort is smaller. Further reduction can result by using symmetric half structure.

3. THE NUMERICAL ALGORITHM

The numerical algorithm has two phases, since in a simplified manner, the horizontal and vertical behaviour of the CWR track are considered independently [4].

3.1 The numerical algorithm for vertical loadings

This computational model is linear elastic consisting of a beam on elastic springs. The nodes of the structure are considered at the sleepers. Each node has two degrees of freedom: the vertical translation w and the rotation θ_y. The system of equations of equilibrium is:

$$\mathbf{Ka} = \mathbf{F} \tag{1}$$

where:

K is the stiffness matrix of the structure and results by assembling the stiffness matrices **k** of the beams and the vertical stiffness of the sleepers;

a is the displacement vector of the nodes of the structure;

F is the vector of forces at the nodes of the structure, which (in this case) results by assembling the vectors $\mathbf{f_0}$ of the forces on the beams.

The stiffness matrix $\mathbf{k}_{(4x4)}$ of a beam is given by:

$$\mathbf{k} = \mathbf{B}^T \mathbf{k}^d \mathbf{B} \tag{2}$$

where $\mathbf{B}_{(2x4)}$ is a transformation matrix, which links the vector of displacements of the beam and the reduced vector of displacements of the beam, and $\mathbf{k}^d_{(2x2)}$ is the reduced stiffness matrix of the beam reported only to the reduced set of nodal displacements and forces.

The stiffness matrices and the load vectors of the beams are assembled by the relations (3):

$$\mathbf{K}_{ind,ind} = \mathbf{K}_{ind,ind} + \mathbf{k}, \quad \mathbf{F}_{ind} = \mathbf{F}_{ind} + \mathbf{f}_0 \tag{3}$$

and the vertical stiffness of the sleepers is assembled with the help of the equation (4):

$$\mathbf{K}_{jnd,jnd} = \mathbf{K}_{jnd,jnd} + R_z L \tag{4}$$

The constraints of the structure are introduced by setting to zero the displacements of the supports. The free displacements of the nodes result by solving the system of linear equations:

$$\mathbf{a}_{id} = (\mathbf{K}_{id,id})^{-1} \mathbf{F}_{id} \tag{5}$$

Using the vertical displacements *w*, the vertical force on each sleeper can be computed and then the transversal, longitudinal and torsional resistances are corrected taking into account the forces *Q* on each sleeper.

3.2 The numerical algorithm in the horizontal plane

This computational model is a straight or curved beam on elastic supports with misalignments (fig. 1). The nodes of the structure are considered at the sleepers. At each node are introduced longitudinal, transversal and rotational spring elements which are modeling the sleepers. The infinite boundary elements at the ends of the model have equivalent characteristics (Young modulus and thermal expansion coefficient) in order to replace the theoretical infinite rail [5]. The loading of the model is an increase of the temperature in the rail. The characteristics of the beams and of the springs correspond to the two rails of the track panel.

A node has three degrees of freedom: two linear displacements in the horizontal plane, u and v and the rotation θ_z around the vertical axis. In the analysis of the structure the goal is to obtain the displacement - variation of temperature curve. The problem is solved through an updated Lagrangian formulation by a displacement control based incremental process. The behaviour of the system is determined as a sequence of increments of the state parameters (forces and displacements). In the current increment j, characterized by a small control displacement δv_{cj}, the nonlinear behaviour of the system can be approximated by linear relations:

$$\mathbf{a}_{j+1} = \mathbf{a}_j + \delta\mathbf{a}_j, \quad \delta\mathbf{F}_j = \mathbf{K}_j \delta\mathbf{a}_j \tag{6}$$

In the above equation $\mathbf{a}_j$ is the displacement vector in the current configuration, $\delta\mathbf{a}_j$ is the increment of the displacements, $\delta\mathbf{F}_j$ is the incremental load vector and $\mathbf{K}_j$ is the incremental (tangent) stiffness matrix of the structure.

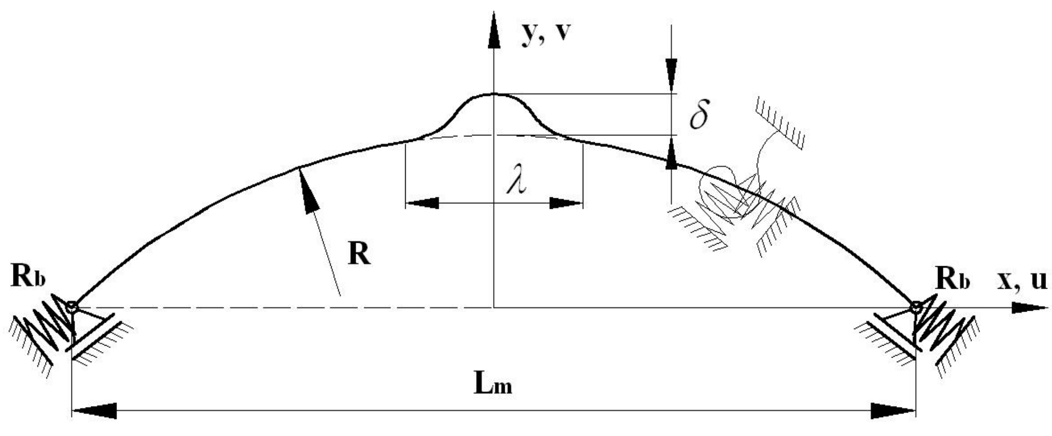

Fig. 1 The model for horizontal displacements

From equations (6), results the incremental scheme:

$$\delta\mathbf{a}_j = (\mathbf{K}_j)^{-1} \delta\mathbf{F}_j, \quad \mathbf{a}_{j+1} = \mathbf{a}_j + \delta\mathbf{a}_j \tag{7}$$

For better performances of the incremental algorithm, Heun's midpoint rule has been adopted [2]. The algorithm uses a *displacement control* technique. The vector $\delta\mathbf{F}_j$ is not explicitly expressed and the displacement increment $\delta\mathbf{a}_j$ results from an unknown variation of the temperature, which produces a known increase of the control displacement.

The tangent stiffness matrix $\mathbf{K}_j$ in the j-th increment depends on the parameters of the system in the current step, which results by assembling the stiffness matrices $\mathbf{k}_{t(6x6)}$ of the beam elements and of the springs which model the sleepers is:

$$\mathbf{k}_t = EA/L \cdot \mathbf{r}^T\mathbf{r} + \mathbf{B}^T(\mathbf{k}^d + \mathbf{k}_G^d)\mathbf{B} + N_j/L \cdot \mathbf{z}^T\mathbf{z} \tag{8}$$

Matrices $\mathbf{k}^d$ and $\mathbf{k}_G^d$ are the material and geometric stiffness matrices, respectively. They are expressed with the reduced set of displacements which produce deformations and they are not containing the rigid body displacements of the beam. This reduced form of the stiffness matrices needs less computational effort and speeds up significantly the computation. Equation (8) introduces the non-linear effect of the axial force N_j. The complete tangent stiffness matrix in the updated lagrangian formulation has two more terms corresponding to the variation of the length of the beam in bending and to the effect of the shear force [1]. Since in the current cases the structure is divided in a sufficient number of beams, the errors are very small, when neglecting these two terms. In a comparative study using the complete tangent stiffness matrix and equation (8) the differences between the resulting limit temperatures were only at the fifth digit [1].

4. ABOUT THE SAFETY CRITERION AGAINST THE BUCKLING OF CWR TRACK

For every set of date which contain the physical and geometrical parameters of the CWR track introduced in the SCFJ model it will result (Fig. 2) a buckling response curve of track [8].

This curve is characterized by two points [5]:

$T_{b,max}$ - the maximum increase of temperature for which the buckling certainly starts, and

$T_{b,min}$ - the minimum increase of temperature which occurs in the post-buckling domain.

For a railway track safety conditions, Tallow is the maximum allowable temperature above the neutral temperature of the rail that is considered safe as far as track buckling is concerned.

The safety concepts and criteria proposed by researchers are based on one of the following situations [8]:

a) evaluation of $T_{b,max}$;
b) evaluating of $T_{b,min}$;
c) simultaneous quantification of $T_{b,max}$ and $T_{b,min}$.

Since the first situation leads to imprudent results in terms of safety, and the second method leads to too conservative results, it appears that the third method is the most rational, and therefore it is based on safety criteria developed by UIC.

For this reason the criterion of safety implemented in SCFJ program is provided by the new Leaflet UIC 720 through the ERRI D 202 Specialists' Committee [9]:

$$T_R - T_N \leq Tallow \tag{9}$$

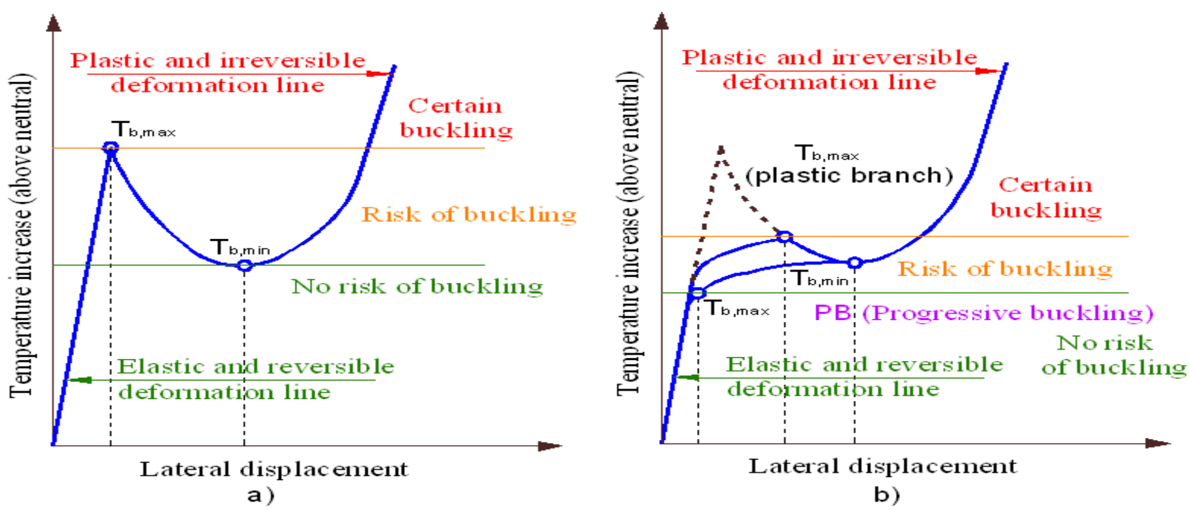

Fig. 2 The curve of buckling response for a track: a) with good ballast (explosively buckling), b) with poor ballast or in a sharp curve (progressive buckling)

where T_R is temperature of rail at a specific moment, T_N is neutral temperature of rail and the T_{allow} is computed as follows:

If $\Delta T > 20$ °C:

$$T_{allow} = T_{b,min} + 0{,}25\Delta T \tag{10}$$

If 5 °C $< \Delta T < 20$ °C:

$$T_{allow} = T_{b,min} \tag{11}$$

If 0 °C $< \Delta T < 5$ °C:

$$T_{allow} = T_{b,min} - 5\ °C \tag{12}$$

If $\Delta T < 0$ °C: it is not allowable in main lines,

where:

$$\Delta T = T_{b,max} - T_{b,min} \tag{13}$$

5. THE EVALUATION OF BUCKLING PROBABILITY

The essential aim of structural design is to ensure that in all sections the "minimum" sectional strengths are at least equal to the "maximum" structural effects of the loads [10].

The buckling "load" can be expressed in terms of the rail temperature increase over the neutral, and the "strength" is expressed in terms of the allowable temperature increase, ΔT_{all}.

The buckling evaluation can be approached in a deterministic or in a probabilistic manner [4].

In a deterministic approach the above criterion for buckling safety is satisfied or not. Hence, the track will either buckle out or not, and the "probability" of buckling is either 1 or 0.

In a probabilistic approach, the probability of load exceeding the strength is the "failure probability of the structure" and it can be evaluated on the basis of the so-called "convolution" integral [11] given below.

If $f_R(x)$ and $F_R(x)$ are the probability density function and the cumulative density function of the random sectional resistance, R, and, similarly, $f_S(x)$ and $F_S(x)$ are the probability density function and the cumulative distribution function of the random sectional load effect, S, the probability of failure is [10]:

$$P_f = P(S > R) = \int_0^{+\infty} [1 - F_S(x)] \cdot f_R(x) dx \tag{14}$$

or

$$P_f = P(R \le S) = \int_0^{+\infty} F_R(x) \cdot f_S(x) dx \tag{15}$$

The integrals of equations (14) and (15) are called integrals of convolutions and they are solved by using the following relations for discrete distributions [11]:

$$P_f = \sum_{x_i = 0}^{x_n} \frac{1}{2} [F_R(x_i) \cdot f_S(x_i) + F_R(x_{i+1}) \cdot f_S(x_{i+1})](x_{i+1} - x_i) \tag{16}$$

or

$$P_f = \sum_{x_i = 0}^{x_n} \frac{1}{2} \{[1 - F_S(x_i)] \cdot f_R(x_i) + [1 - F_S(x_{i+1})] \cdot f_R(x_{i+1})\}(x_{i+1} - x_i) \tag{17}$$

The input data of the SCFJ program are the histograms of key parameters of CWR track stability and the output data are the histogram of allowable temperature T_{allow} and the histogram of difference between temperature of the rail at a specific moment and the neutral temperature of the rail ($T_R - T_N$). Then these results are introduced in the expressions of convolution integral to obtain the buckling probability versus rail temperature [4].

6. CONCLUSIONS

In the free market conditions, but also because of the global heating phenomenon, which generates many problems on the operation and maintenance of the CWR track in a safe and profitable manner, it is very useful to develop one numerical model for the CWR track buckling simulation on purpose to the evaluations of the safety degree against of CWR track buckling [1, 4].

SCFJ model is different from the others, because it includes a correction of the torsional resistance of the fastenings, which takes into account the vertical loads of the vehicle. The computations are simplified by decoupling the horizontal and the vertical behaviour of the CWR track, and by including the possibility of use of a symmetric half of the structure [1]. The model was validated by comparative analyses with similar models and with the results found in the literature. The agreement with these results is very good. In addition, the SCFJ program has shorter running times than similar programs [4].

CWR buckling under vehicle and thermal loads can be predicted using deterministic and probabilistic approaches. The deterministic approach will decide whether the CWR track with given parameters will buckle out or not. If it does not buckle, the "safety assurance" in terms of a buckling margin of safety can also be evaluated. The probabilistic approach introduces the statistical variability in the input parameters. For given statistical distributions of these parameters, the probabilistic approach gives the probability of buckling as a function of anticipated maximum rail temperature.

The probabilistic approach developed for CWR track buckling evaluations provides more flexibility in the maintenance of CWR tracks. Tradeoffs are possible between ballast lateral resistance, CWR neutral temperature and other parameters for more cost-effective maintenance for the same level of buckling risk.

A computational procedure for the determination of buckling probabilities has been formalized into a comprehensive buckling safety analysis program called SCFJ. The program incorporates both the deterministic and probabilistic analysis modules.

The SCFJ probabilistic method presented here can provide a rational basis for speed reductions for buckling risk mitigation when the rail temperature is above a "critical temperature". Allowable speed levels can also be determined using the method.

The main application of this probabilistic computational model is to assessment of temporary train speed limits using the simulation of the CWR track buckling in a probabilistic approach. The temporary train speed limits disturb normal passenger and freight traffic set in train schedule and determine losses due to the decrease of circulation capacity on the railway. An estimation of the allowable temperature limit under which it is possible to circulate in safety conditions with a known speed limit on the railway sector studied is an imperious necessity. In view of the great variability of main parameters which govern the stability of the CWR track it must use the algorithm for probabilistic assessment of the CWR track buckling developed in [3], [4] and [12] to estimate these temporary train speed limits.

The SCFJ model opens perspectives for future researches, about:

- the development of the SCFJ model to allow quantification of the influence of the different neutral temperatures of rails and the influence of different wear of the rails;
- the possibility to take into consideration different fixing temperatures of the rails along the track;
- the possibility to analyze the CWR tracks buckling in dynamic conditions.
- the modeling of the complete rail temperature variation cycles and the modeling of the boundary conditions with expansion joints, which allow to take into consideration the effect of breathing zones of the CWR track;
- the use of discrete element method for model the interaction between sleepers and ballast.

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Received September 20, 2010